WORKING GROUP REPORT: THE POWER OF COMPUTATIONAL THINKING IN MATHEMATICS AND DATA SCIENCE EDUCATION

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The "Power of Computational Thinking in Mathematics and Data Science Education" working group held its inaugural meeting at PME-NA 45 in Reno, Nevada. The skills and practices of CT can empower teachers to emphasize abstraction, automation, modeling, and simulations as their students investigate relationships in mathematics and data science. The focus of the three sessions was to advance conversations about the integration of CT in mathematics and DS education with aims to launch new collaborations. Our overarching goal of providing more equitable access to authentic mathematical problem solving through guided the design and facilitation of the working group sessions. Participants experienced three CT-integrated data science tasks on Day 1, created working visuals of the synergies across the disciplines on Day 2, and proposed directions for future research on Day 3.

Keywords: Computational Thinking, Computing and Coding, Data Analysis and Statistics, Modeling

A Brief History and Motivation of the Working Group

This working group journey started at the PME-NA 44 conference in Nashville in 2022. During a session focused on computational thinking (CT) in mathematics education, the founding members (Yilmaz et al., 2023a) embarked on a journey, laying the groundwork for this thematic group. Mr. Alegre and Dr. Yilmaz presented on the transformative integration of mathematical thinking (MT) and computational thinking (CT) within the "Cultural Quilts" coding project. They shared how the coding efforts of high school students not only offered valuable insights into mathematical strategies (e.g., defining functions, using geometric transformations) within their code but also served as a means for students to express their cultural identities and values through uniquely personalized quilt designs (Alegre et al., 2022). Dr. Galanti (2022) presented on how elementary teachers in an online graduate-level CT course engaged in mathematical sensemaking using block-based programming. She shared how teachers modified parameters in a Scratch project to explore the dynamic relationship between changing height and changing volume to create a box with maximum volume from a sheet of paper. Teacher's coding artifacts modeled the reciprocal relationship between CT (algorithmic thinking, abstraction, and automation) and MT (pattern seeking and generalization). These two presentations contribute to the growing knowledge base on the integration of CT and mathematics and its potential to deepen learning in both disciplines (e.g., Brady et al., 2021; Goldenberg et al., 2021; Kallia et al., 2021; Weintrop et al. 2016).

Our shared passion for exploring the integration of CT and mathematics extended informal conversations at the PME-NA 44 conference to more format collaborations. Dr. Lawler invited the presenters from the CT session at PME-NA 44 to co-host a mini-symposium at Kennesaw State University supported by the National Science Foundation (https://research.kennesaw.edu/cistemer/culturally-relevant-integration-cs-mathematics.php) This two-day event was attended by over 30 computer science and mathematics education scholars from diverse backgrounds and contexts. They considered ways in which culturally relevant pedagogy might enhance integrated approaches to learning computation and mathematics.

Furthermore, our discussions extended to wonderings about the role of CT in data science education. We draw on several definitions of data science as an overlap of the skills of multiple disciplines including mathematics, business, statistics, and computer science (Lee et al., 2022). Acknowledging that data science requires not only CT but also MT, we were excited to expand our exploration to the synergies among these three disciplinary domains. Dr. Yilmaz envisioned extending these efforts beyond our one-time symposium event. She facilitated our submission of a thematic group proposal for PME-NA 45 by its founding members. This proposal created a "community focused on advanced conversations about synergies between CT in mathematics and data science education with the aim to launch new collaborations" (Yilmaz et al., 2023a, p. 674). Recognizing the importance of CT as a crucial yet underemphasized aspect of K-16 education in an increasingly technological world, this working group aims to challenge the mathematics education community to advance the teaching and learning of CT within both mathematics and data science.

Progress Made Throughout the PME-NA 45 Working Group

Day 1 Progress and Outcomes

Workshop participants collaboratively engaged in three integrated problem-solving tasks. These tasks, all focused on statistical concepts (measures of central tendency), were selected to provide multiple entry points to our discussion of the synergies among CT, MT, and data science without an assumption of prior knowledge. The discussion questions were designed to stimulate conversations about the ways in which the concepts and practices of CT could be integrated in mathematics and data science teaching.

