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Legged Robot State Estimation within Non-inertial Environments

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I. INTRODUCTION

This work investigates the robot state estimation problem within a non-inertial environment. The proposed state estimation approach relaxes the common assumption of static ground in the system modeling. The process and measurement models explicitly treat the movement of the non-inertial environments without requiring knowledge of its motion in the inertial frame or relying on GPS or sensing environmental landmarks. Further, the proposed state estimator is formulated as an invariant extended Kalman filter (InEKF) [1] with the deterministic part of its process model obeying the group-affine property, leading to log-linear error dynamics. The observability analysis confirms the robot's pose (i.e., position and orientation) and velocity relative to the non-inertial environment are observable under the proposed InEKF.

II. METHODOLOGY

This section presents the proposed process and measurement models. The proposed InEKF aims to estimate the robot's position $\mathbf{p}_t$, velocity $\mathbf{v}_t$, and orientation $\mathbf{R}_t$ relative to the dynamic ground frame $\{D\}$, expressed in $\{D\}$. The references frame is shown in Fig. 1a). The sensors considered are an inertial measurement unit (IMU) attached to the robot, another one fixed to the dynamic ground, and joint encoders.

We express the state on a matrix Lie group $\mathcal{G} \subset \mathbb{R}^{9\times 9}$ as:

$$\mathbf{X}_t := \begin{bmatrix} \mathbf{R}_t & \mathbf{v}_t & \mathbf{p}_t \\ \mathbf{0}_{1,3} & 1 & 0 \\ \mathbf{0}_{1,3} & 0 & 1 \end{bmatrix}. \tag{1}$$

Let $\mathfrak{g}$ be the associated Lie algebra. Using the IMU motion dynamics, we obtain the process model as:

$$\frac{d}{dt}\mathbf{X}_t = -{}^D\tilde{\mathbf{U}}_t \mathbf{X}_t + \mathbf{X}_t {}^B\tilde{\mathbf{U}}_t + ({}^D\mathbf{w}_t)^\wedge \mathbf{X}_t - \mathbf{X}_t ({}^B\mathbf{w}_t)^\wedge, \tag{2}$$

where the isomorphism, $(\cdot)^\wedge : \mathbb{R}^{\dim\mathfrak{g}} \to \mathfrak{g}$, maps any vector $\boldsymbol{\xi} \in \mathbb{R}^{\dim\mathfrak{g}}$ to the $\mathfrak{g}$, and for $i \in \{D, B\}$

$${}^i\tilde{\mathbf{U}}_t := \begin{bmatrix} [{}^i\tilde{\boldsymbol{\omega}}_t]_\times & {}^i\tilde{\mathbf{a}}_t & \mathbf{0}_{3,1} \\ \mathbf{0}_{1,3} & 0 & 1 \\ \mathbf{0}_{1,3} & 0 & 0 \end{bmatrix}, \tag{3}$$

and ${}^i\tilde{\mathbf{a}}_t$, ${}^i\tilde{\boldsymbol{\omega}}_t$ are the linear acceleration and angular velocity data returned by IMUs, ${}^i\mathbf{w}_t$ is additive white Gaussian noise, $[\ ]_\times$ denotes the skew-symmetric matrix of a vector, and $\mathbf{0}_{m\times n}$ is an $m \times n$ zero matrix.

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The robot's stance foot position relative to $\{D\}$ is defined as $\mathbf{d}_t$. When the robot's foot has static contact with the ground of the non-inertial environment, the foot velocity satisfies $\frac{d}{dt}(\mathbf{d}_t) = \mathbf{0}_{3,1}$ [2]. Taking the first time derivative of the kinematics relationship associate with the leg odometry, the measurement model can be expressed as:

$$-\mathbf{v}_t + \left[{}^D\tilde{\boldsymbol{\omega}}_t\right]_\times \mathbf{p}_t = \left(\mathbf{R}_t \left[{}^B\tilde{\boldsymbol{\omega}}_t\right]_\times - \left[{}^D\tilde{\boldsymbol{\omega}}_t\right]_\times \mathbf{R}_t\right) s(\tilde{\mathbf{q}}_t) + \mathbf{R}_t \mathbf{J}(\tilde{\mathbf{q}}_t)\dot{\tilde{\mathbf{q}}}_t, \tag{4}$$

where $\mathbf{J}(\mathbf{q}_t)$ is the Jacobian of leg odometry $s(\mathbf{q}_t)$ and $\dot{\mathbf{q}}_t$ is the time derivative of the joint angle $\mathbf{q}_t$.

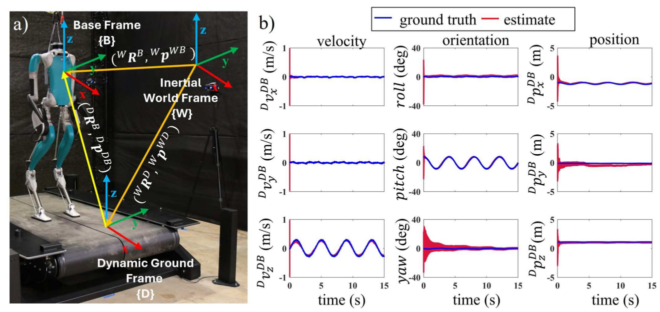

Fig. 1. Experiment setup and results. a) Reference frames and notations used in the filter derivation. b) Estimation results of the proposed InEKF.

III. RESULTS

To simulate the dynamic ground motion, the treadmill is programmed to simultaneously perform a sinusoidal pitch $10^\circ \sin\frac{\pi t}{2}$ and a sway motion $0.05\text{m}\cos\frac{\pi t}{2}$. During the experiment, robot stands on the treadmill. The experiment results in Fig. 1b) illustrates that the proposed filter exhibits a fast convergence rate and small estimation errors due to the explicit treatment of the ground motion. Notably, under the proposed filter, the robot's relative base yaw and position converge to the ground truth, supporting the observability analysis results that they are observable during ground motion. In contrast, the absolute yaw and position are not observable under the previous filter design [3].

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