Locking-It-In and Doubling: Mediating Second-Grade Students' Activity with Multiplicative Thinking

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Abstract

Using activity theory as a lens, we aimed to understand what second-grade students' interactions revealed about their thinking and what mediated students' engagement with important multiplicative ideas. In this setting, students interacted with multiplicative thinking using a coding robot and other artifacts as mediating tools. Through qualitative analysis, we found that students interacted with three concepts related to multiplicative thinking (i.e., composite units, doubling, iterating), and the lead mediators in their interactions included the robot's remote, dry erase marker and table, and peers/teacher. Students gravitated to artifacts that made sense to them, and the implication is that students need agency in opportunities to use artifacts and have interactions with rules and the community to make meaning of complex mathematical ideas.

Purpose

Our research is about children's doing and using of mathematics when integrated with computational thinking. Specifically, we conduct design studies of integrated math-coding activities with dynamic technological tools, such as coding robots, to study children's mathematical and computational thinking (Clarke-Midura et al., 2024; Shumway et al., 2021). This purpose of this paper is to explore the Botley coding robot as a mediating tool for students' multiplicative thinking and ways their interactions within and between activity systems lead to changes in and development of their thinking.

Perspectives

Multiplicative Thinking and Loops

Multiplication is often presented simply as an extension of repeated addition, however, multiplicative thinking involves the ability to form and recognize equal groups (also known as composite units or unitizing) and coordinate groups of quantities in more complex ways, such as iterating (or copying/replicating), shrinking, creating and interpreting an array, and describing factors and multiples (Cheeseman et al., 2020; Götze & Baiker, 2021; Hurst & Hurrell, 2016; Mulligan & Mitchelmore, 1997). The shift from additive to multiplicative thinking in elementary school is subtle yet challenging (Lannin et al., 2013).

Research suggests important conceptual synergies between mathematical and computation thinking, which may address innovative ways to teach challenging mathematical ideas (e.g., Weintrop et al., 2016). One example is the related concepts of *repeats* in mathematics (e.g., repeated patterns, repeated addition as multiplication, and repeated multiplication as exponents) and *repeat* loops in coding (i.e., the loop is a code that repeats a specified sequence of instructions). Beck et al. (2024) designed fifth-grade lessons anchored in the common concept of "repeats" to teach exponents in math and the repeat loop in Scratch coding. We drew on this "repeats" synergy for second-grade students in their early interactions with multiplicative thinking because the loop code and the concept of repeat are related to ideas such as composite units and repeated iteration of these units. These ideas are important in early experiences related to multiplicative thinking.

Cultural-Historical Activity Theory

Cultural-historical activity theory (Sannino & Engeström, 2018) is a useful lens for understanding human-technology interaction (Kaptelinin & Nardi, 2017) and interpreting mathematics education as a system of activities (Godino et al., 2024). Activity theory suggests that the human mind is not bound within the individual but is a product of the actions and activities of humans engaging with the world with signs, tools, and other cultural artifacts (Sannino & Engeström, 2018). Figure 1 provides our adaptation of Engeström's (1987) human activity system to suit our research context of students pursuing an object (e.g., coding, mathematics) through mediating artifacts (e.g., coding robot) within a community that interacts according to (or in conflict with) each other, rules, and division of labor.

Methods

In this study, we aimed to understand the collective and social nature of students' grappling with a new mathematics concept (multiplicative thinking) and a new programming code (loops). We zoomed in and out of collective actions (children, teachers, class, and tools) and observed the students' meaning-making practices and their evolution of multiplicative thinking and its related ideas. Our questions were: 1) What do students' interactions and expressions reveal about their thinking with multiplicative concepts? What mediated students' engagement with important multiplicative ideas?

