Transformations in the Plane: Towards Interpretations and Proofs of Linearity

Anairis de la Cruz Benito
Florida State University

Christine Andrews-Larson
Florida State University

Keywords: linear algebra, linear transformations, proof

In recent years, student reasoning in linear algebra has been a rich area of research, but relatively few studies have focused on linearity and proof in this area (Stewart et al., 2019). In this study, our research questions are: (1) What are students' evoked concept images of linear transformations in the context of a Desmos module focused on transformations in the plane? (2) What do students report finding helpful in this module?

Existing literature has examined how students conceive of linear transformations in relation to their interpretations of functions in prior coursework, and how students shift from local to global views of linear transformations (Zandieh et al., 2017; Andrews-Larson et al., 2017), but there is limited work on how students make sense of the properties of linearity, particularly in relation to their proof activity. For our analysis, we draw on the notion of concept image from Tall and Vinner (1981) who described a person's concept image for a particular concept as "the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes" (Tall & Vinner, 1981, p. 152).

The data for this report comes from Desmos activity completed by 54 U.S. undergraduate linear algebra students at the end of a unit on linear transformations. The goals of this activity were to help students identify and prove whether or not given transformations in the plane were linear. We analyzed our data by first developing data-driven codes (deCuir-Gunby et al., 2011) for students' responses to the questions "Explain in your own words what it means for a transformation to be linear and why it matters whether a transformation is linear," and "Which of the examples or slides in this Desmos activity did you find most helpful for your learning and why?" We aim to develop theory-driven codes to relate our findings to those of Zandieh et al. (2017), particularly identifying aspects of students' concept images of linear transformations in terms of: computations, properties, and clusters of metaphorical expressions.

We identified three broad categories of responses regarding the meaning of linear transformations that relate to students' concept images: references to the formal definition, preservation of straightness and parallelism of lines, and references to uniformity in how an image is transformed (e.g. points affected equally, retaining proportional changes in position and magnitude, reversibility, and preservation of orientation). We identified four primary aspects of the Desmos activity that students reported finding helpful: features that helped students develop intuition for linear transformation (e.g. via visualization and distinguishing examples from non-examples), features that helped students relate algebraic to geometric interpretations of linear transformations, and features that helped students make sense of the formal definition and proof methods (e.g., separating out interpretations of the two properties of linear transformations; providing examples of correct proofs and prompts to write a proof). In our poster, we will examine relationships among the meanings students ascribe to linear transformations, what students found helpful, and existing literature.

References

Andrews-Larson, C., Wawro, M., & Zandieh, M. (2017). A hypothetical learning trajectory for conceptualizing matrices as linear transformations. *International Journal of Mathematical Education in Science and Technology, 48*(6), 809–829. http://dx.doi.org/10.1080/0020739X.2016.1276225

DeCuir-Gunby, J. T., Marshall, P. L., & McCulloch, A. W. (2011). Developing and using a codebook for the analysis of interview data: An example from a professional development research project. *Field Methods*, *23*(2), 1361–55. https://doi.org/10.1177/1525822X10388468

Stewart, S., Andrews-Larson, C., & Zandieh, M. (2019). Linear algebra teaching and learning: themes from recent research and evolving research priorities. *ZDM*, *51*, 1017–1030. https://doi.org/10.1007/s11858-019-01104-1

Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics, with special reference to limits and continuity. *Educational Studies in Mathematics, 12*(2), 151–169.

Zandieh, M., Ellis, J., & Rasmussen, C. (2017). A characterization of a unified notion of mathematical function: the case of high school function and linear transformation. *Educational Studies in Mathematics, 95*(1), 21–38. https://doi.org/10.1007/s10649-016-9737-0

Transformations in the Plane: Towards Interpretations and Proofs of Linearity

Anairis de la Cruz Benito ad19cc@fsu.edu
Christine Andrews-Larson

NSF grant #1914793

MOTIVATION

In recent years, student reasoning in linear algebra has been a rich area of research, but relatively few studies have focused on linearity and proof in this area (Stewart et al., 2019).

RESEARCH QUESTIONS

(1) What are students' evoked concept images of linear transformations in the context of a Desmos module focused on transformations in the plane?

(2) What do students report finding helpful in this module?

PARTICIPANTS

53 undergraduate linear algebra students.

METHODS

We analyzed developed data-driven codes (deCuir-Gunby et al., 2011) for students' responses to prompts in the Desmos module.

Take a picture to access the Desmos unit on Linear Transformations

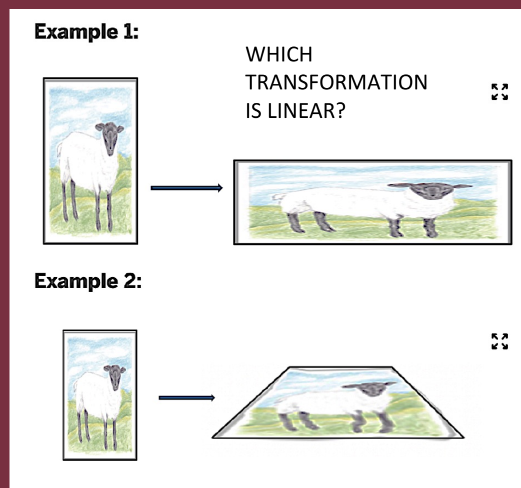
WHAT DOES IT MEAN FOR A TRANSFORMATION TO BE LINEAR?

P005: This means that the order of operations of T(V1) + T(V2), and T(V1 +V2) does not matter. You can transform then add or add then transform. Also applies to scaling with constant c. Scale before the transformation or after, it should be the same in a linear transformation.

P009: For a transformation to be linear, it has to be able to be written linearly in an algebraic form. This is made evident in the formal definition.

P0028. A linear transformation is a transformation that keeps the origin the same, straight lines remain straight, and parallel lines remain parallel. It also follows the properties T(V1+V2) = T(V1)+T(V2) and cT(u)= T(cu). It matter whether a transformation is linear because it helps us understand the nature of transformation and it does not completely change the original thing.

P0018. A transformation is linear when a vector's position relative to other vectors is the same from before the transformation to after, both additively and multiplicatively. It matters because it means the transformation is reversible, and that the relationships between vectors are maintained from before to after the transformation.



Visualization can help students make sense of the formal definition of linear transformations; students take away a variety of interpretations from visualization activities.

RESULTS

We identified three categories of responses that relate to students' concept images based on:

- references to the formal definition,
- preservation of straightness and parallelism of lines, and
- references to uniformity in how an image is transformed (e.g. points affected equally, retaining proportional changes in position and magnitude, reversibility, and preservation of orientation).

Based on student reports, useful features of the module were those that helped students:

- develop intuition for linear transformations (e.g. via visualization; distinguishing examples from non-examples),
- relate algebraic to geometric interpretations of linear transformations, and
- make sense of the formal definition
- model appropriate proof methods (e.g., separating interpretations of two properties of linear transformations; providing examples of correct proofs)

SIGMAA on RUME
MAA
MATHEMATICAL ASSOCIATION OF AMERICA

FLORIDA STATE UNIVERSITY
VIRES ARTES MORES
1851

1471