Optimal PMU Placement for State Estimation with Grid Parameter Uncertainty

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***Abstract*—We address the problem of optimal experimental design (OED) for Bayesian nonlinear inverse problems governed by power flow models. The inverse problem consists of inferring the values of the unknown (or uncertain) susceptance parameters and voltage angles parameter that characterize the power flow from available observations (i.e., power flow and potential Phasor Measurement Units (PMU) measurements). To quantify the uncertainties in the reconstructions, we invoke a Bayesian framework. Under the assumption of Gaussian noise and prior probability densities (for both susceptance and voltage angles) and after linearizing the parameter-to-observable map (describing the power flow), the posterior density becomes Gaussian and can therefore be characterized by its mean and covariance. The mean is given by the solution of a nonlinear least squares optimization problem, which is solved via an inexact Newton method. The posterior covariance matrix is given (in this case) by the inverse of the Hessian of the least squares cost objective function. Following an A-optimal experimental design strategy, we then seek to minimize the posterior variance of the parameter estimates, which is given by the trace of the posterior covariance (i.e., the inverse Hessian operator). To solve the A-optimal design problem, we adopt a greedy approach to cope with the binary structure of the weights. We demonstrate the effectiveness of the A-optimal design approach by comparing the OED results with random designs for 14- and 118-bus power flow problems. The results reveal the OED's potential to significantly reduce the uncertainty in the estimation by optimally placing PMUs.**

***Index Terms*—Bayesian inverse problems, Uncertainty quantification, A-optimal experimental design (OED), PMU placement, Power flow.**

I. Introduction

Power system state estimation provides crucial information for system controls. Failures in the state estimate have led to catastrophic consequences. In the Northeast blackout of 2003, over 508 generating units across 265 power plants ceased operation in various states in the US and Ontario, Canada, affecting 55 million individuals for several hours. This blackout was attributed to inaccuracies in state estimation, highlighting the severe repercussions of such failures. Phasor Measurement Units (PMUs) offer a promising avenue for enhancing state estimation by leveraging time synchronization. This aligns real-time measurements from multiple remote grid points with the assistance of GPS technology. Although PMUs can help prevent such events, randomly placing them over a fraction of the buses in a system is unlikely to exploit all the benefits this technology offers.

Hybrid PMU-Supervisory Control and Data Acquisition (SCADA) state estimation techniques have been developed to leverage PMU measurements and enhance accuracy. Skok *et al.* [1] utilized the traditional weighted least square (WLS) method with SCADA measurements and used PMU measurements to improve the transmission line parameters, which directly enhance the WLS state estimation accuracy. Watki *et al.* [2] developed a genetic algorithm that minimize the number of PMUs needed so that the mean absolute error of state estimate reached the desired threshold. While these efforts yielded improvements, the selected PMU locations are likely to be suboptimal. X. Li *et al.* [3] devised a hybrid PMU-SCADA approach, considering convergence-observability-performance metric in their semidefinite program with relaxed integer constraint to identify optimal PMU placements. Although optimal positions were determined, the uncertainties in the estimates were not quantified. Alternatively, Q. Li *et al.* [4] utilized Optimal Experimental Design (OED) to identify optimal PMU locations with A-, E-, and D-optimal design criteria, employing a greedy algorithm to address the OED problem. However, their approach assumed exact knowledge of line susceptances and used a *linearized model* of the state estimation problem, neglecting the joint effect of errors in susceptance priors and angle estimates.

This paper builds upon [5], where a Bayesian inverse problem is used to quantify the uncertainties of their nonlinear differential-algebraic system model parameter given noisy measurements described by an additive Gaussian noise model. In our paper, we present a Bayesian inverse problem formulation to quantify uncertainties in state estimation ($\boldsymbol{\theta}$)

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and susceptance parameter $(\mathbf{b})$, which we refer to as DC parameters, from a nonlinear DC power flow estimation model given noisy DC power flow measurements described by an additive Gaussian noise model. These uncertainties are integrated into the OED problem using Bayesian A-optimal design to reduce parameter uncertainties and determine optimal PMU locations. A greedy algorithm is employed to solve the OED problem, with two sets: the active set comprising chosen buses and the candidate set containing remaining buses. The algorithm iteratively introduces PMU measurements from the candidate set along with those from the active set to solve the Bayesian inverse problem. The outcome is the identification of K local optimal PMU locations with high confidence in the DC parameters.