Participants and Setting

Participants were three second-grade students in an elementary classroom in the western United States—pseudonyms Edison, Lila, and Ricardo. Their teacher, Dr. Koz (a co-researcher and co-author), assigned them to work together as a group to solve mathematics-coding tasks, which took place within a larger classroom of 25 7- and 8-year-old students. The students in this case study worked together at a long table with a Botley robot, its remote control, dry erase markers that were used on the table, and paper/pencil for completing the teacher-created recording sheets. The three students in this small group also engaged with the teacher and/or other students during whole-class discussions.

Lessons and Materials

Dr. Koz designed two lessons for multiplicative thinking, which contained several tasks centered on using the Botley 2.0 robot (see Figure 2). The tasks introduced Botley's Loop code, which Dr. Koz considered to be a conceptual move from one-to-one correspondence to one-to-many correspondence as students used the Loop code to iterate a composite unit (such as the two Forward codes in Figure 3). Other artifacts were dry erase markers and table surfaces, paper task sheets and pencils, and long hard surfaces for Botley's movements (see Figure 4). The tasks for Days 1-2 were about students creating programs for moving Botley eight forward movements using only Forward and Loop codes. The tasks for Day 3 were about analyzing pre-created programs (see Table 1) and "doing the math" to determine how far the program instructs Botley to travel.

Data Sources and Analysis

We analyzed 86 minutes of video data using qualitative methods. Content logging and memoing in Maxqda facilitated familiarity with the video data and led to the identification of video segments that illustrated students' multiplicative thinking. We then went backwards from these critical moments to look at the evolution of students' ideas. Drawing on the CHAT perspective, we used thematic analysis to understand how the students interacted with the mathematical ideas and developed their early multiplicative thinking within the activity system.

Results and Discussion

Edison, Lila, and Ricardo all interacted with early multiplicative ideas as they worked together to solve the tasks. While their efforts for solving the tasks were collaborative and around a shared outcome, each student gravitated toward artifacts for their individual sense-making and grappled with multiplicative ideas in their own ways. The key ideas the students grappled with were composite units, doubling, and iterating, and next we provide examples of what mediated students' engagement with these multiplicative ideas.

"Locking It In"

The unit or composite unit was an important multiplicative concept that emerged in students' interactions. Students discussed the composite unit as the "locking it in" rule for making the Loop code work. Figure 5 shows Lila's ways of interacting with the composite unit in which she notated the unit with a bracket connecting the two Loop codes. Lila's notations of composite units and extras seemed additive in nature (i.e., there is a *loop* part and an *extras* part to the program), however her language-gestures provided a window into her thinking about the iteration of these composite units (i.e., "twice" represents an iteration).

Figure 6 expresses Lila's interactions with composite units as mediated by the writing artifacts. Lila was often holding the marker and notating the program as Edison used the remote or after Ricardo stated a program for her to write. Even when it was Lila's responsibility to input the program, she immediately gave up the remote and took back the writing artifact when she finished. The figure also shows that Lila's actions with the writing artifact varied as she sometimes did this notating quietly to herself, shared the notation with the whole class, or discussed the notation with Edison.

"Double": Doubling or Iterating?

Students also interacted with iteration, which is the key multiplicative idea of repeating copies of a composite unit. An entry point for iteration is the concept of doubling. Doubling in the context of a Loop code is a natural connection to the "doubles facts" (e.g., 4 + 4 = 8) in second grade. Edison stayed with this "doubling" understanding through several tasks and considered the Loop code on the Botley remote as a command that doubles an amount. Figure 7 provides two examples of his language-gestures with this concept: 1) Edison correctly uses the term "double" when referring to the first iteration of the unit, 2) he says "double" when actually he means "copy" of the composite unit—though he used "double" incorrectly, his language-gestures are about iteration.

During the Day 3 lesson, Edison expressed his understanding with the word "repeat" rather than "double" as seen in Figure 8 as well as in his Figure 9 notation which distinguished the 3 as a unit that is repeated as 3 + 3. We identified this as a critical moment because "double" had limited Edison's expression of his multiplicative understanding. We considered these examples in Figures 8 and 9 a subtle, yet important development in his thinking.

