A COLLEGE ALGEBRA INSTRUCTOR'S TRANSITION FROM PROCEDURAL TO CONCEPTUAL THROUGH THE CODESIGN OF FORMATIVE ASSESSMENTS

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This study explores how an instructor designing assignments in an online homework system for a college algebra course considers the development of procedural fluency in connection with conceptual foundations. Findings reveal shifts in the instructor's perspective, highlighting the importance of connecting procedural fluency to conceptual foundations. The study underscores the potential of co-design activities in reshaping instructors' beliefs and instructional practices, offering insights into enhancing mathematics education in college algebra courses.

Keywords: Instructional Activities and Practices; Undergraduate Education; Technology

The instruction of college algebra courses has long been a concern given the barrier it presents for many students (Tunstall, 2018) – both those for whom it acts as a terminal math course and those who need it as a prerequisite for another course within their major. Mathematics education organizations have long called for undergraduate mathematics instructors to support students' deeper engagement with mathematical ideas and habits of reasoning (e.g., NCTM, 1980; AMATYC, 2006; MAA, 2018). This means attending to students' conceptual understanding, procedural fluency, and how the latter builds on the former. In the Mathematical Association of America (MAA) *Instructional Practices Guide* (2018), the authors noted,

> Conceptual understanding involves knowing what to do and why it works, while procedural fluency involves deciding and knowing how to do it…. When students learn procedures in such a way that they are connected to conceptual foundations, they will have more success in using these procedures, will recall them for a longer period of time, and will be able to use these procedures flexibly and effectively in a problem-solving situation. (p. 42)

In undergraduate mathematics courses, like college algebra, there is widespread emphasis on traditional lecture focusing on procedural knowledge, rather than conceptual understanding (e.g., Duffin et al., 2019; Khasawneh et al., 2023; Veith et al., 2023).

College algebra courses are typically quite large, often having well over 50 students enrolled in a single section. In these contexts, web-based homework systems are often used to support students and instructors by providing immediate feedback to both. Research has shown that these systems can make learning more active and adaptive while also focusing on improving conceptual understanding and problem-solving skills (e.g., Porter et al., 2015; Rochelle et. al, 2016; Twigg, 2009). However, there is little research on how instructors might use these systems to improve their practice – especially as it relates to supporting their students in learning procedures in ways that are connected to conceptual foundations. The purpose of this study is to answer the following research question: How does an instructor who is designing the

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assignments and supports for an online homework system for a college algebra course consider the development of procedural fluency as connected to conceptual foundations?

Context of the Study

This study is in the context of a larger project in which the instructor is leading the co-design of a set of online homework problems – and their supports – for undergraduate college algebra using the online homework system ASSISTments (www.assistments.org). Online homework systems are commonly used in college mathematics courses (e.g., ALEKS (Hagerty et al., 2005), MyMathLab (Duffin et al., 2023), WeBWork (Roth et al., 2008)), in addition to learning management systems that have mechanisms for creating online assignments (e.g., Canvas, Moodle). These programs are similar in that they make assigning homework problems easy, students can resubmit multiple times until their response is deemed correct, and students are given immediate feedback with respect to the correctness of their final answers. However, in addition to this immediate feedback, students also have the ability to access carefully designed supports. What sets ASSISTments apart are the supports that instructors can add at the problem level (up to 3 different supports), rather than just using publisher-supplied supports available to students as they work on assigned problems. Supports can be in the form of videos, worked solutions, or scaffolding hints. While ASSISTments has been used widely at the middle school level (Feng and Heffernan, 2006; Heffernan et al., 2012; Heffernan and Koedinger, 2012), this is the first use of it in undergraduate college algebra meaning that though the platform exists, the problems and supports for those problems needed to be created.

Methods

This is an intrinsic case study (Yin, 2018) of one college algebra instructor, Michael (the third author of this paper), who is also leading the co-design of the college algebra assignments in ASSISTments at a large southeastern university. This unique context provides an opportunity to learn about how Michael is making sense of "building procedural fluency from conceptual understanding" (MAA, 2018, pg. 42) through co-design (Severance et al., 2016).

The data for this study include a series of interviews as well as analysis of artifacts used in the creation of assignments and supports to use in ASSISTments assignments. Michael took part in 6 semi-structured interviews over the course of a year. The interviews were between 30 and 60 minutes long, took place via Zoom, and were recorded. Artifacts include Google documents and sheets in which Michael kept meeting notes and planned the problems he was going to add to an ASSISTments assignment as well as the associated supports. Given that the rest of the local research team is made up of mathematics education researchers, Michael's practice was to ask for their feedback on all supports as well as problems he had labeled as "conceptual" in nature. As such, discussions about these designs occurred in the shared documents using the "comment" features.

Interview transcripts and design artifacts were coded (by the first two authors) for *attending to conceptual foundations*. All of the quotations assigned this code were then read for emerging themes. Throughout this process, the research team would share the emerging findings with Michael to get his feedback and to make sure we were representing his ideas appropriately. Those emerging themes are what we report on in this study.

Preliminary Findings

In what follows, we share three phases of Michael's ongoing development: his stance that conceptual understanding is not a priority, coming to understand what conceptual understanding of a procedure means, and advocating for making connections between procedures and their conceptual foundations.