The remainder of this paper is structured as follows: Section II introduces the power flow model with Gaussian noise and the Bayesian inverse problem for inferring $\boldsymbol{\theta}$ and $\mathbf{b}$. Section III outlines the OED problem formulation and the greedy algorithm to solve the OED problem. In Section IV, defines the pruning process, followed by the resolution of the OED problem for IEEE 14-bus and 118-bus system. Finally, Section V presents our conclusions and direction of future research.

II. Problem Formulation

A. Power Flow Model

We model the power flow via the DC power flow equations [6]. In this setting, the power grid is represented as a directed graph with a set of edges, denoted as $\mathcal{E}$, as illustrated in Fig. 1 (right). Each edge e in the graph symbolizes a power line, while each node represents a bus.

The inverse problem consists of using available observations p_e to infer the values of the unknown (or uncertain) susceptance parameter and voltage angles that characterize the power flow. Mathematically this inverse relationship is expressed as

$$p_e = b_e(\theta_{i(e)} - \theta_{j(e)}) + \eta_e, \qquad \text{for all } e \in \mathcal{E}. \tag{1}$$

Here p_e represents the power flow on line e, $i(e)$ is the *from* node on e, $j(e)$ is the *to* node of e, b_e is the *susceptance* on e, $\theta_{i(e)}$ and $\theta_{j(e)}$ are voltage angles at the endpoints of e. The observations p_e contain noise due to measurement uncertainties and model errors [7], [8]. Hence we use an additive Gaussian noise model to capture this, i.e., we add a noise term η_e to each observation. In vector form, (1) reads

$$\mathbf{p} = \mathbf{f}(\boldsymbol{\theta}, \mathbf{b}) + \boldsymbol{\eta}, \tag{2}$$

where $\mathbf{p}$, $\mathbf{b}$, $\boldsymbol{\theta}$ are vectors of their respective variables, and $\boldsymbol{\eta} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Gamma}_{noise})$, with $\boldsymbol{\Gamma}_{noise} \in \mathbb{R}^{|\mathcal{E}|\times|\mathcal{E}|}$ a diagonal noise covariance matrix. The so-called parameter-to-observable map $\mathbf{f}(\boldsymbol{\theta}, \mathbf{b})$ is defined as $diag(\mathbf{b})(A^T\boldsymbol{\theta})$ and corresponds to the first term in the right hand side of (1); its evaluation involves the solution of the power flow equations given $(\boldsymbol{\theta}, \mathbf{b})$. Here, $diag(\mathbf{b})$ is a diagonal matrix with $\mathbf{b}$ on the diagonal, and $A \in \mathbb{R}^{N\times|\mathcal{E}|}$ is the incident matrix providing a mapping between nodes and edges, where N and $|\mathcal{E}|$ are the number of nodes and edges. Specifically, for each edge e, the entry $A_{l,e} = 1$ if $l = i(e)$, the entry $A_{l,e} = -1$ if $l = j(e)$, or 0 otherwise.

In this paper, we assume that all graphs under consideration consist of many cycles, and may additionally have dangling trees originating from nodes within those cycles. In the dangling trees, $\boldsymbol{\theta}$ can be solved for by using (1) with the power flow measurements $\mathbf{p}$, θ at the root of each dangling trees and using the prior $\mathbf{b}$ information. For this reason, our method focuses solely on solving for parameters $\boldsymbol{\theta}$ and $\mathbf{b}$ within the cycles, with a fixed value for θ at the slack bus[1]. Thus we remove the dangling trees by pruning, since they are not required in our method. The pruning process is described in Section IV.