In Figure 10 shows Edison's activity with the Botley remote control as a leading mediator for interacting with the doubles concept and the community as the leading mediator for interacting with the repeating concept. Edison consistently held the remote, only being without it when program input was a peer's responsibility. The arrow shading and colors highlight Edison's individual tinkering with the remote, separate from the community (gray arrow), which emerged as an important interaction for his grappling with doubles and iterating concepts and how to work the rule of lock-it-in. Edison's whole-class participation was interesting in that he participated often and sometimes with mistakes, which he brought back to his small group with statements like, "that was weird." These participation opportunities in the whole class followed by individual tinkering with the remote or discussions with his small group around the rules for coding seemed to support the evolution of his ideas.

Iterating: It Means "Do It Again"

Like Edison, Ricardo interacted with the iteration concept, and like Lila, this was mediated by writing artifacts. Different from Lila, Ricardo did not notate as much as write and erase program ideas on the table with the markers to think of new ways to come up with 8 movements for Botley. Figure 11 shows a sequence of images and transcript segments from the video data that show this interaction as he used the marker to think about programs that were not working. Ricardo seemed further along in his iteration concept development and took a lead role understanding the syntax complexities of this new Loop code. This is reflected in Figure 12, which also highlights the marker/table as a lead mediating artifact.

Ricardo and his group were stuck on a new way to make 8 for more than 15 minutes. After time to grapple with a new solution, the teacher joined this group and Ricardo's interactions with him led to a shift in his understanding of the rules for the Loop code. Figure 13 shows a portion of this interaction, and Figure 14 indicates that the dry erase marker/table, the rules for the Loop, and an interaction with Dr. Koz were the leading mediators for Ricardo's interactions with iteration.

Significance of the Study

As students in this study grappled with different objects (composite units, doubling, and iterating), they had some agency to choose artifacts and interactions to develop their thinking. Open-ended tasks with a thinking-oriented goal lend themselves to providing students agency for their learning. From a lesson design perspective, the coding robot was not as central in students' interactions with multiplicative ideas as we imagined (see Figure 1) and in ways that we observed in previous research (e.g., Shumway et al., 2021). Rather, the robot was mainly used for testing solutions to the tasks and served as an engaging tool for exploring math. However, the remote control, dry erase marker/table, and pencil/paper task sheet emerged as the leading mediating artifacts (see Figures 6, 10, 12, and 14). These artifacts allowed students to directly interact with the codes and programs and share ideas with their peers as they made sense of a new code (the Loop code) and new ways of mathematical thinking. More work is needed to expand the scope of this case study for building knowledge about socio-material interactions, for example, multiple cases to explore patterns in how students engage with artifacts to mediate learning and more detailed analysis of how socio-material interactions influence cognition.

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Figures and Tables

Figure 1. An adaptation from Engeström's (1987) image of an activity system from Shumway, et al. (2023).