Task 1 was adapted from a set of freely available elementary CT-integrated mathematics modules (Education Development Center, 2021). Participants used an unplugged CT approach (without a computer) to write algorithms to move two bots to calculate the mean and median of a data set (See Figure 1). They considered the following synergistic questions:

- How does CT contribute to the development of the mathematical concepts of mean, median, and spread in this task?
- How does the context of a mathematics task create opportunities to develop CT skills and practices?
- How do the algorithms developed by the students in this task relate to concepts and techniques commonly used in data science?

Task 2 was an adaptation of a plugged YouCubed K-12 Data Science activity (2020). Participants accessed an open-source Common Data Analysis Platform (CODAP) to explore mean, median, and mode for a mammals data set See Figure 2). They considered the following synergistic questions:

- How does data science contribute to the development of the mathematical concepts of mean, median, and spread in this task?
- How does the context of a data science task create opportunities to develop CT skills and practices?
- What shifts do you see in CT skills and practices used when students transition from Task 1 (algorithmic design as problem-solving) to Task 2 (data exploration and visualization as problem-solving)?

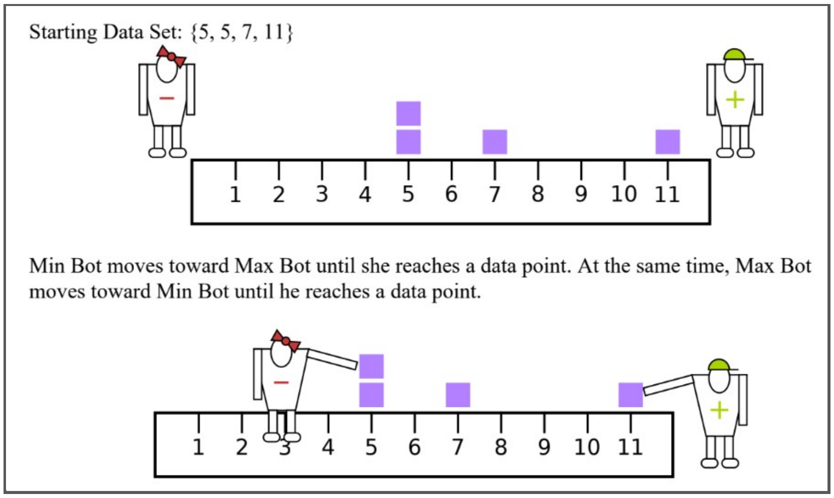

Figure 1: Excerpt from Task 1 - Algorithm Writing to Compute Mean and Median) (Education Development Center, 2021)

Mammals

Mammals (27 cases)

in-dex	Mammal	Order	LifeSpan (years)	Height (meters)	Mass (kg)	Sleep (hours)	Speed (km/h)	Habitat	Diet
1	Africa...	Probosc...	70	4	6400	3	40	land	plants
2	Asian...	Probosc...	70	3	5000	4	40	land	plants
3	Big B...	Chiropt...	19	0.1	0.02	20	40	land	meat
4	Bottl...	Cetacea	25	3.5	635	5	37	water	meat
5	Chee...	Carnivora	14	1.5	50	12	110	land	meat
6	Chim...	Primate	40	1.5	68	10		land	both
7	Dom...	Carnivora	16	0.8	4.5	12	50	land	meat
8	Donk...	Perisso...	40	1.2	187	3	50	land	plants

Figure 2: Excerpt from Task 2 - Using CODAP to Explore Attributes of Mammal Data Set

Task 3 used a simple computer program (Figure 3) to explore the interactions of CT and MT in block-based programming approaches to calculating the mean of a set of numbers. Participants had the opportunity to modify code to compare and contrast two approaches to a statistical calculation: 1) the use of a single variable to accumulate the sum of numbers as they are entered by the user and divided by the number of entries; and 2) the use of a list to first store all entered values before calculating the mean by iterating over a list of numerical entries.