B. Bayesian Power Flow Inverse Problem

To infer the DC parameters, $\mathbf{b}$ and $\boldsymbol{\theta}$, for the power flow model and quantify the uncertainties in the reconstruction, we invoke a Bayesian formalism. In this formulation, we state the inverse problem as a problem of statistical inference over the space of uncertain parameters, which are to be inferred from data and the power flow model. The solution of the resulting Bayesian inverse problem is a posterior probability density function (pdf). Bayes' Theorem states the posterior pdf explicitly as

$$\pi_{post}(\boldsymbol{\theta}, \mathbf{b}) \propto \pi_{like}(\mathbf{p}|(\boldsymbol{\theta}, \mathbf{b}))\pi_{prior}(\boldsymbol{\theta}, \mathbf{b}), \tag{3}$$

where $\pi_{post}(\boldsymbol{\theta}, \mathbf{b})$ denotes the posterior that results from updating the prior probability $(\pi_{prior}(\boldsymbol{\theta}, \mathbf{b}))$ with information summarized by the likelihood $(\pi_{like}(\mathbf{p}|(\boldsymbol{\theta}, \mathbf{b})))$ [7], [8].

The likelihood represents the probability of observing the noisy measurement $\mathbf{p}$ given $\boldsymbol{\theta}$ and $\mathbf{b}$. As discussed above, we assume that the noise due to errors in measurements and model errors are additive and Gaussian. Hence we can then express the pdf for the likelihood model explicitly as

$$\pi_{like}(\mathbf{p}|(\boldsymbol{\theta}, \mathbf{b})) \propto \exp(-\|\mathbf{p} - \mathbf{f}(\boldsymbol{\theta}, \mathbf{b})\|^2_{\boldsymbol{\Gamma}^{-1}_{noise}}), \tag{4}$$

where $\boldsymbol{\Gamma}_{noise}$ is the noise covariance matrix.

The prior encodes any knowledge about the parameter space that we may wish to impose before the measurements are considered. In this paper, we assume that $\boldsymbol{\theta}$ and $\mathbf{b}$ are independent and Gaussian. Therefore, we write

$$\pi_{prior}(\boldsymbol{\theta}, \mathbf{b}) \propto \pi_{prior}(\boldsymbol{\theta})\pi_{prior}(\mathbf{b}). \tag{5}$$

More specifically, the prior distribution for $\boldsymbol{\theta}$ is defined by

$$\pi_{prior}(\boldsymbol{\theta}) \propto \exp(-\|\boldsymbol{\theta} - \tilde{\boldsymbol{\theta}}\|^2_{\boldsymbol{\Gamma}^{-1}_{\theta,pr}}), \tag{6}$$

where $\tilde{\boldsymbol{\theta}}$ and $\boldsymbol{\Gamma}_{\theta,pr}$ are the mean and covariance, respectively. The prior distribution for $\mathbf{b}$ is defined by

$$\pi_{prior}(\mathbf{b}) \propto \exp(-\|\mathbf{b} - \tilde{\mathbf{b}}\|^2_{\boldsymbol{\Gamma}^{-1}_{b,pr}}), \tag{7}$$

where $\tilde{\mathbf{b}}$ and $\boldsymbol{\Gamma}_{b,pr}$ are the mean and covariance, respectively.

[1] We neglect dangling trees because when working with DC power flow equations, flows on dangling trees are equivalent to transportation flows. However, dangling trees should be considered if implementing the described methods using AC power flow equations or approximations with voltage magnitudes.

We recall that even if Gaussian priors and noise probability distributions are invoked, the posterior probability distribution may not be Gaussian due to the nonlinearity of $\mathbf{f}(\boldsymbol{\theta}, \mathbf{b})$ [7], [8]. In this paper, we use a Gaussian approximation of the posterior centered at the maximum a posteriori (MAP) point of the Bayesian inverse problem, namely we approximate the posterior by $\mathcal{N}((\boldsymbol{\theta}_{MAP}, \mathbf{b}_{MAP}), \boldsymbol{\Gamma}_{post})$. The MAP point is obtained by solving

$$\min_{\boldsymbol{\theta}, \mathbf{b}} \; -\log \pi_{post}(\boldsymbol{\theta}, \mathbf{b}), \tag{8}$$

and the posterior covariance $\boldsymbol{\Gamma}_{post}$ is given by

$$\boldsymbol{\Gamma}_{post} = \mathbf{H}^{-1}(\boldsymbol{\theta}_{MAP}, \mathbf{b}_{MAP}), \tag{9}$$

where $\mathbf{H}$ is the Hessian of $-\log \pi_{post}(\boldsymbol{\theta}, \mathbf{b})$ [7].

