Mining Gems to Inspire Teacher Reflection

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The authors present an instructional strategy designed to facilitate teacher reflection and focus teachers' attention and curiosity when students are working on problems.

Much like gem miners search for gemstones, teachers, too, can search for "gems" when students work on problems. Gems are students' mathematical contributions, which includes solution strategies or attempts, questions or statements as they work on problems, or complete solutions. Gem Mining is similar to monitoring, one of the 5 *Practices for Orchestrating Productive Mathematics Discussion* (Smith & Stein, 2018); however, we view the purpose of Gem Mining differently. Monitoring is "the process of paying attention to the thinking of students during the actual lesson as they work individually or collectively on a particular task" (p. 54). Monitoring is typically enacted within the full sequence of the 5 Practices—anticipating, monitoring, selecting, sequencing, and connecting (Smith & Stein, 2018)—with the ultimate goal of driving meaningful classroom discussions.

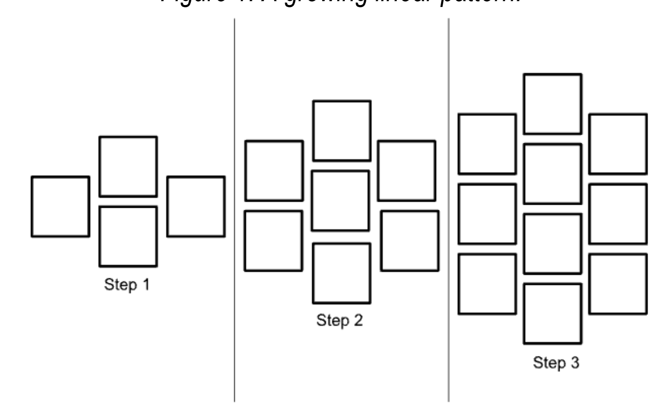
In contrast, Gem Mining is a standalone practice meant to facilitate teacher reflection and focus teachers' attention and curiosity as they circulate during class, especially when students work individually or in small groups. Unlike monitoring, where teachers search for student work to orchestrate a class discussion around a task, Gem Mining provides space for teachers to formatively assess students' thinking, reflect on current instruction, and consider what it might mean for future instruction. Gem Mining is small in scope, easy to implement, and is a practice that can be extended in accordance with teachers' goals but can just as easily be implemented as the name suggests, simply mining for gems. In this article, we offer three ways in which teachers can use Gem Mining in their classroom.

Searching for Similar Gems

One approach to Gem Mining is to focus on commonalities among students' work. This is like finding two gems of the same type. For instance, consider a growing linear pattern (Figure 1). As students[1] work to make sense of the task, a teacher might notice two students who decided to rearrange the blocks to make sense of the pattern (Figure 2).

Ali shifted the pattern up to align the top blocks for each step. In doing so, Ali recognized the original set of four blocks remained constant across the steps, as three new blocks were added at the bottom of each step. This led Ali to the solution $4 + 3(n - 1)$. Dionne also rearranged the blocks, but Dionne aligned the blocks across the bottom. Dionne noticed there was always one block on top and the rest of the blocks formed three columns that were the same height as the step number. This led Dionne to the expression $1 + 3n$. Although both Ali's and Dionne's final expressions were not in the same

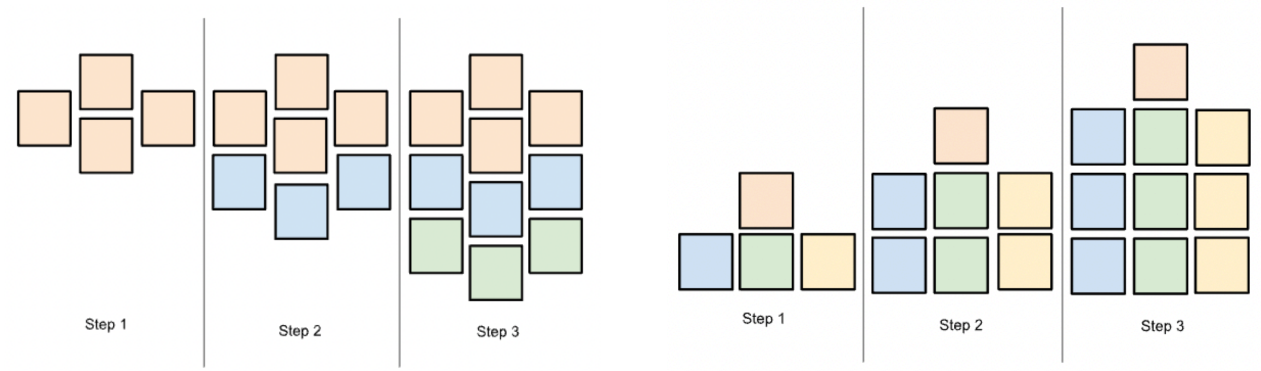
Figure 1. A growing linear pattern.