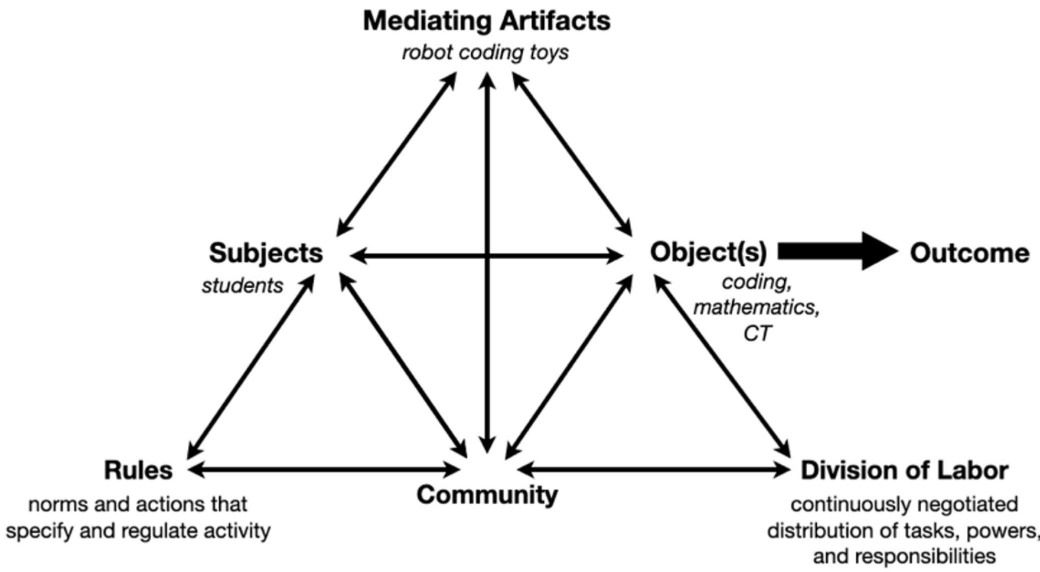


Figure 2. The Botley 2.0 coding robot and its remote control for programming its movements. The view of the remote shows the Forward and Loop codes.

Figure 3. The Loop code as a context for multiplicative thinking because of the iterative nature of the Loop. In this example, the program tells the Botley robot to move 9 forward movements via 4 copies of 2 plus 1 forward (2 + 2 + 2 + 2 + 1 or 2 x 4 + 1).

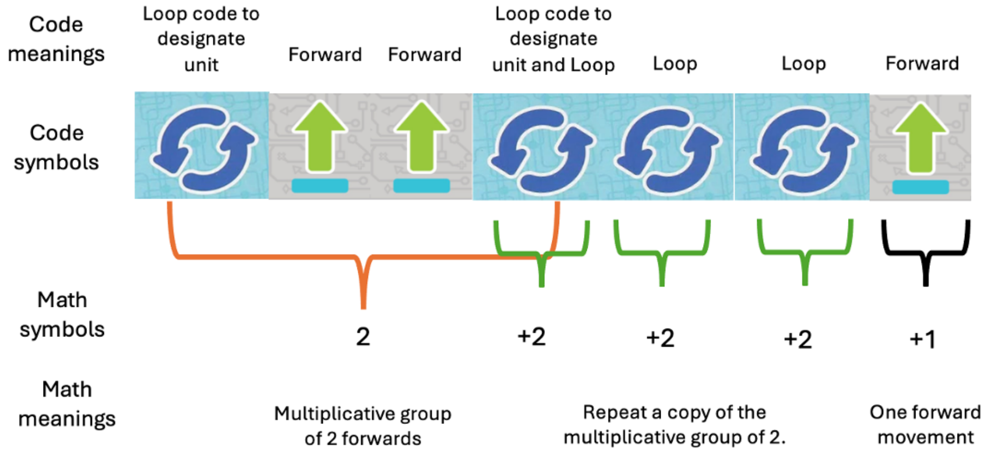


Figure 4. The students in this group often wrote their program ideas on the table, and after testing it, wrote their correct programs on the task sheet.