III. Proposed Approach

A. A-optimal Experimental Design Problem Formulation

Following an A-optimal design strategy, we seek to minimize the average posterior variance of the parameter estimates, which is given by the trace of the posterior covariance [10]–[13]. In particular, the design is introduced in the Bayesian inverse problem through a vector of weights for possible PMU locations.

In what follows, we denote by $x_i, i = 1, ..., N$, the potential locations for PMUs. The nodes are referred to as *candidate sensor locations*. For each candidate location x_i, we assign a binary indicator w_i, with $w_i = 1$ indicating a PMU should be placed at node i and $w_i = 0$ otherwise. The A-optimal experimental design problem can be formulated as follows: find the optimal binary indicators $\mathbf{w} = [w_1 \dots w_N]^\top$ within the feasible space $\mathcal{W} := \{0, 1\}^N$ such that the trace of the posterior covariance (i.e., uncertainty in the reconstruction) is minimized.

The w-weighted negative log posterior reads

$$\begin{aligned} \mathcal{J}(\boldsymbol{\theta}, \mathbf{b}; \mathbf{w}) \;\; = \;\; & \frac{1}{2}\|\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}\|^2_{W(\mathbf{w})^{1/2}\boldsymbol{\Gamma}^{-1}_{OED}W(\mathbf{w})^{1/2}} + \\ & \frac{1}{2}\|\mathbf{p} - \mathbf{f}(\boldsymbol{\theta}, \mathbf{b})\|^2_{\boldsymbol{\Gamma}^{-1}_{f,noise}} + \\ & \frac{1}{2}\|\boldsymbol{\theta} - \tilde{\boldsymbol{\theta}}\|^2_{\boldsymbol{\Gamma}^{-1}_{\theta,pr}} + \frac{1}{2}\|\mathbf{b} - \tilde{\mathbf{b}}\|^2_{\boldsymbol{\Gamma}^{-1}_{b,pr}}, \end{aligned} \tag{10}$$

where $\hat{\boldsymbol{\theta}}$ denotes the measurements at the PMUs, $W(\mathbf{w})$ is a diagonal matrix with the design $\mathbf{w}$ on the diagonal, and $\boldsymbol{\Gamma}_{OED}$ is the covariance matrix of the PMU noise measurements.

The A-optimal experimental design problem can be summarized as

$$\min_{\mathbf{w} \in \mathcal{W}} \Phi(\mathbf{w}) := \mathrm{tr}(\boldsymbol{\Gamma}_{\text{post}}(\mathbf{w})), \tag{11}$$

where $\boldsymbol{\Gamma}_{\text{post}}(\mathbf{w})$ is the inverse of the Hessian of the cost function $\mathcal{J}$ with respect to $(\boldsymbol{\theta}, \mathbf{b})$.

B. Computational Methods

To compute the MAP point efficiently, we apply an inexact Newton approach [14]. Therefore, we use gradient and Hessian information, and hence are able to compute the covariance apply (i.e., inverse of the Hessian apply at the MAP). For all numerical experiments we set a tolerance for Newton's method of 10^{-8}. The number of iterations the Newton method took to converge for this tolerance was about 8 and 11 for the 14-bus and the 118-bus system.

To solve the A-optimal design problem (11), we adopt a greedy approach, which allows us to cope with binary weights. This approach is attractive especially when the number of sensors is of moderate size. The reason for this is that at the j^{th} step of the greedy algorithm, $N - j - 1$ OED objective evaluations are required. The total cost, measured in OED objective evaluations, for placing K PMUs is $C(K, N) := KN - K(K-1)/2$, hence it scales with the number of sensors. We note that one OED objective evaluation requires solving minimizing (10). We summarize the greedy approach in Algorithm 1.

Algorithm 1 Greedy approach for solving the OED problem.