Conceptual Understanding is Not a Priority

Early in the project, Michael repeatedly noted that attending to the connections between procedures and their conceptual foundations was not a priority for him. This was evident in the design of both the questions he included in the ASSISTments assignments and their associated supports. For example, approximately 6 months prior to the pilot semester, Michael and his team began identifying problems to include in the first ASSISTments assignments. In one of these early meetings, he shared that he knew he needed to include what he referred to as "conceptual problems", but he was not sure what that looks like. He mentioned many times that he was only including this type of problem because he thought he had to due to the goals of the overarching project. He did not think they were a priority because they are not tested on the course common exam. Michael explained, "I don't think I would highly emphasize any feedback that I give for conceptual questions in the current course format for College Algebra. Conceptual questions tend to not be emphasized on the common final." He also shared that "using space on my tests in order to ask the conceptual questions will take that space away from the procedural questions that are going to be on the final."

Coming to Understand What Conceptual Understanding of a Procedure Means

As Michael started to draft what he referred to as "conceptual problems", there is evidence that he felt he was learning what conceptual understanding of a procedure might mean. When he drafted these items, there was a lot of back and forth with the mathematics educators on the team about what makes a question "conceptual". For example, the first draft of a "conceptual question" for a unit on exponents stated, ″When can you subtract two exponents?" After some discussion about the fact that the question is simply asking students to identify when they can use a particular procedure, and was missing the conceptual "why", Michael revised the question to, "Provide an example of when subtracting two exponents is appropriate. Explain your reasoning." This revised question is focused on procedural fluency - recognizing when a procedure is appropriate to use - and by asking students to explain their reasoning it is asking for the underlying concept as well.

Figure 1: Sample Supports for an ASSISTments Item

Support Draft 1	Support Draft 2
Similar to the graph of a parabola, the quadratic formula, $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$, can determine the solutions to a quadratic equation. The discriminant, b^2-4ac, is part of the quadratic formula. The value of the discriminant indicates how many times the parabola intersects the x-axis and how many real solutions a quadratic equation has.	As a reminder, the discriminant, b^2-4ac, is the part of the quadratic formula that is under the square root symbol, $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$. Given a square root can have two (if > 0), one (if = 0), or no real solutions (if < 0), the value of the discriminant determines how many solutions there are for a quadratic equation.

If the discriminant is positive, i.e., b^2-4ac > 0, how many solutions does the quadratic equation have?	If the discriminant is positive, i.e., b^2-4ac> 0, how many solutions does the quadratic equation have? Write the answer as a number.

At the same time as he was creating the questions in the assignments, Michael was also creating the supports for those questions. We saw a similar shift in this work. For example, when designing a support for the problem: "Based on the discriminant, state how many real solutions there are to the following quadratic equation: $2x^2 - 6x + 7 = 0$." Michael's first support draft was very procedural, asking students to recall a rule they learned about the discriminant (see the example on the left side of Figure 1). After some back-and-forth discussion with the mathematics educators to identify the foundational concept – that a square root can have one, two, or no real solutions, he revised the support to include the conceptual foundation (Figure 1 right side). Michael later noted that this back and forth was helping him understand what conceptual understanding means in the context of a procedure-heavy course like college algebra.

Advocating Connecting Procedures to their Conceptual Foundations

After a full semester of pilot and design work, there was evidence that Michael not only had a deeper understanding of how to develop procedural fluency from conceptual understanding but also thought it was important. Michael shared that based on his engagement in this design work, he has started to change his practice. He explained, "I started to really think about how I should give an explanation in class…I've gotten better over time, I think my instruction has gotten better." At the same time, there is evidence that Michael has begun to advocate for an emphasis on providing conceptual foundations in the supports for ASSISTments assignments as well. He explained that he reached a point where he thought "those explanations I was giving a class are just so much better than the ones that we were putting into the assignments." This made him realize that,

> If I had to give this to my students, I would want to make sure that they had a support that was the closest thing to what I could provide them if they were actually in person with me…So that's why I started saying that these things need to be better. These things can't just be railroaded through and just kind of shoved down their throats. They actually need to understand what is going on.

Michael is currently redesigning many of the ASSISTments assignments to align with what he now understands about the importance of connecting procedures to their conceptual foundations.

Discussion and Conclusion

The case of Michael provides an interesting example of how engaging in co-design, like designing the ASSISTments assignments and supports, might shift instructors' beliefs about the importance of attending to the conceptual foundations of procedures when developing procedural fluency in a college algebra course. We emphasize the co-design aspect of such work, as it seems as if the interaction with mathematics educators was an important part of Michael's journey. Our results are consistent with research on curricular co-design efforts (e.g., Severence et al., 2016). We recognize that engaging in this kind of design is not something that all instructors get to do, but the findings here suggest it might be helpful to consider ways of possibly engaging them in similar co-design activities. In fact, unlike online homework products that are prepopulated with

questions, ASSISTments allows all instructors to add their own questions and supports making such work possible. We are curious about how instructors who engage with the assignments and supports that Michael has created will, or will not, take up his stance on the importance of including the conceptual foundations – consistent with MAA (2018) recommendations – in their own explanations and attention to student responses on conceptual questions within ASSISTments to inform their instruction. This is an area ripe for future research.

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