Student reasoning about determinants with GeoGebra

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Linear algebra is a rich setting for students to explore geometric and conceptual connections between determinants, linear transformations, and other topics such as linear dependence and invertibility. The Inquiry-Oriented Linear Algebra (IOLA) determinants task sequence leverages students' knowledge of linear transformations in $\mathbb{R}^2$ and $\mathbb{R}^3$ to support an understanding of determinant as a measure of (signed) multiplicative change in the area or volume caused by the transformation. The sequence includes GeoGebra applets designed to support students' explorations of how the matrix components relate to the transformed object's graph and to the determinant; the applets' dynamic feedback also facilitates student conjecturing and generalizing about determinant properties. Our research question is: Through interacting with the applets, what relationships do students observe between matrices, determinants, and geometric objects transformed by the associated linear transformation?

Theoretical background and literature

Adopting the instructional design theory of Realistic Mathematics Education (Freudenthal, 1991), the IOLA curricular materials build from experientially real task settings to foster active student engagement in the guided reinvention of mathematics through student and instructor inquiry (Rasmussen & Kwon, 2007). Our research is guided by the Design Research Spiral (Wawro et al., 2022), in which we iteratively draft, implement, and refine task sequences, guided by various design research theories at each phase. Dynamic geometry software (DSG) can help students investigate, visualize, predict, and generalize certain situations (e.g., Gol Tabaghi & Sinclair, 2013), which are central to inquiry. DSG applets can also help students make connections to their existing geometric and algebraic knowledge. For example, Donevska-Todorova (2012) found that DGS's ability to simultaneously display numeric and geometric feedback helped students connect to their prior geometric understanding and build connections between the 2x2 determinant and conceptions of area.

Methods

This poster focuses on students' work on Task 4 of the IOLA determinants task sequence (US NSF DUE #1915156, 1914841, 1914793), in which students explore the geometric interpretation of matrix transformations and their determinants via GeoGebra applets for 2x2 and 3x3 matrices. Each applet consists of a matrix, sliders to change the matrix entries, the corresponding determinant value, and a dynamic parallelogram or parallelepiped showing the image of the unit square or cube under the matrix transformation. The participants were 13 volunteers (mostly second-year engineering students) from an introductory linear algebra class at a US research university. The data are written responses to a prompt that asked students for at least two observations they made from using the applets and initial ideas for why the observations seemed sensible. We first coded phrases in students'

responses that align with applet features, namely changes in: the matrix entries [M], the determinant value [D], or the graph of the transformed object [G]. Second, we coded phrases paired with logical structure and supportive justifications as observations. For example, "when I change this entry in the matrix, the determinant stays the same" was coded as an [M⇒D] observation. The authors independently analyzed the data, reaching near-total agreement.

Results and Discussion

In our analysis, we found 43 [M], 41 [D], and 24 [G] phrases, as well as 42 observations, 37 of which began with [M]. The most common observation type was [MDG], meaning "[M] implies [D], which makes sense because of [G]." For example, one student wrote, "If any of the columns in the 3x3 are LD with another, the det(M) will be zero due to there being no volume, only area or a line." As design researchers, reflecting on student data in light of the overall learning goals informs our revisions to the task sequence. For example, it is sensible that most student observations began with [M] because the most interactive aspect of the applet was adjusting the matrix component values. This potentially hindered students from conjecturing the biconditionality of the determinant generalizations. Thus, we refined the applets to include two additional features: swapping rows and columns, and dragging the graphed images of the basis vectors that define the parallelepiped or parallelogram. We conducted four interviews with 2-3 students in March 2023 to investigate what observations and generalizations the students made with the refined applets. In addition to summarizing some results from Wawro et al. (2023), this poster will present results from the new data that used the refined applets. In addition to analyzing this new data, future work includes exploring and theorizing the alignment between specific aspects of RME, such as emergent models, and student exploration and engagement in DGS.

References

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