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DEVELOPING ADAPTIVE EXPERTISE: EXAMINING OPPORTUNITIES FOR REFLECTIVE PRACTICES DURING A STUDIO DAY CYCLE

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Engaging teachers in reflective practices is recognized as a crucial component of their adaptive expertise development. Drawing on this perspective of adaptive expertise development, we qualitatively examined how the design and structure of a Studio Day professional learning cycle afforded opportunities for reflective practice for secondary in-service mathematics teachers. We found that small group reflections, immediate reflections-on-action, and the use of videos afforded notable instances of reflective practices throughout the Studio Day Cycle that supported teachers' development of adaptive expertise of equity-based, language-responsive teaching. We suggest that Studio Day Cycles are one avenue to better support in-service teachers' development of adaptive expertise of mathematics language routines and multilingual learner core practices.

Keywords: Professional Development, Middle School Education, Equity, Inclusion, and Diversity

Supporting adaptive expertise development is one way the field can better prepare mathematic teachers to face the evolving challenges of teaching and attend to the diverse and emergent needs of students (Anthony et al., 2015). To effectively respond to the increasing linguistic diversity and growing proportion of multilingual learners in K-12 classrooms (Meyer et al., 2020), we contend that an adaptive expertise of equity-based, language-responsive pedagogy positions teachers to support all students in mathematics, especially multilingual learners (Roberts & Olarte, 2023). Despite the growing focus on equity-based pedagogies and curriculum for multilingual learners (e.g., de Araujo & Smith, 2021), the existing scholarship base on adaptive expertise has generally focused on pre-service teachers (e.g., Anthony et al., 2015). We argue that it is equally important to examine and identify best practices to support in-service teachers' development of adaptive expertise of mathematics instruction for multilingual learners. In the present work, we report on how teachers' participation in a professional learning cycle, Studio Days (Von Esch & Kavanagh, 2018), focused on mathematics language routines (Zwiers et al., 2017) and multilingual learner core practices (Roberts & Olarte, 2023) supported opportunities for the development of adaptive expertise of equity-based, language-responsive mathematics teaching.

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Engaging teachers in reflective practices (e.g., Cavanagh & Prescott, 2010) – broadly referring to opportunities that teachers can be reflective and metacognitive about their teaching practices – is recognized as an integral dimension of adaptive expertise development (Anthony et al., 2015; Wetzel et al., 2015). Tsui (2009) asserted that “the process of reflection and conscious deliberation in which practical knowledge is theorized and theoretical knowledge is interpreted in practice” (p. 437), is how teachers develop adaptive expertise. Moreover, effective professional learning programs have been characterized as those that allow teachers to explore, inquire, experiment, and reflect (Wise et al., 1999). Although many scholars have reported on professional learning interventions that prompt mathematics teachers to engage in reflective practices and inquiry (e.g., Gningue et al., 2014), there is still limited research examining professional learning efforts that specifically encourage teachers to reflect on mathematics instruction for multilingual learners (de Araujo et al., 2018). The research question that guided this study was: How did opportunities for reflective practices within a Studio Day Cycle support mathematics teachers’ development of adaptive expertise of mathematics instruction for multilingual learners?

Conceptual Framework

We draw on reflective practice (i.e., Muir & Beswick, 2007) and adaptive expertise (Yoon et al., 2015) to share how the design and structure of the Studio Day Cycle (Von Esch & Kavanagh, 2018) afforded teachers opportunities to engage in reflection that supported their adaptive expertise development.

Reflective Practice

Existing literature has widely emphasized the importance of teachers being metacognitive about their practice and how looking inward and reflecting on that practice is crucial to their development and change (Cavanagh & Prescott, 2010). Hayden et al. (2013) wrote, “Reflection on critical incidents in teaching and on feedback received can become the catalyst for transformative change in teaching practice” (p. 144), highlighting the salience of teachers both considering important events in their classrooms and receiving support to unpack those events. Encouraging teachers to reflect on their students’ use of language in mathematics classrooms and their current language-responsive practices within a professional learning community can be conducive to developing their adaptive expertise. We align our work with Muir and Beswick (2007), who conceptualized reflective practice as “reflection that is deliberate and can be focused on events or incidents, and personal experiences” (p. 77). They offered a three-level model to examine in-service teachers’ reflective practices: (1) *technical description*, or teachers recalling general accounts of classroom practices, focusing on technical aspects, and omitting value judgements to the experiences; (2) *deliberate reflection*, or teachers identifying ‘critical incidents’ and providing rationales for past and future actions; and (3) *critical reflection*, or teachers moving beyond identifying ‘critical incidents’ to consider others’ perspectives and offer alternatives. We consider these forms of reflective practices to uniquely support teachers’ development of dimensions of adaptive expertise.