Input: The target number of sensors K.
Output: The design vector $\mathbf{w}$.

1. $\mathbf{w} \leftarrow \mathbf{0}$.
2. $\mathcal{I}_{\text{candidate}} \leftarrow \{1, \cdots, N\}$.
3. $\mathcal{I}_{\text{active}} \leftarrow \emptyset$.
4. **For** $l = 1$ **to** K:
5. Evaluate $\Phi(\mathbf{w} + \mathbf{e}_j)$ for all $j \in \mathcal{I}_{\text{candidate}}$. $\{\mathbf{e}_j$ is the jth coordinate vector in $\mathbb{R}^N$.$\}$
6. $i_l \leftarrow \arg\min_{j \in \mathcal{I}_{\text{candidate}}} \Phi(\mathbf{w} + \mathbf{e}_j)$.
7. $\mathcal{I}_{\text{active}} \leftarrow \mathcal{I}_{\text{active}} \cup \{i_l\}$.
8. $\mathcal{I}_{\text{candidate}} \leftarrow \mathcal{I}_{\text{candidate}} \setminus \{i_l\}$.
9. $\mathbf{w} \leftarrow \mathbf{w} + \mathbf{e}_{i_l}$.

IV. Numerical Experiments

A. Problem Setup

We have two type of measurements: one type is on the edges ($\mathbf{p}$) and the other type is on nodes (PMU). To generate measurements on the lines $\mathbf{p}$ we use (1), where η is drawn from a normal distribution with 1% noise with the ground truth $\boldsymbol{\theta}_{true}$ and $\mathbf{b}_{true}$. The ground truth are obtained from [15]. To generate measurements on the nodes, we perturb $\boldsymbol{\theta}_{true}$ with random draws from a Gaussian distribution centered at $\mathbf{0}$ with standard deviation 0.02, which corresponds to the 1μ second GPS synchronization accuracy (according to [4]). The voltage angle (θ) corresponding to reference bus is set to 0, without loss of generality.

Next we discuss the choice of the priors needed for Bayesian inference for $\mathbf{b}$, and $\boldsymbol{\theta}$. For our numerical experiment we assume $\mathbf{b}$ and $\boldsymbol{\theta}$ are independent following Gaussian distributions. Prior knowledge of $\mathbf{b}$ suggest a more conservative variance and a known mean. In particular, we follow [9], and choose a prior distribution $\mathcal{N}(\tilde{\mathbf{b}}, \boldsymbol{\Gamma}_{b,pr})$, where

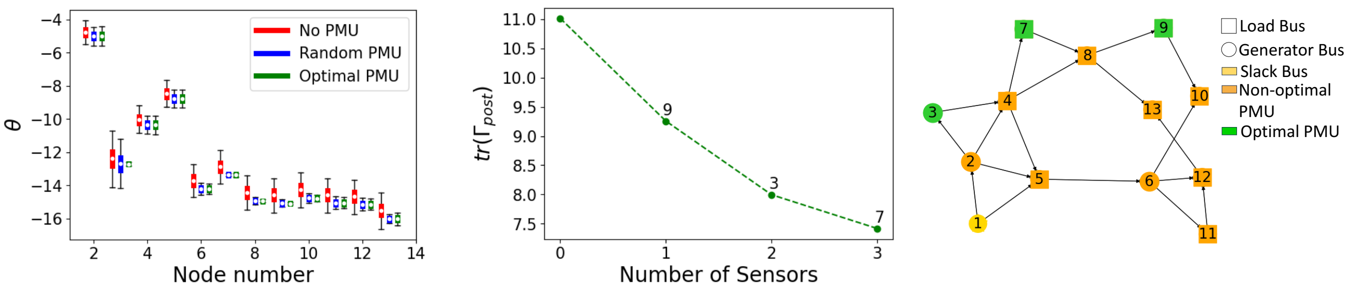

Fig. 1. OED results for the 14-bus system. Left: the plot of $\boldsymbol{\theta}$ confidence intervals with no PMU (red), randomly placed PMU (blue) and optimal PMUs placement (green). The white dot in each interval is the MAP point of their respective problems. Middle: the OED objective versus the number of PMUs (sensors). The number (9,3, and the 7) on the graph corresponds to the optimal choice for PMU placement. Right: the optimal PMUs on the graph for the post-pruned 14-bus system.