All three of these tasks were intended to highlight abstraction, decomposition, pattern recognition, algorithmic thinking, logical thinking, modeling, and automation as productive aspects of CT within mathematics education (Kallia et al., 2021). The participants were able to draw upon these different problem-solving experiences within a data science context to think about how dynamic computer models and the underlying algorithmic thinking can support generalization and evaluation in problem solving.

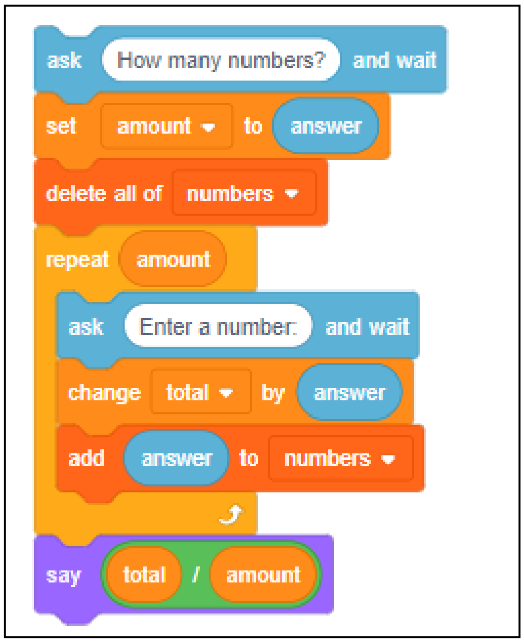

Figure 3: Excerpt from Task 3 (Using a Block-based Scratch Computer Program to Calculate Mean)

Day 2 Progress and Outcomes

The three tasks on Day 1 provided a shared experience from which to elicit participants' own teaching and learning experiences with CT, MT, and data science. After examining a series of published visual conceptualizations of relationships between the three disciplines (e.g., Lee et al., 2022; Sneider et al., 2014), the participants worked in small groups to create their own visuals (see Figure 4 for examples). To contextualize these visualizations of disciplinary relationships, Mr. Alegre and Dr. Lawler presented research that challenges us as mathematics education researchers to reflect on whose ways of knowing are valued in CS and mathematics education. Mr. Alegre described a research practice partnership between Louisiana State University and secondary teachers to integrate CS in mathematics. Dr. Lawler shared work in after school programs designed to engage middle grades students in imagination, creativity, reasoning, and discourse through integrated coding and mathematics experiences. These presenters encouraged conversations about the need to draw upon teachers' and students' cultural assets in considering future directions for disciplinary integration.