Adaptive Expertise

Adaptive expertise broadly refers to the process of teachers’ recognizing and identifying emergent needs, making sense of multiple perspectives, and orchestrating multiple teaching approaches to meet the demands of different situations (Hatano & Inagaki, 1984; Yoon et al., 2015). We utilize Yoon et al.’s (2015) characterization of three dimensions of adaptive expertise Kosko, K. W., Caniglia, J., Courtney, S., Zolfaghari, M., & Morris, G. A., (2024). *Proceedings of the forty-sixth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Kent State University.

to describe teachers' mathematics instruction more explicitly for multilingual learners (See Table 1). Like other scholars who have brought the theories of reflective practice and adaptive expertise together (e.g., Anthony et al., 2015; Tsui, 2009), we consider opportunities of reflective practice as conduits of teachers' adaptive expertise development. With our goal of supporting the development of their adaptive expertise of mathematics instruction for multilingual learners, we aimed to engage in-service teachers in a myriad of reflective practices within a Studio Day Cycle.

Table 1: Adaptive Expertise of Mathematics Instruction for Multilingual Learners

Dimension	Description
Flexibility	Awareness of students and context, particularly multilingual learners. Ability to constantly adapt practice and respond to unexpected issues as related to students' needs, particularly multilingual learners.
Deeper Level of Understanding	Able to assimilate information and to implement or make connections that builds or addresses deeper level of knowledge. Able to bring in variations from outside the present system of activity as related to instruction for multilingual learners. Able to describe the affordances and constraints of mathematics language routines. Considers contexts in which to apply and integrate instructional practices for multilingual learners.
Deliberate Practice	Demonstrates an ability to show motivation, focus, and repeated effort to monitor their practice, and devises and subsequently attempts revamped attempts to improve implementation, as related to multilingual learners. Improves, assesses, and reflects on their own and others' implementation of language-responsive practices. Explicit evidence of reflecting on how to improve as related to mathematics language routines and other language-responsive practices. Describes how they are motivated to continue to develop their practice.

Method

Context

The present study is part of a large, multi-year funded project focused on supporting the development of mathematics teachers' adaptive expertise of mathematics language routines and data science instruction in a school district in the West Coast of the United States. We designed our professional development intervention around Studio Days (Von Esch & Kavanagh, 2018; See Figure 1 below). Adapted from Lesson Study, two teachers develop and study a single lesson (not necessarily the same lesson/content). However, their lessons are focused on the same focal mathematics language routine (MLR) paired with a multilingual learner core practice (Roberts & Olarte, 2023). Other teachers observe a live enactment of the lesson and reflect on the observed lesson. In the 2023-2024 academic year, we planned three Studio Day Cycles with participating

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teachers. Each Studio Day Cycle began with a Pre-Studio Day where the research team introduced the focal MLR and multilingual learner core practice. We outlined the stages of the routines, highlighted considerations for enacting the routines, and participating teachers experienced the routines as students. About one week after the Pre-Studio Day, two teachers volunteered to enact the focal routine in their classroom during the Studio Day, and the research team and other participating teachers observed these classroom enactments. The teachers then debriefed their enactment following their lesson, briefly sharing their experience, and receiving feedback from the observers. Then, about a week after the Studio Day, we reflected on that experience in a Post-Studio Day, where teachers reflected on video clips of the enactments, copies of student work from the Studio Day, and the MLR and multilingual learner core practice.