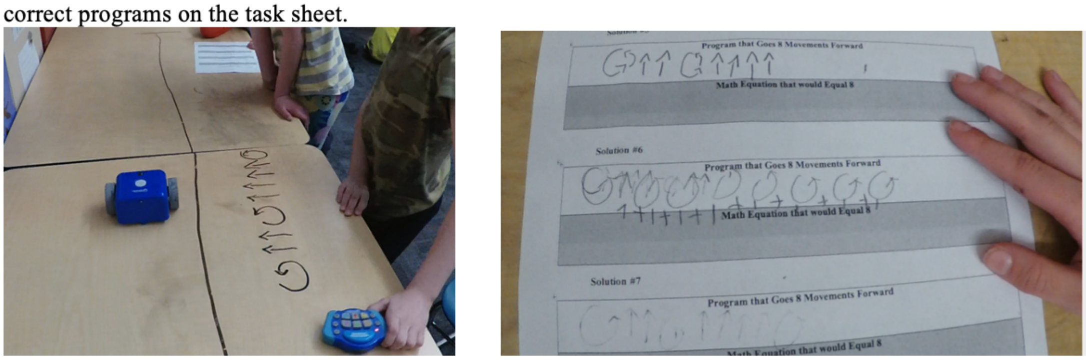
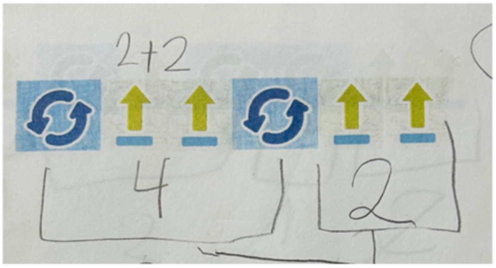


Table 1

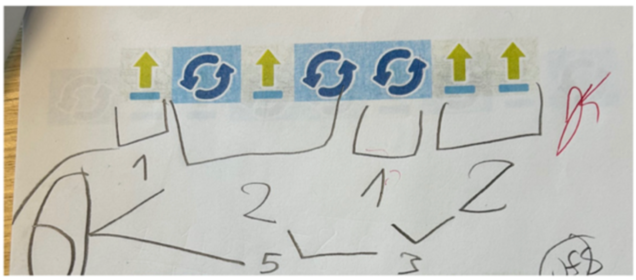
Pre-Created Iteration Programs with Increasing Level of Mathematical and Coding Complexity

Pre-Created Program	Targeted Coding Complexity	Mathematical Equivalence
	Basic unit Iteration	(2)2
	Unit multi-iteration	(1)4
	Unassigned iteration in middle	3+0+1
	Iteration with extras on end	(2)2+2
	Iteration with extras at beginning	2+(3)2
	Iteration with extras on end and beginning	2+(2)2+1
	Multi-iteration with extras on beginning and end	1+(1)3+2
	Unassigned iteration at beginning	0+6
	Multiple iterating units with a single coefficient	(1)2+1+(2)2
	Multiple iterating units with multiple coefficients	(2)3+1+(1)2
	Multiple iterating units with extras at beginning and end	1+(1)4+1+(1)2+1

Figure 5. Lila's notations and transcript segments about these notations.



These two [pointing to the two Loop codes] *doubled this* [pointing to the two forwards inside the Loop codes] *because they locked it in*.



This [pointing to the second Loop] *made the one go twice…and then this one* [pointing to the third Loop] *just made another one*.

Figure 6. Leading mediators for Lila's concept of "locking in" the unit and how it is used for repeating.