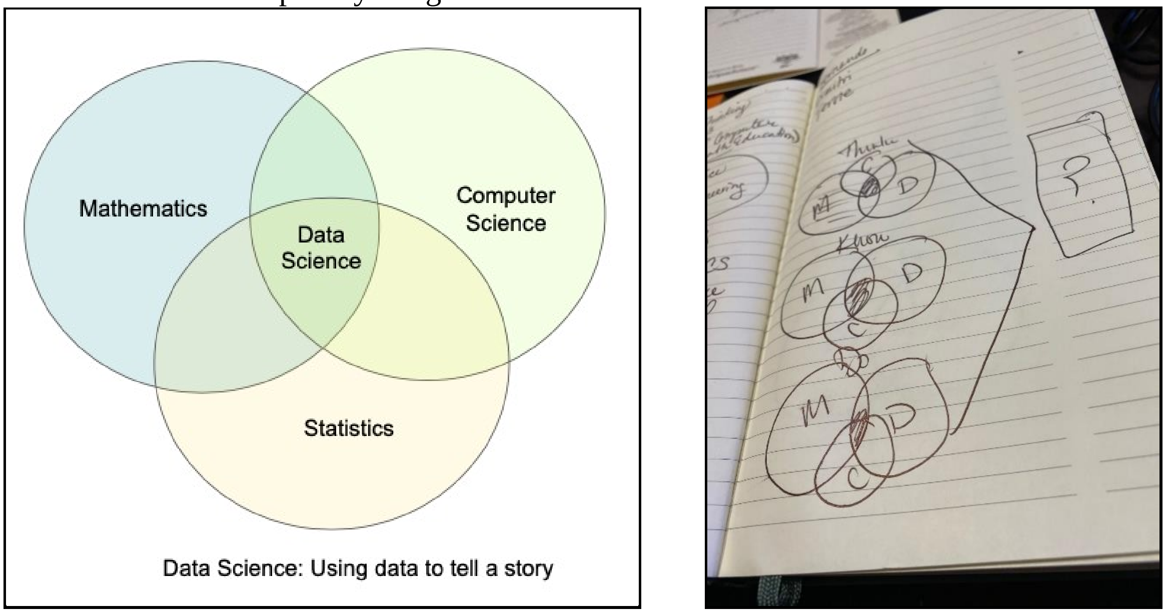

Figure 3: Participant-created visualizations of CT, MT, and Data Science

Day 3 Progress and Outcomes

The final session started with participants' reflection on the visuals created on Day 2 and organic conversation across two main areas: 1) Computational Thinking: Knowing, Doing, Thinking and 2) Equity and Access: Practical Applications of Computation and Mathematics in Schools.

Computational Thinking: Knowing, Doing, Thinking.

The participants delved one of the visuals created on Day 3 (See Figure 3 above). This visual emphasized the three aspects of computational integration as knowing, doing and thinking. They suggest computational knowing could include understanding algorithms and data structures and computational doing could include coding. They argued that both knowing and doing are both essential for CT. They also discussed concrete examples on the misconception that learning to code with an application such as Scratch (a visual block-based programming language) equates to CT. They shared the need for a broader conceptual understanding of what CT encompasses. The participants generated two questions for inquiry and reflection.

- Is computational doing limited to coding, or are there other ways to enact CT?
- What constitutes computational knowing, thinking and how can it be effectively taught and assessed in mathematics and data science education?

Examination of various visuals from Day 1 and Day 2 motivated participants to consider the benefit of creating a unified visual showing the synergies between CT, MT, and data science. They suggested such a visual could support educators, policymakers, and practitioners in conceptualizing the varied approaches to integrating computation in K-16 mathematics and data science education and identifying areas for cross-disciplinary collaborations.

The participants acknowledged the challenges in creating a unified visual. One significant challenge is the establishment of common language and frameworks across disciplines. Each discipline may have its own terminology, methodologies, and epistemologies, making it challenging to develop a shared understanding. For instance, while mathematicians may approach problem-solving through abstraction and proof, computer scientists may focus on algorithms and computational complexity. Bridging these diverse perspectives requires careful negotiation and collaboration among experts from different disciplines. Additionally, the rapid evolution of technology and research methods further complicates efforts to create a unified representation that remains relevant over time.

Equity and Access: Practical Applications of Computation and Mathematics in Schools

The participants discussed how mathematics could serve as a pathway to increase access to computer science and related fields, potentially addressing issues of equity. Unlike elective courses like computer science, which may have prerequisites or limited availability, mathematics is a core subject taught to all students. One of the participants described how some students, particularly those underperforming in traditional subjects like mathematics and English, may be denied access to elective courses like computer science. This lack of access denies them the opportunity to explore practical applications in which they can use both CT and MT. Concrete applications of CT and MT skills, such as pattern making and measurement in fields like interior or fashion design, could improve students' understanding of mathematical concepts. Similarly, engaging students in computational activities rooted in mathematics might not only enhance their computational skills but also deepen their mathematical understanding. By incorporating CT into mathematics, schools can provide students with opportunities to explore real-world problem-solving and creative expression, potentially empowering students who might otherwise struggle with traditional math instruction.

Consideration of future research on equity and access to CT within K-12 mathematics and data science education settings yielded two questions for further inquiry. The first question was “Can students succeed in computer science courses without first meeting traditional grade-level math standards?” This question delves into providing alternative pathways or interventions to support more diversity in engagement with computer science in K-12 settings. The second question was “How can we as mathematics educators promote equitable access to CT in K-12 settings, particularly for students from marginalized backgrounds?” This question underscores the need to address systemic barriers and biases that may hinder students participation in CT programs.