$\boldsymbol{\Gamma}_{b,pr} = diag(\boldsymbol{\sigma}_{pr,b})$ with $\boldsymbol{\sigma}_{pr,b} = 0.12 \times diag(|\mathbf{b}_{true}|)\boldsymbol{\epsilon}_b$, $\boldsymbol{\epsilon}_b \sim \mathcal{N}(\mathbf{0}, I_{|\mathcal{E}|})$, $I_{|\mathcal{E}|}$ is identity matrix of size $|\mathcal{E}|$, $|\mathbf{b}_{true}|$ is the vector $\mathbf{b}_{true}$ where we take the absolute value of each element, and $\tilde{\mathbf{b}} = \mathbf{b}_{true} + \boldsymbol{\sigma}_{pr,b}$. For $\boldsymbol{\theta}$ we set a large variance modeling no prior knowledge. In particular we choose $\mathcal{N}(\mathbf{0}, \boldsymbol{\Gamma}_{\theta,pr})$ where $\boldsymbol{\Gamma}_{\theta,pr} = diag(\boldsymbol{\sigma}_{pr,\theta})$ with $\boldsymbol{\sigma}_{pr,\theta} = 100 \times diag(|\boldsymbol{\theta}_{true}|)\boldsymbol{\epsilon}_\theta$, $\boldsymbol{\epsilon}_\theta \sim \mathcal{N}(\mathbf{0}, I_N)$, I_N is identity matrix of size N, and $|\boldsymbol{\theta}_{true}|$ is the vector $\boldsymbol{\theta}_{true}$ where we take the absolute value of each element.

B. Pruning Process

For the numerical results, we will employ the 14-bus and 118-bus system. The graphs associated with the 14-bus and 118-bus system respectively resemble portions of the American Electric Power System (in the Midwestern US) as of February 1962 and December 1962. This information was obtained from [15].

Our physical model assumes that each node in the graph lies on a cycle. Therefore we prune leaves and branches from the dangling trees which are unnecessary components, and delete their associated information from the vectors $\boldsymbol{\theta}$ and $\mathbf{b}$. Then we re-index the nodes so that node 1 is the slack bus. The pruned graph for the 14-bus system is shown on the right of Fig. 1. After applying the pruning process to the 118-bus system, we reduced the graph from 118 nodes 186 edges to 109 nodes and 177 edges.

C. Numerical Experiments

We conduct two experiments to illustrate the OED approach. Experiment one focuses on the 14-bus system to help visualize the results while experiments two focus on the 118-bus system to demonstrate that the OED approach can be scaled to a larger system.

In the first experiment, we first solve the inverse problem with no PMUs and illustrate the MAP estimate for $\boldsymbol{\theta}$ with their associates uncertainties. To further reduce these uncertainties we add PMU measurements and apply the OED procedure. On the left of Fig. 1, we show the reduction of the OED objective (uncertainty in the reconstruction) with three optimal PMUs. In the center plot in Fig. 1 the corresponding MAP point and uncertainty are shown in green. For comparison we also show result with three random PMUs in red. These results reveal that adding PMU measurement reduce uncertainty and optimal is superior to random. Note that we do not show the MAP estimates for $\mathbf{b}$, since the uncertainty for these parameters are set small and hence there is not much improvement with additional PMUs. The right figure in Fig. 1 shows optimal location of the PMU in the graph.

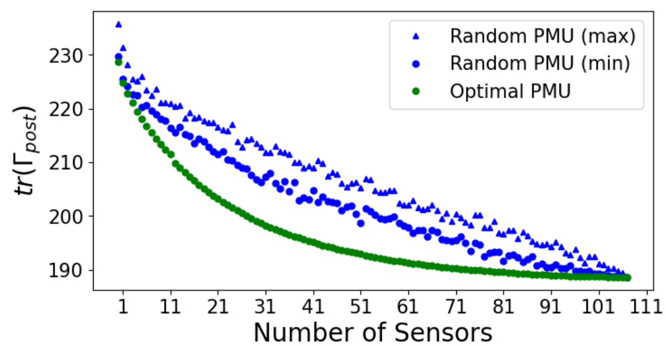

Fig. 2. OED results for the 118-bus system. Comparison of the optimal (green) and random (blue) PMU placement. For each number of sensors we choose 20 random designs and show the max (blue triangles) and min (blue circles) of the set of 20 random design OED objective evaluations.