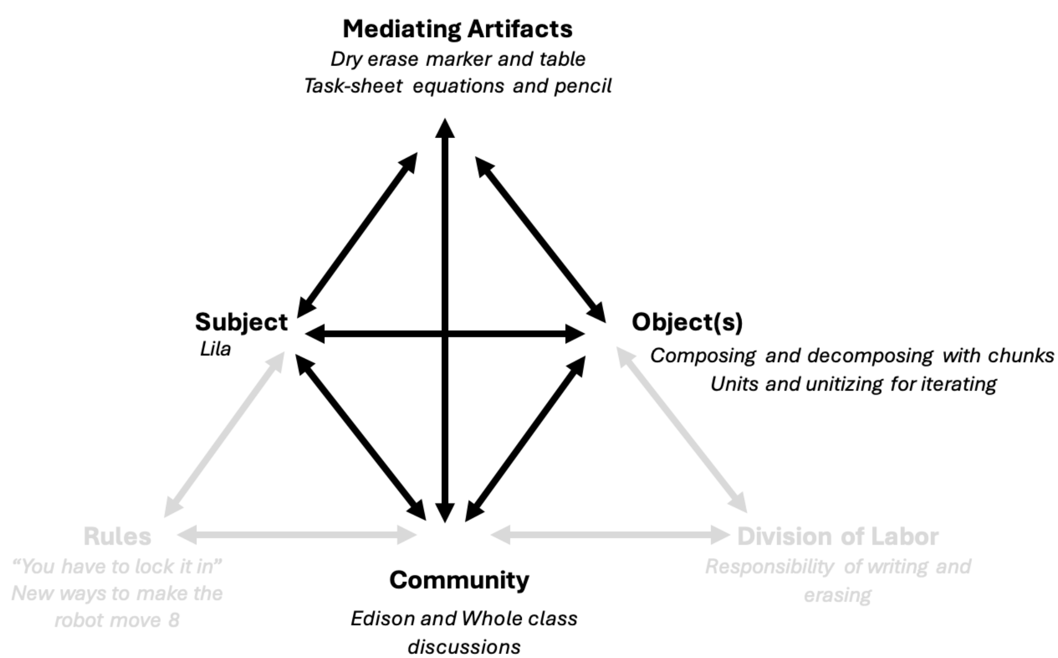


Figure 7. Edison's transcript segments for "double".

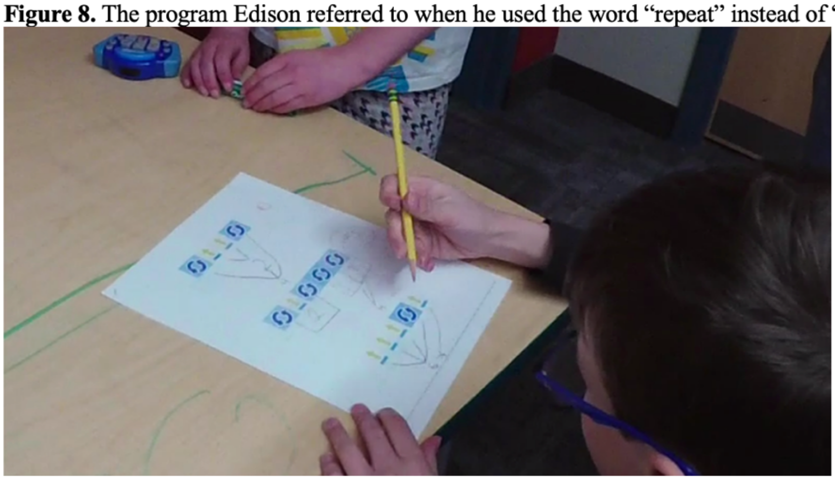
On Day 2, Edison participated in a whole-class discussion about another group's program: *Loop Forward Forward Loop Loop Forward Forward*. He was called on to explain why it worked:

EDISON: Double this [pointing to the two forwards in the Loop]
DR KOZ: Do the 2 [pointing to the two forwards in the Loop]
EDISON: And then it gets 4
DR KOZ: And then you double it to get 4
EDISON: And then you double the 2 again to get 6
DR KOZ: "And then you double the 2 again to get 6." So you had 2 4 6
EDISON: And then there's 2 extra at the end

Later, about a different solution within his small group he explained:

EDISON: I thought…double the 2 to get 2 and double the 2 again to get 4 and double the 2 again to get 6. Then double the 2 again to get 8.

Figure 8. The program Edison referred to when he used the word "repeat" instead of "double."



Because this is useless [pointing to the Loop]. *You have to lock it in to make it repeat*.

Figure 9. Edison's notations on a task.