Progress Made Following the October 2023 Working Group

Our working group convened virtually in November 2023 and in January 2024 to continue to discuss directions for research and collaboration at the intersection of CT, MT, and data science. The participants collapsed our working list of 13 researchable topics generated on Day 3 of the October meeting to a set of seven topics for collaborative research (See Table 1). The attendees also considered methodologies to support exploration of these topics. A literature review seemed well aligned with Topic 1, while an investigation of existing curricula seemed to be a productive entry point for Topic 4. Discussions of potential research centered on the stakeholders in disciplinary integration, specifically students and classroom educators. The ways in which students and educators experience the integration of CT and MT was of continuing interest, with multiple theoretical frameworks (e.g. identity, content knowledge and pedagogical content knowledge) and mathematical domains (e.g., geometry, algebra, and number) guiding the conversations.

Our working group has continued its effort to define a near-term collaborative project by reading a 2023 systematic literature review on the integration of CT in K-12 mathematics education with a focus on instruction and learning (Ye et al., 2023). The central question of this literature review was, ”How do students' CT and mathematics learning interact when they are involved in CT-based mathematics instruction, and what are the consequences of such interactions?” (p. 13). Participants met virtually in January 2024 to share their key takeaways and their key wonderings based upon their reading. The following questions emerged:

- Ng & Cui (2021) offered the descriptor of “computationally-enhanced mathematics education”. Would this description attract more interest in innovating traditional school curriculum with mathematics as a standalone subject?.
- There was no mention of the role that artificial intelligence or machine learning could play in the teaching and learning of CT and MT. Should we expand our focus to these potential integrations?
- Dick and Hollebrands (2011) define a mathematical action technology that can “perform mathematical tasks and/or respond to the user’s actions in mathematically defined ways” (p. xii). How does computer programming fit within this definition?
- Dynamic geometry software applications are mathematical action technologies. What CT skills and practices are teachers engaging in with students using dynamic algebra and geometry software applications?

Table 1: Directions for Collaborative MT-CT- Data Science Research

Direction for Collaborative Research	Number of Interested Researchers
Synergies between MT, CT, and Data Science	9
Integration of MT, CT, and Data Science in K-12 Education: Curriculum, Implementation, and Challenges	6
Integration of MT, CT, and DS: Effect on Underserved Students	3
Affordances of Data-Driven Math Curricula	2
Teaching Mathematics, Data Science, and Computer Science Simultaneously	4
STEM Teachers as “Integrators” of computational thinking	3
STEM Parenting Identity	1

Ye et al. (2023) stated that “future research on teacher professional development concerning the emergent competency of CT-based mathematical thinking is indispensable” (p. 24), but research on preservice and practicing teachers was an exclusion criterion for their literature review. We are tentatively working toward a collaborative literature review on preparing pre-service and practicing teachers to integrate CT in mathematics instruction.

Conclusions and Looking Ahead

Based on existing literature that explores the connections between CT and MT (e.g. Brating & Kilham, 2021; Hickmott et al., 2021) , as well as the integration of CT into mathematics as a school subject (e.g., Chan et al., 2023; Rich et al., 2020), our group is motivated to delve deeper into identified gaps, needs and challenges in these studies. Limited empirical studies have explored the synergies between CT and MT (Hickmott et al., 2018). Only a few studies have incorporated the expertise of mathematics educators to explore the integration of CT in mathematics (Hickmott et al., 2018; Kallia et al., 2021).

Another identified gap in the research is evidence of what resources (e.g., knowledge, curricular materials, tools) teachers need to effectively integrate CT into their mathematics teaching (Wu et al., 2021; Yadav et al., 2016) and how to assess teacher CT learning (Galanti & Baker, 2023). These gaps highlight the need for more professional learning opportunities for teachers. However, only a limited number of PD studies (e.g. Ahamed et al., 2010; Hart et al., 2008; Wu et al., 2021; Yilmaz et al., 2023b) have focused on the integration of CT and MT. This indicates a need for more tailored PD initiatives designed through interdisciplinary collaboration of mathematics and CS educators (Dahshan & Galanti, in press; Menekşe, 2015).