For the second experiment, we apply the OED process to the 118-bus system. For this problem, there are 109 possible choices for PMUs. To compare the OED results (optimal design) and random designs, we generate 20 random designs for each number of sensor and evaluate the OED objective. The results are shown in Fig. 2. The results reveal the following: the uncertainty decays exponentially meaning there is a significant reduction in the small number of sensors regime, but this reduction decreases as more sensors are added. We see a similar behavior for the random designs in the very low sensor regime (i.e., $K < 10$). However as the number of sensors increases, the uncertainty reduction with the random designs behaves more linearly. The optimal design clearly outperforms the random designs in the mid 75 percentile interval (i.e., $10 \leq K \leq 80$). For the large number of sensors regime (i.e., $K \geq 90$), where data are collected at nearly every sensor, the

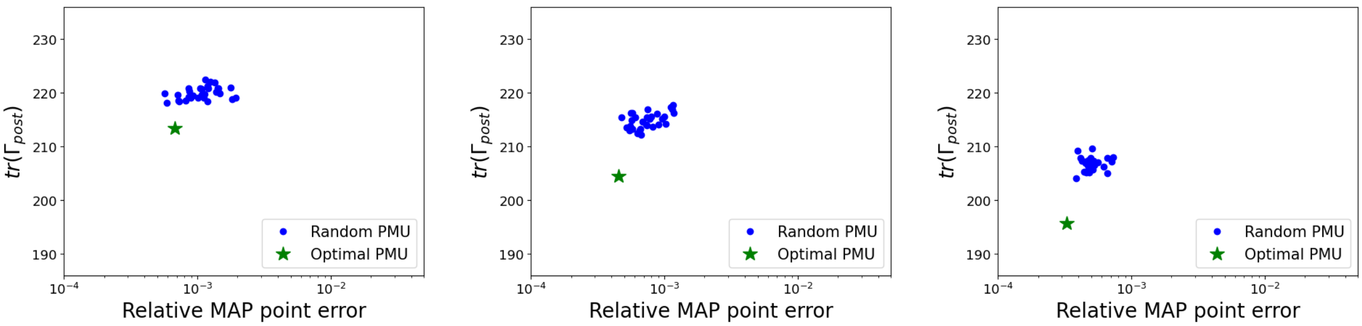

Fig. 3. OED results for the 118-bus system. Relative error of the MAP point versus OED objective for the optimal design (green star) and for 30 random designs (blue dots) for 10 (left), 20 (center), 40 (right) sensors.

performance of the random designs and the optimal design are comparable. Another observation from these results is the diminishing returns as the number of sensors is increased. For the optimal designs using more than 70 sensors results in only negligible decrease in the OED objective.

To further assess the effectiveness of the computed A-optimal PMU sensor placement, in Fig. 3 we compare the relative error of the MAP point as well as the OED objective obtained using the optimal design (green star) and 30 randomly generated designs (blue dots) with 10 (left), 20 (center), 40 (right) sensors. These results are obtained by solving the Bayesian inverse problem (for $\boldsymbol{\theta}_{MAP}(\mathbf{w})$) described in Section II-B with the negative log posterior given in equation (10). The relative error is computed using

$$\frac{\|\boldsymbol{\theta}_{MAP}(\mathbf{w}) - \boldsymbol{\theta}_{true}\|_2}{\|\boldsymbol{\theta}_{true}\|_2}.$$

These results are consistent with the results shown in Fig. 2, namely that the A-optimal sensor placement outperforms the random designs in terms of uncertainty reduction. With respect to the relative error, the results show that adding more sensors the MAP point is improved. We also note that on rare occasion the random design leads to a smaller error. This is expected since the OED procedure does not minimize this error but the variance.