[1] *Note:* the student work pictured throughout the article is hypothetical student work, inspired by what the authors have experienced with students.

form, they were equivalent. The gem is that students thought to move the blocks to reveal the pattern.

Figure 2. Ali and Dionne's student work.



Ali *Dionne*

Finding similar gems can provide teachers with valuable insights into instruction and students' learning. If a teacher notices most students use the same strategy, it may indicate the teacher has funneled students toward a particular solution strategy. This might be on purpose. In the example above, if the teacher had emphasized the importance of moving blocks around to determine the pattern, then we expect students to take this approach. Finding many similar gems may also indicate students are most comfortable with a given strategy, meaning teachers may need to encourage other strategies. Knowing students' have taken similar approaches provides formative assessment data for the teacher to drive subsequent instruction.

Searching for Different Gems

Imagine a teacher went searching for gems as students worked on the problem, 5/3÷1/2. As the teacher circulated, they came across two students who used different strategies to get the correct answer. Jordin found common denominators and Rony instead inverted and multiplied. Both students used distinct, valid strategies to correctly solve the problem (Figure 3).

Sometimes multiple solution strategies can get lost. One particular strategy might get privileged due to who shared it, the mode by which it was shared, its ease of use, or a variety of other social or mathematical reasons. When this happens, opportunities for learning become less robust. Often, students believe there is just one way to solve a problem; however, if we want students to be flexible problem solvers, they need to have multiple solution strategies at their disposal. Searching for different gems is one way to allow for multiple solution strategies to surface. In the prior example with Jordin and Rony, they both got the answer of 10/3, yet they did so very differently. A teacher might consider the benefits of making both students' work public (with permission). They might reflect on their presentation of the material and, going forward, try to purposefully incorporate both solution strategies as options. Finally, a teacher might consider instances in which one solution strategy is a better choice than the other and think about how to promote this consideration with students. By actively searching for different gems, teachers can formatively assess their instruction in relation to students' thinking, ensuring that multiple solution strategies are given the space they deserve.

Figure 3. Jordin and Rony's student work.

$\frac{5}{3} \div \frac{1}{2}$	$\frac{5}{3} \div \frac{1}{2}$
$\frac{10}{6} \div \frac{3}{6}$	$\frac{5}{3} \cdot \frac{2}{1}$
$\frac{10 \div 3}{6 \div 6} = \frac{10 \div 3}{1}$	$\frac{5 \cdot 2}{3 \cdot 1}$
$\frac{10}{3}$	$\frac{10}{3}$
Jordin	*Rony*

Searching for a Unique Gem

A unique gem is a solution strategy that distinguishes itself from the others. It might be a novel approach to a task or a one-of-a-kind strategy within your class. Unique gems give teachers a great opportunity to anchor discussions. As an example, a teacher might search among the class for a novel approach to finding the area of a non-standard closed figure (Figure 4).

A common strategy might be to break the figure into common shapes (e.g., two rectangles and one triangle), find the area of each shape, and sum the areas (Figure 5).

Figure 4. A non-standard closed figure.