Our group is also interested in how the integration of CT in mathematics education and data science education in fostering teacher and student problem-solving identities. By looking across the literature in CT identity (Kong & Lai, 2022) and use of coding to foster creativity in mathematical thinking (Castle, 2023), we seek to understand how CT integration can build an individual sense of self as a “doer” of mathematics or data science.

As we continue engaging in generative inquiry and collaborative research, our thematic group is committed to seeking ways to address these gaps and needs and to broaden interest in this work in the PME-NA community.

Acknowledgments

This work was partially supported by Gordon A. Cain Center for STEM Literacy, NSF awards 2031490 and CNS- 1923573, and the U.S. DOE awards U411C190287 and S423A20001. We acknowledge that the first and second authors contributed equally to writing this report.

References

Ahamed, I., Brylow, D., Ge, R., Madiraju, P., Merrill, S. J., Struble, C. A., & Early, J. P. (2010). Computational thinking for the sciences: A three day workshop for high school science teachers. In *Proceedings from SIGCSE’10: The 41st ACM Technical Symposium on Computer Science Education* (pp. 42–46).

Alegre, F., Yilmaz, Z., Moreno, J., & Kendrick, R. (2022, November). Culturally relevant computing tasks: Evidence of synergies between students’ mathematical and computational thinking. In A. E. Lischka, E. B. Dyer, R. S. Jones, J. Lovett, J. Strayer, & S. Drown (2022). *Proceedings of the 44th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1977 – 1981). Nashville, TN: Middle Tennessee State University.

Brady, C. Vogelstein, L., Gresalfi, M. & Knowe, M. (2021). Circular reasoning: Shifting epistemological frames across mathematics and coding activities. In D. Olanoff, D., K. Johnson,, & Spitzer, S. *Proceedings of the 43rd annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1182–1190). Philadelphia, PA.

Bråting, K., & Kilhamn, C. (2021). Exploring the intersection of algebraic and computational thinking. *Mathematical Thinking and Learning, 23*(2), 170–185. https://doi.org/10.1080/10986065.2020.1779012

Castle, S. D. (2023, March). Leveraging computational science students' coding strengths for mathematics learning. In *Proceedings of the 54th ACM Technical Symposium on Computer Science Education V. 1* (pp. 263–269). https://dl.acm.org/doi/10.1145/3545945.3569861

Chan, S. W., Looi, C. K., Ho, W. K., & Kim, M. S. (2023). Tools and approaches for integrating computational thinking and mathematics: A scoping review of current empirical studies. *Journal of Educational Computing Research, 60*(8), 2036–2080.https://doi.org/10.1177/07356331221098793

Dahshan, M., & Galanti, T. M. (in press). *Teachers in the loop: Integrating computational thinking and mathematics to build early place value understanding.* Education Sciences.

Dick, T. P., & Hollebrands, K. F. (2011). *Focus in high school mathematics: Technology to support reasoning and sense making* (pp. xi–xvii). National Council of Teachers of Mathematics.

Educational Development Center. (2021). *Integrating Computational Thinking*. https://sites.google.com/site/stemcwithct/home

Galanti, T. M. (2022). Thinking outside the box: Preparing elementary teachers to integrate computational thinking and mathematics. In A. E. Lischka, E. B. Dyer, R. S. Jones, J. Lovett, J., J. Strayer, J., & S. Drown, S. (2022). *Proceedings of the 44th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1995 – 1999). Nashville, TN: Middle Tennessee State University.

Galanti, T., & Baker, C. (2023). Transforming assessment in online learning: Preparing teachers to integrate computational thinking in elementary classrooms. *Contemporary Issues in Technology and Teacher Education, 23*(4). https://citejournal.org/volume-23/issue-4-23/current-practice/transforming-assessment-in-online-stem-learning-preparing-teachers-to-integrate-computational-thinking-in-elementary-classrooms

Goldenberg, E. P., Carter, C. J., Mark, J., Reed, K., Spencer, D., & Coleman, K. (2021). Programming as language and manipulative for second-grade mathematics. *Digital Experiences in Mathematics Education, 7*(1), 48–65. https://doi.org/10.1007/s40751-020-00083-3

Hart, M., Early, J. P., & Brylow, D. (2008, March). A novel approach to K-12 CS education: Linking mathematics and computer science. In *Proceedings of the 39th SIGCSE technical symposium on Computer Science Education* (pp. 286–290).