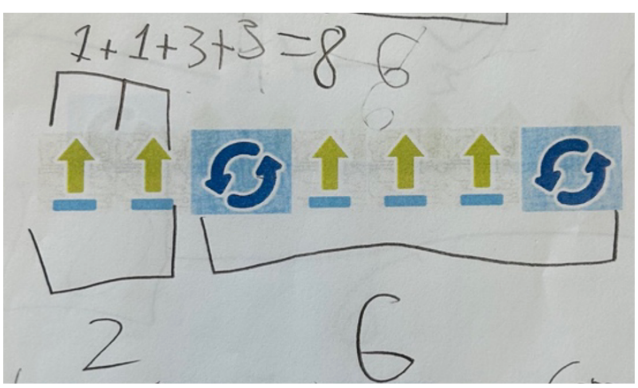


Figure 10. Leading mediators for Evan's concept of doubling and then "repeating" (iterating).

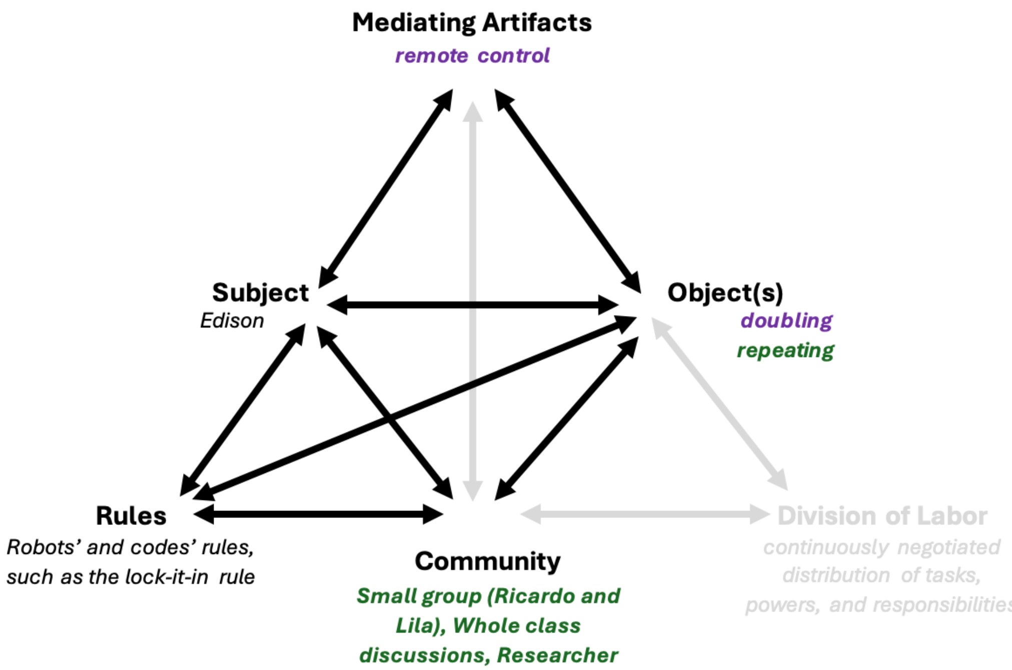
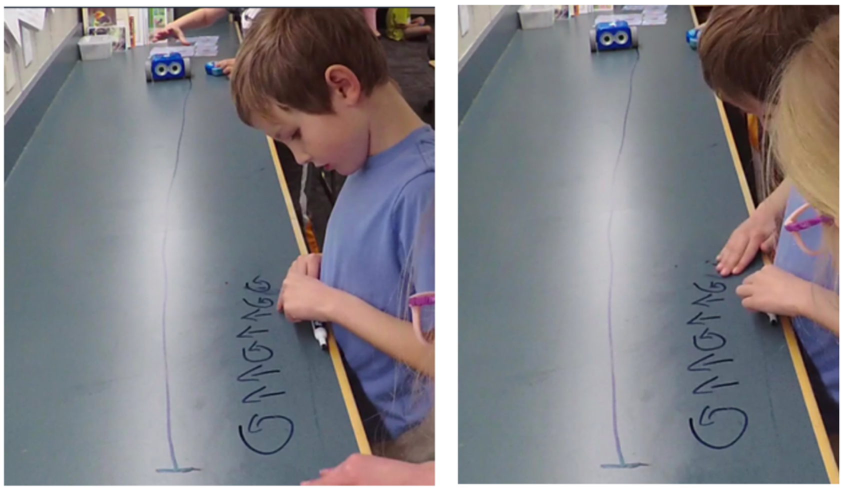


Figure 11. Sequence of images and transcript as Ricardo interacts with the meaning of the Loop code.