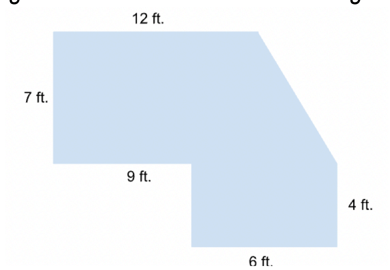
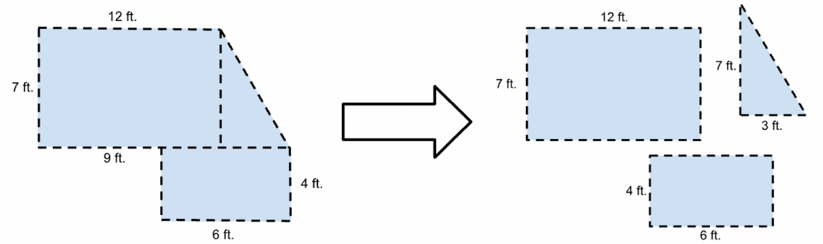


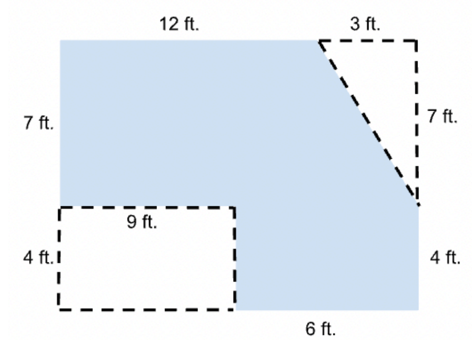
Figure 5. One strategy for finding the area of a non-standard closed figure.



Circulating around the room, a teacher might notice nearly all of the class used this decomposition strategy, except for one student, Fernanda. Rather than partitioning the irregular shape, Fernanda decided instead to imagine the completed rectangle and then account for the negative space. What makes this solution a unique gem is that it was novel in relation to the other solution strategies the students employed (Figure 6).

When teachers find a unique gem there are some questions worth considering. One is, do I understand the student's thinking? Answering this question might require initiating a conversation with the student, which in turn reinforces the idea students are knowledgeable and their ideas are valued in the classroom. When unique gems surface, they provide an opportunity for teachers to assign competence to students (Cohen et al., 1999), which can be generative for the classroom community. Another reflective question would be, is this a strategy that I want all my students to know and use regularly? In the example above, Fernanda used negative space to turn a non-standard closed figure into a more recognizable one, a 15 ft. by 11 ft. rectangle. This might be a strategy you want your students to add to their repertoire, so are there other problems that necessitate Fernanda's strategy in order to find the solution?

Figure 6. Fernanda's strategy for finding the area of a non-standard closed figure.



Gem Mining to Inform Instruction

Knowing which solution strategies are commonly or rarely used among students provides teachers with valuable formative assessment data and allows teachers to reflect on future instruction. Gem Mining allows for interesting or alternative strategies to surface, which might otherwise go unnoticed. It is worth noting in this article, we only shared correct student solutions. However, there is value in searching for incorrectly applied strategies or strategies that lead to incorrect or incomplete solutions. Errors are opportunities for learning and reflecting on instruction, so these gems are also valuable! A caution when using student errors is ensuring the student is not embarrassed when the error is shared. One way to do this is to recreate the work yourself to keep anonymity before sharing with students. To help teachers get started with this instructional nudge, we provide some reflection questions in the table below. Whether searching for correct or incorrect solutions, Gem Mining is a great way to surface students' ideas and gauge student thinking in the classroom.

Table 1. Reflection Questions and Follow-Up Actions

Cases	Reflection Questions
Similar Gems	• Did I funnel my students, intentionally or otherwise, towards a certain strategy? • Students seem most comfortable with this one strategy. Are there other options they should know? • Do I need to revisit a topic, highlight alternative strategies, or make adjustments based on what I'm noticing?
Different Gems	• How can I validate each student's gem as viable and productive? • Is there a particular distinction I want to highlight among different gems? • When would one strategy be preferred over the other?
Unique Gem	• Would other students benefit from engaging with this unique gem? • Do I understand their thinking? • Why might others not have used this strategy? • Is this a strategy I want all my students to know and be able to use regularly?