Hickmott, D., Prieto-Rodriguez, E., & Holmes, K. (2018). A scoping review of studies on computational thinking in K–12 mathematics classrooms. *Digital Experiences in Mathematics Education, 4*(1), 48–69. https://doi.org/doi:10.1007/s40751-017-0038-8

Kallia, M., van Borkulo, S. P., Drijvers, P., Barendsen, E., & Tolboom, J. (2021). Characterising computational thinking in mathematics education: a literature-informed Delphi study. *Research in Mathematics Education, 23*(2), 159–187. https://doi.irg/10.1080/14794802.2020.1852104

Kong, S. C., & Lai, M. (2022). Computational identity and programming empowerment of students in computational thinking development. *British Journal of Educational Technology, 53*(3), 668–686.

Lee, H.S., Mojica, G. F., Thrasher, E.P., & Baumgartner, P. (2022). Investigating data like a data scientist: Key practices and processes. *Statistics Education Research Journal, 21*(2). https://iase-web.org/ojs/SERJ/article/download/41/457

Menekse, M. (2015). Computer science teacher professional development in the United States: A review of studies published between 2004 and 2014. *Computer Science Education, 25*(4), 325–350.

Ng, O., & Cui, Z. (2021). Examining primary students' mathematical problem solving in a programming context: Towards computationally enhanced mathematics education. *ZDM - Mathematics Education, 53*(4), 847–860. https://doi.org/ 10. 1007/ s11858-020-01200-7

Rich, K. M., Yadav, A., & Larimore, R. A. (2020). Teacher implementation profiles for integrating computational thinking into elementary mathematics and science instruction. *Education and Information Technologies, 25*, 3161–3188. https://doi.org/10.1007/s10639-020-10115-5

Sneider, C., Stephenson, C., Schafer, B., & Flick, L. (2014). Exploring the science framework and NGSS: Computational thinking in the science classroom. *The Science Teacher, 38*(3), 10–15.

Weintrop, D., Beheshti, E., Horn, M., Orton, K., Jona, K., Trouille, L., & Wilensky, U. (2016). Defining computational thinking for mathematics and science classrooms. *Journal of Science Education and Technology, 25*(1), 127–147.

Wu, S., Peel, A., Bain, C., Horn, M.S., Wilensky, U. (2021). Different paths, same direction: Dow teachers learn computational thinking in stem practices through professional development. *Proceedings of the 5th APSCE International Computational Thinking and STEM in Education Conference*, (pp. 52–57).

Yadav, A., Gretter, S., Hambrusch, S., & Sands, P. (2016). Expanding computer science education in schools: understanding teacher experiences and challenges. *Computer Science Education, 26*(4), 235–25.

Ye, H., Liang, B., Ng, O. L., & Chai, C. S. (2023). Integration of computational thinking in K-12 mathematics education: a systematic review on CT-based mathematics instruction and student learning. *International Journal of STEM Education*, *10*, Article 3. https://doi.org/10.1186/s40594-023-00396-w

Yilmaz, Z., Galanti, T., Lawler, B. & Alegre, F. (2023a). The power of computational thinking in mathematics and data science education. In T. Lamberg & D. Moss (2023). *Proceedings of the 45th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (vol. 2, pp. 674–676). Reno, NV: University of Nevada.

Yilmaz, Z., Williams, K, Alegre, F., Kendrick, R & Moreno, J. (2023b). Teachers' understanding of synergies between computational and mathematical thinking after a summer professional development. In *Proceedings of the 7th APSCE International Conference on Computational Thinking and STEM Education* (pp. 13–19). Taiwan: National Central University

Youcubed K-12 Data Science. (2020). *Unit 3: Measures of Center & Spread*. https://www.youcubed.org/wp-content/uploads/2020/05/Unit-3-v1.pdf