After testing *Loop Forward Loop Loop Forward Forward Forward Loop*, which resulted in only 6 movements, Ricardo erases and write this new program *Loop Forward Forward Loop Forward Forward Loop*, which will again result in 6 because the last two loop codes are "useless." In the first image, Ricardo says: "Two [pointing to the two forwards], four [pointing to the first loop], six [pointing to the next two forwards], eight [pointing to the next loop]." In the second image he erases the last loop since his counting landed on 8. When the group tests the program, and it does not work, the last images shows that he erases the chunk of forwards and loop on the right and looks around for the teacher.

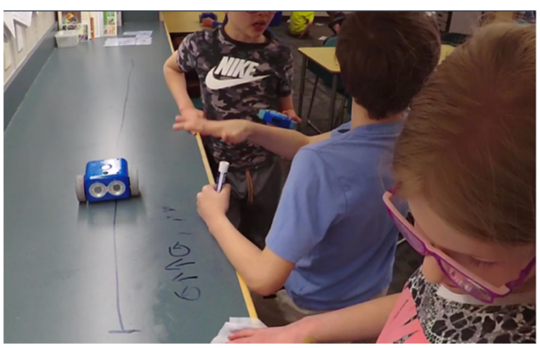


Figure 12. Leading mediators for Ricardo's objective to find a new way to make 8.

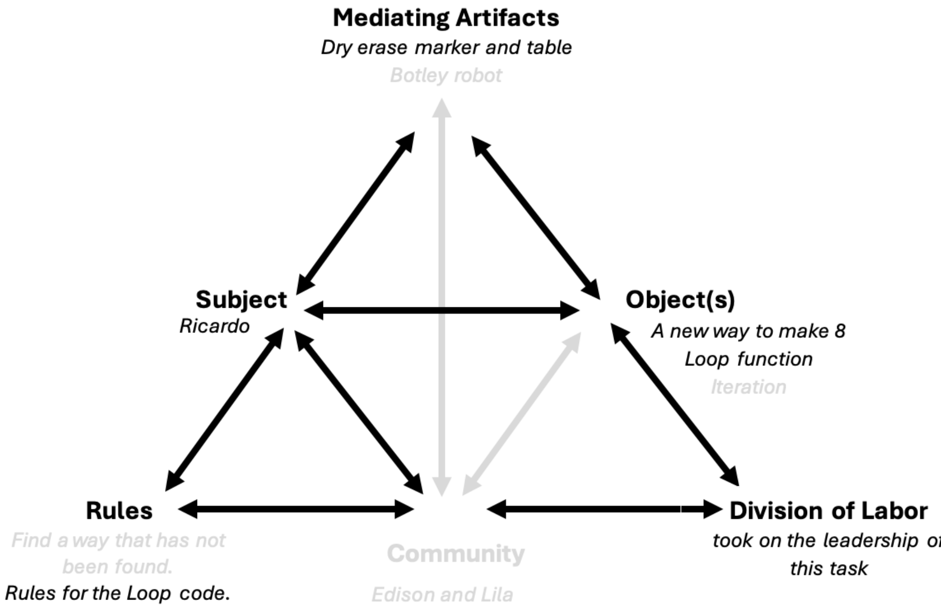


Figure 13. Sequence of images of Ricardo's interaction with the teacher.

Dr. Koz: *So, I'm wondering about this one*
[pointing to the Loop after the "lock-it-in" loop code]

Dr. Koz: *What do you think this means?*
Ricardo: *Do it again.*

Figure 14. Leading mediators for Ricardo's objective to understanding the Loop syntax.