V. Conclusion

In this paper, we introduce an A-optimal design of experiments for nonlinear Bayesian inverse power flow problems. The Bayesian inverse problem is formulated and solved to infer the DC power flow parameters, e.g., voltage angle and susceptance, and quantify the uncertainties in the reconstruction. The OED problem is aimed at finding an optimal PMU sensor configuration with the scope of reducing the uncertainty. We show results for two power flow problems, one with 14 buses and one with 118 buses. We demonstrate the effectiveness of the A-optimal design approach, by comparing the OED results with random designs. Future research efforts will focus on extending the approach to more complex power grid models.

References

[1] S. Skok, I. Ivankovic, and Z. Cerina. "Hybrid state estimation model based on PMU and SCADA measurements." IFAC-PapersOnLine 49.27, pp 390-394, 2016.

[2] M. H. Wakti, L. M. Putranto, S. P. Hadi, *et al.* "PMU location determination in a hybrid PMU-SCADA system." 2020 12th International Conference on Information Technology and Electrical Engineering (ICITEE). IEEE, 2020.

[3] X. Li, A. Scaglione and T. -H. Chang, "A Framework for Phasor Measurement Placement in Hybrid State Estimation Via Gauss–Newton," in IEEE Transactions on Power Systems, vol. 29, no. 2, pp. 824-832, March 2014, doi: 10.1109/TPWRS.2013.2283079.

[4] Q. Li, R. Negi, and M. Ilic, "Phasor measurement units placement for power system state estimation: A greedy approach," in Proc. IEEE Power and Energy Soc. Gen. Meeting, pp. 1–8, 2011.

[5] N. Petra, C. G. Petra, Z. Zhang, *et al.* "A Bayesian approach for parameter estimation with uncertainty for dynamic power systems." IEEE Transactions on Power Systems 32.4 (2016): 2735-2743.

[6] X. F. Wang, S. Yonghua, and I. Malcolm. "Modern power systems analysis". Springer Science & Business Media, 2010.

[7] A. Tarantola, "Inverse Problem Theory and Methods for Model Parameter Estimation". Philadelphia, PA, USA: SIAM, 2005.

[8] J. Kaipio, and S. Erkki. "Statistical inverse problems: discretization, model reduction and inverse crimes." Journal of computational and applied mathematics 198.2, pp. 493-504, 2007.

[9] S. Vlahinic, D. Frankovic, M. Z. Durovic, *et al.* "Measurement uncertainty evaluation of transmission line parameters." IEEE transactions on instrumentation and measurement vol. 70, pp. 1-7, 2021.

[10] A. Alexanderian, N. Petra, G. Stadler, *et al.* "A-optimal design of experiments for infinite-dimensional Bayesian linear inverse problems with regularized ℓ_0-sparsification." SIAM Journal on Scientific Computing 36.5, 2014: A2122-A2148.

[11] A. Alexanderian. "Optimal experimental design for infinite-dimensional Bayesian inverse problems governed by PDEs: A review." Inverse Problems 37.4, 2021: 043001.

[12] A. Alexanderian, N. Petra, G. Stadler, *et al.* "Optimal design of large-scale Bayesian linear inverse problems under reducible model uncertainty: Good to know what you don't know." SIAM/ASA Journal on Uncertainty Quantification 9.1, pp. 163-184, 2021.

[13] A. Alexanderian, N. Ruanui, and N. Petra. "Optimal design of large-scale nonlinear Bayesian inverse problems under model uncertainty." arXiv preprint arXiv:2211.03952, 2024.

[14] J. Nocedal and S. J. Wright, Numerical Optimization, 2nd ed. New York,NY, USA: Springer, 2006.

[15] R. D. Zimmerman, C. E. Murillo-Sanchez, and R. J. Thomas, "MATPOWER: Steady-State Operations, Planning and Analysis Tools for Power Systems Research and Education," Power Systems, IEEE Transactions on, vol. 26, no. 1, pp. 12–19, Feb. 2011.

[16] NetworkX. https://networkx.org/.