References

Cohen, E. G., Lotan, R. A., Scarloss, B. A., & Arellano, A. R. (1999). Complex instruction: Equity in cooperative learning classrooms. *Theory into Practice, 38*(2), 80-86. https://doi.org/10.1080/00405849909543836

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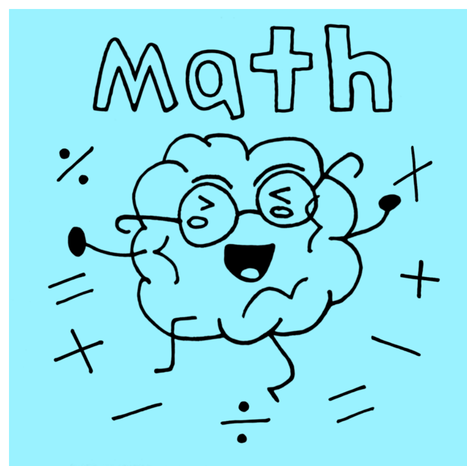
2024 NCCTM Logo Contest

Reported by James Sapp, Swannanoa, NC

Each Spring, NCCTM sponsors the Mathematics Logo Contest. The NCCTM Board selects the winning logo at its Spring meeting. The 2024 winning logo, pictured, will be available on shirts at the NCCTM State meeting in November.



We are pleased to announce this year's winner after a three-year hiatus!

Congratulations to

Luna Muro. Third Grade. Randleman Elementary School

Art Teacher Megan Clapp
Classroom teacher Katelynn Greene

Innovator Award Nominations

The North Carolina Council of Teachers of Mathematics accepts nominations for the Innovator Award at any time. The Committee encourages the nomination of organizations as well as individuals. Any NCCTM member may submit nominations. The nomination form can be obtained from the "awards" area of the NCCTM Website, www.ncctm.org. More information can be obtained from: Dr Ana Floyd, afloyd@randolph.k12.nc.us.

Problems2Ponder

Holly Hirst, Boone, NC

In each issue of The Centroid, Problems2Ponder presents problems similar to those students might encounter during elementary and middle school Olympiad contests.

Problem submissions: If you have an idea for a problem, email Holly Hirst (hirsthp@appstate.edu) a typed or neatly written problem statement, along with a solution. Include your name and school so that we can credit you.

Solution submissions: If teachers have an exceptionally well written and clearly explained correct solution from a student or group of students, we will publish it in the next edition of The Centroid. Please email Holly Hirst (hirsthp@appstate.edu) a PDF document of the solution, with the name of the school, the grade level of the students, and the name of the teacher.

Fall 2024 P2P Problems

Problem A: Amy picks a whole number, squares it, and then subtracts 1. She gives the number to Brian, who adds 3 to it and then doubles that result to arrive at a final answer of 54. What was Amy's original number?

Problem B: Three students are asked to choose two sides of a triangle and add the lengths. The three sums found are: 27, 32, and 35. Is this enough information to determine the perimeter of the triangle? If not, why not? If so, what is the perimeter?

Spring 2024 P2P Problem Solutions

Problem A: We call a number funny if it is the product of three prime numbers, two of which are the same e.g.,12 = 2x2x3 Is funny). How many funny numbers are between 30 and 60?

Solution: One way to determine this is to find the prime factorization for all numbers from 30 to 60. For example, 30 = 2x3x5, and so is not "funny." We can be a little more efficient, though, by listing factorizations for each prime that might be doubled and keeping those that are between 30 and 60, and we find four: 44, 45, 50, and 52.

Double 2: ~~2x2x2, 2x2x3, 2x2x5, 2x2x7~~, **2x2x11, 2x2x13**, ~~2x2x17~~ (too big)
Double 3: ~~3x3x2,~~ **3x3x5**, ~~3x3x5~~ (too big)
Double 5: **5x5x2,** ~~5x5x3~~ (too big)
Double 7: ~~7x7x2~~ (too big)

Problem B: 1x1x1 blocks will be placed in a 4x4x4 box with an open top. A red block is placed in the corner of the box. Then the least number of green blocks needed to hide the red block from view are placed in the box. Lastly the least number of yellow blocks needed to hide the green blocks from view are placed in the box. After this, how many blocks are in the box?

Solution: We need to place a green block on top of the red block to hide it, but we can also see the sides of the red block (as shown by the arrows), so we need two more green blocks (for the exposed sides). Similarly, we need 6 yellow blocks to completely hide the green block tops and sides, so there will be 10 blocks in the box.