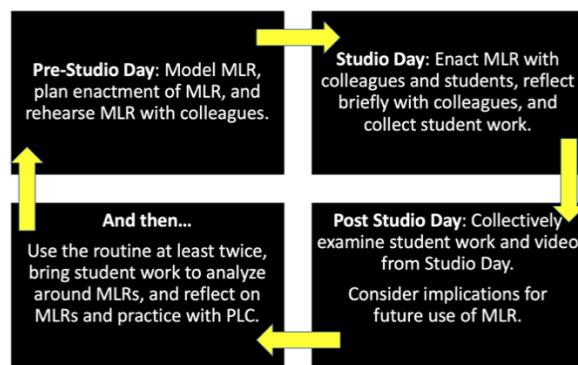


Figure 1: Studio Day Professional Learning Structure

Participants

The research team designed the Studio Day Cycle for the 2023-2024 academic year. Purposeful sampling (Miles et al., 2020) was used to recruit the district's mathematics instructional support specialist, three Math 7 teachers, and two Math 8 teachers from the three district junior high schools (See Table 2). For the present study, we report on the first Studio Day Cycle, where we focused on the mathematics language routine *Collect & Display*, paired with the multilingual learner core practice: *identifying disciplinary language demands and supports*. In *Collect & Display*, teachers capture students' oral words, ideas, phrases into a stable reference. The intent of the routine is to stabilize students' language in order to use their output as a reference for developing their mathematical language (Zwiers et al., 2017). The multilingual learner core practice of identifying disciplinary language demands and supports, refers to teachers employing or identifying language supports for students. They also adequately scaffold or produce language while attending to aspects of language that may be challenging for students (Aguirre & Bunch, 2012).

Table 2: Participant Profiles

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Name (pseudonym)	Grade Level	Race/ Ethnicity (self- described)	Years Teaching Mathematics	Bilingual/ Multilingual (self- described)
Ms. Ruth	7th grade	White	26	—
Ms. Severn	7th grade	Caucasian	4	Semi Fluent in Spanish
Ms. Taylor	7th grade	White	34	—
Ms. Foster	8th grade	White	3	Russian
Ms. Penny	8th grade	White	2	—
Ms. Hope	Instructional Specialist	Caucasian	25	—

Data Collection and Analysis

We video- and audio-recorded each meeting of the Studio Day Cycle, and utilized the videos and transcriptions of each meeting to examine the types of reflective practices that teachers engaged in. First, we created content logs of the videos to identify notable instances of reflective practices. We then coded (Miles et al., 2020) the transcriptions for the type of reflective practice using Muir and Beswick's (2007) three-level model. This allowed us to describe how the participants reflected during the Studio Day Cycle. Next, we drew on Yoon et al.'s (2015) characterization of the three dimensions of adaptive expertise (again, see Table 1) to make sense of how the moments of reflective practice supported teachers' development of adaptive expertise. We met as a research team to discuss themes that we observed in the data and wrote analytic memos (Miles et al., 2020) to better understand how the opportunities of reflective practices within the Studio Day Cycle afforded or constrained teachers' adaptive expertise development.

Findings

To illustrate how the reflective practices within a Studio Day Cycle supported teachers' adaptive expertise development, we describe the structure of each day of the cycle and highlight the notable instances of reflective practices taken up by the participants.

Pre-Studio Day

Our goal for the Pre-Studio Day was to introduce teachers to the focal mathematics language routine Collect & Display, and to the multilingual learner core practice, identifying disciplinary language demands and supports. Although we observed instances of all three types of reflective practice during the Pre-Studio Day, we found that teachers primarily engaged in *technical descriptions*. This was expected given that this first day of the Studio Day Cycle was designed to introduce teachers to the core practice and MLR, as well as to get a sense of how teachers noticed their students' language use in the classroom and to discern what they already did to support students to read, write, and speak about mathematics. For example, Ms. Foster said,

We've identified disciplinary language demands and supports, like making sure kids truly understand the words that we're saying mathematically. Like, just making sure if I'm saying "solve", what does that mean?...So, just making sure kids truly understand the words that we're saying and using their language to help bridge the gap [motions bringing hands together] between academic [language] and their every day [language].

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This *technical description* of her students' language demands and her practice of building on students' language also revealed Ms. Foster's *flexibility*, or awareness of students' needs.

Reflective practices in small groups. One exemplar opportunity for reflective practice during the Pre-Studio Day was placing participants in small groups and having them reflect on the following questions: (1) Why do students use language in mathematics classrooms? (2) How do your students use language and communicate in the mathematics classroom? (3) What tools do your students use to use language and communicate in the mathematics classroom? The questions were purposefully posed to get a general sense of what teachers presently noticed and understood about their students' use of language. They were first given time to think individually, and this afforded opportunities to engage in *technical descriptions*. However, once the teachers were discussing in the small groups, we observed instances where teachers engaged in *deliberate reflection* as they identified critical incidents of students using language in the classroom, and *critical reflection* as they expanded beyond identifying critical incidents and considered the perspectives and experiences of the members of their small group. They used a Jamboard to record their ideas and display them to the whole group, where they shared such ideas as: "They use hands or drawings to clarify because they don't have formal language, so we try to get them to have language to explain" and "language is used to explain their thinking and to clarify their understanding." These ideas illustrated that the teachers were able to work in a group with each other to develop generalizable ideas in their *critical reflections*.

During the small group time, teachers went back and forth sharing their ideas, and it was in these rich discussions that we observed teachers engage in both *deliberate reflection* and *critical reflection*. For example, in one small group we observed teachers collectively reflecting on critical incidents of students' language use, such as students gesturing or asking each other questions in the classroom. Then, Ms. Hope demonstrated evidence of *critical reflection* as she considered the perspectives of the other members of the group, synthesized their group reflections, and articulated that students' language could be broadly categorized as "input and output." In these reflective practices, we again primarily saw evidence of *flexibility*, as evidenced by awareness of students' language in mathematics.

Studio Day

The Studio Day occurred one week after the Pre-Studio Day. On this day, Ms. Ruth and Ms. Taylor enacted the routine Collect & Display in one of their class periods. The other teacher participants took on participant-observer roles during the lessons, walking around, taking observation notes, and interacting with students. The Studio Day began with a pre-brief of Ms. Ruth's lesson, where she provided details of her lesson plan, her classroom, and her expectations of what the research team/other teachers should do during her lesson. After Ms. Ruth enacted the routine, the participants and the research team met to debrief Ms. Ruth's lesson. After this debrief, we held a similar pre-brief for Ms. Taylor's upcoming lesson. The other teachers took on similar participant-observer roles during Ms. Taylor's classroom enactment, and at the end of the day, we debriefed Ms. Taylor's enactment of Collect & Display. The reflective practices of the Studio Day privileged *deliberate reflection* and *critical reflection* because teachers observed actual classroom enactments through which they identified and reflected on critical incidents shortly after each teachers' enactment.

Immediate reflection-on-action. Key reflective features of the Studio Day were the pre-brief sessions that oriented teachers to details of the upcoming lesson, classroom dynamics, and

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student engagement prior to observing Ms. Ruth and Ms. Taylor enact the routine with their students, and the debriefs immediately after the lesson that engaged teachers in reflection-on-action (Manrique & Abchi, 2015). Reflection-on-action refers to teachers' purposeful reflection after their practice, and in our debriefs, we prompted teachers to reflect on Ms. Ruth's and Ms. Taylor's enactment of Collect & Display. For the enacting teachers, we asked them questions such as: (1) How did you feel about the lesson?; (2) Did you consider multilingual students during your lesson?; and (3) How did you adapt in real time? For the observing teachers, we asked them questions such as: (1) What did we see students doing during Collect & Display?; and (2) How did students engage with disciplinary language demands? The debriefs allowed the teacher who just enacted the routine to reflect on their teaching practices immediately after class, and provided the other participating teachers opportunities to share insights and feedback based on their observation notes. Moreover, we purposefully oriented teachers' reflections using the MLR and paired multilingual learner core practice.

Again, we observed that teachers were able to take up all three reflective practices during the debriefs. For example, Ms. Ruth engaged in *technical descriptions* and *deliberate reflection* within her own enactment of the routine and shared that the students were "very engaged and [for] kids who have difficulty accessing [the problem], it [Collect & Display] gives them opportunities to access, because there's no penalty for getting it wrong." During this moment of reflective practice, we found that Ms. Ruth exhibited the dimensions of adaptive expertise *flexibility* and *deeper level of understanding*, because she demonstrated an awareness of her students, and she articulated an affordance of Collect & Display – mainly that it was a routine that allowed students to access the mathematics content and language. The immediate reflections-on-action in the debriefs also afforded the observing teachers valuable reflective practices that developed their adaptive expertise. For example, Ms. Severn engaged in *technical descriptions* and *deliberate reflections* as she praised Ms. Ruth's ability to connect students' informal language with the formal mathematics language. She explained, "Highlighting the ways that informal and formal language related to each other was, I think, a good way to marry the different levels of language that the kids need." In this reflection, Ms. Severn exhibited *flexibility* because of her awareness of students' language use, and she also exhibited a *deeper level of understanding* of practice of identifying disciplinary language demands and supports. Through her reflection of Ms. Ruth's lesson, Ms. Severn shared those explicit connections between students' informal and formal language supported their language needs in mathematics.

Post-Studio Day

We held our Post-Studio Day one week after the Studio Day and in between this time, teachers were encouraged to continue to use the MLR in their classrooms. Additionally, to prepare for the Post-Studio Day, the research team purposefully selected video clips from Ms. Ruth's and Ms. Taylor's classroom enactments of Collect & Display. We selected clips from two types of videos: a video from an iPad turned towards the front of the classroom (i.e., focused on the teacher), and videos from Ordro headband cameras that students were wearing during class. We selected moments that would allow for broad reflection, as well as those that would allow for purposeful reflection on teachers' language-responsive mathematics instruction. We found that because a goal of the Post-Studio Day was to discuss how teachers can build on the enactments of the Studio Day and how they can implement the routine in their own classrooms, the moments of reflective practices on this last day primarily encompassed *deliberate reflections* and *critical*

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reflections and afforded development of the *deeper level of understanding* and *deliberate practice* dimensions of adaptive expertise. Participants were able to articulate how they might integrate the routine in their classroom and demonstrated motivation and desire to improve future implementations of the MLR.

Reflecting on videos enactments. The notable opportunity for reflective practice of the Post-Studio Day was when teachers were shown the video clips of Ms. Ruth's and Ms. Taylor's enactment of Collect & Display. Overall, the teachers reported that this was an extremely valuable opportunity not only to see themselves teach, but seeing the videos from the student headband cameras provided new insight into how the students worked with their peers, how they spoke about mathematics, and how they made sense of the task. Both Studio Day focal teachers had utilized the Desmos curriculum (Amplify Education, Inc., 2024) to enact Collect & Display, and the teachers demonstrated evidence of *technical descriptions* and *deliberate reflection* as they articulated details of their enactments and provided rationale for features of their Desmos activity. For example, Ms. Ruth explained, "I don't think I've ever gotten as, as rich variety... What's different? But it's really nice that we have a way for kids to share their thinking that's safe because you can anonymize it." In this moment, we also observed Ms. Ruth's development of a *deeper level of understanding* of the routine Collect & Display, because she described an affordance of facilitating the routine specifically through Desmos – integrating her understanding of the goals of the routine with what Desmos affords for the students. Mathematics language routines are flexible and adaptable, and we observed that teachers developed their adaptive expertise because they described the affordances and constraints of using technology to enact the routine instead of traditional paper/written work.

Discussion and Conclusion

We found that the Studio Day Cycle afforded valuable opportunities for reflective practices that supported teachers' development of adaptive expertise of language-responsive, mathematics pedagogy. Over the course of the cycle, we found notable instances of participants engaging in all three types of reflective practices that supported their development of adaptive expertise. For example, we found that participants most often engaged in *technical descriptions*, consistent with how this reflective practice is considered a lower-level reflection (Muir & Beswick, 2007). Additionally, in the moments of *technical descriptions*, we found teachers to most exhibit and develop *flexibility* as they demonstrated increasing awareness of students or adapted their practice in response to students' needs. Importantly, we found that design features of the Studio Day Cycle privileged specific types of reflective practices – such as viewing videos of classroom enactments encouraging *deliberate reflections*, because participants were oriented to specific critical incidents, and small group reflections encouraging *critical reflections* as teachers considered each other's perspectives and ideas. This is consistent with existing literature on the value of supporting teachers to engage in reflection-on-action (Manrique & Abchi, 2015) as well as the use of video in teachers' professional development (i.e., van Es & Sherin, 2010). The present work demonstrated that professional learning interventions can curate *catalysts of reflective practices* that can specifically support teachers' development of adaptive expertise of language-responsive mathematics instruction. With reflective practices a critical component of adaptive expertise development (Anthony et al., 2015), we suggest that Studio Day Cycles are flexible, adaptable models for interventions that can provide in-service mathematics valuable

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opportunities to develop their adaptive expertise of equity-based, language-responsive pedagogies for multilingual learners.

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