Best of Both Worlds: Developing an Innovative, Integrated, Intelligent, and Interactive System of Technologies Supporting In-Person and Digital Experiences for Early Mathematics

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Abstract

Mathematics is a core component of cognition. Unfortunately, most young children and teachers cannot access research-based early childhood mathematics resources. Building on a quarter-century of research, we are developing and evaluating an innovative, integrated, intelligent, and interactive system of technologies based on empirically validated learning trajectories that provide the best personal and digital tools for assessing and supporting children's mathematics learning. This article reviews the research that guided us, then describes the design principles of the new project, justifying their selection using theory and research, and shares how the design principles helped address challenges in development. The goal is to provide teachers, caregivers, and children with high-quality resources to support early mathematics learning in the context of meaningful, motivating, challenging, and achievable experiences.

Keywords: Educational technology, mathematics, early childhood education, preschool, kindergarten, primary grades

Best of Both Worlds: Developing an Innovative, Integrated, Intelligent, and Interactive System of Technologies Supporting In-Person and Digital Experiences for Early Mathematics

Mathematics is a core component of cognition (Watts et al., 2014) and the foundation of STEM achievement. Unfortunately, most young children and teachers cannot access research-based early childhood mathematics resources. Moreover, access was even more limited during the pandemic, which magnified inequities based on income and race {Kuhfeld, 2020 #10946}.

To address these inequities, we are developing and evaluating an innovative, integrated, intelligent, and interactive system of technologies based on empirically validated learning trajectories that provide the best personal and digital tools for assessing and supporting children's mathematics learning. This article briefly reviews the research that guided us, then describes the new project's design principles, justifying their selection using theory and research. Finally, it shares how the design principles helped address challenges in development.

Learning and Teaching Early Mathematics

High-quality education helps all children learn to *mathematize*— to see the world through a mathematical lens and use mathematics to solve problems {Clements, 2021 #10567;Perry, 2002 #2728`, coined by Hans Freudenthal}. The recipe for equitable, high-quality early mathematics education has three main ingredients, each grounded in research {National Research Council, 2009 #3857;Sarama, 2009 #3380}.

(1) Mathematics content that focuses on big ideas that are mathematically central and coherent {NGA/CCSSO, 2010 #4143}.

(2) Focusing on children’s thinking and learning as the core of planning and implementing educational experiences {e.g.`, \Carpenter, 2014 #2041;Gelman, 1979 #5647;Sarama, 2009 #3380}.

(3) Teaching practices that honor the discipline of mathematics and the children’s cultures, families, individual characteristics, and patterns of thinking and learning {e.g.`, \Carpenter, 2014 #2041;Frye, 2013 #4610;O’Brien, 2023 #11889}.

A framework that combines these components is the learning trajectory (LT) approach {National Research Council, 2009 #3857;Sarama, 2009 #3380}. Based on the theory of Hierarchical Interactionalism, each LT has three components: (a) a mathematical goal, (b) a developmental progression of levels of thinking, and (c) teaching practices. To attain a specific mathematical competence in a given topic (the goal, linked to standards), children learn each successive level (the developmental progression), aided by teaching practices designed to build the mental actions-on-objects (cognitive processes acting on mental representations of concepts) that enable thinking at each higher level.

Goals go beyond behavioral objectives to include a coherent cluster of concepts, skills, and mathematics practices. The LT approach puts children’s development at the core, aligning teaching practices crafted to support each level of this natural development. Just as children learn to crawl, walk, run, skip, and jump with increasing speed and agility – levels of movement – children follow natural developmental progressions in learning mathematics. Teachers who understand the levels of these developmental progressions and base their instruction on their children’s development build mathematics learning environments that are particularly developmentally appropriate, effective, and meaningful.

The third component of LTs is teaching practices, which are based on development progressions and explicitly designed to build each level of thinking. Teaching practices are broad and include educational environments, interactions, and activities—all considering how to address children's learning one level above their current levels of thinking in a progression. Thus, the learning trajectories approach includes all three important components of high-quality mathematics.

Research has validated the efficacy of LTs. The LT approach was the core of the influential TRIAD project that produced effect sizes from 1 to 2 standard deviations in small- and medium-scale studies {Clements, 2007 #2091;Clements, 2008 #2785} and .72 standard deviation in large-scale studies of 100s of teachers and 1000s of children {Clements, 2011 #4177}, with effects decreasing but lasting through the primary {Clements, 2013 #4346} and intermediate grades {Clements, 2023 #8019}. Also, teachers demonstrated higher levels of fidelity from 2 {Clements, 2015 #6812} to 6 years {Sarama, 2016 #8119} past the end of the external intervention. Explicit and rigorous empirical tests have supported the assumptions of the LT approach {Clements, 2021 #10608;Clements, 2020 #9997;Clements, 2019 #9686;Sarama, 2021 #10988}. Significantly, children from marginalized communities benefit as much or more than those from other groups. Because all instruction builds on what the child knows and can do, the learning trajectory approach is *asset-based, supporting more equitable instruction* {National Research Council, 2009 #3857}.

Educational Technology and Early Childhood Mathematics Education

What are the contributions and possible problems of using digital technology to teach early mathematics? Fortunately, there has been a half-century of research on the use and impacts of educational technology {e.g.`, \Herodotou, 2018 #8859;Istiana, 2023 #12295;Lieberman, 2009 #8940;Paul, 2023 #11903;Sarama, 2020 #7337}.

Contributions of Technology

When used in a supportive, interactive, research-validated context, educational technology can make substantial contributions to early childhood education across the domains of development.

Research has reported particularly positive impacts on early mathematics learning {e.g.`, \Ariff, 2022 #11769;Guss, 2022 #11295;Sarama, 2020 #7337}. For example, in the TRIAD research, digital activities were a main moderator of positive effects {Clements, 2011 #4177}. Multiple studies show positive results of research-based apps, including addressing equity issues {e.g.`, \Foster, 2018 #8351;Resnick, 2023 #12357;Reyna, 2023 #12384;Tepho, 2023 #12326}. A recent review identified 8 studies identified that instruction leveraging technological tools like iPads and speech generating devices supported instructional adaptations for children with disabilities {Paul, 2023 #11903}.

The concern that technology isolates and prevents children from having communal experiences and developing social-emotional skills has been alleviated. With the proper arrangements and uses, such as cooperative problem-solving and play with technology, children collaborate the same amount or more on digital than other centers {e.g.`, \Alkhawaldeh, 2023 #12697;Mantilla, 2019 #11814}. Educational technology can function as a “new frontier” of play, expanding children’s play world with new environments and possibilities {Mantilla, 2019 #11814;Sarama, 2016 #8031}.

Multiple studies show that digital activities built upon learning trajectories are particularly effective {Ceylan, 2023 #12003;Foster, 2018 #8351;Guss, 2022 #11295;Sarama, 2020 #7337;Tazouti, 2023 #12107;Thai, 2022 #12877}. This stands in contrast to mathematics interventions that fail because they do not target a level of challenge appropriate to children's current level of mathematics thinking {Libertus, 2023 #12825}, highlighting the importance of formative assessment using learning trajectories {see also \Thai, 2022 #12877}. However, some or all of the three components of LTs are missing in most educational technology apps {Callaghan, 2018 #8299;Tazouti, 2023 #12107}. When the three components of LTs are used, the apps have been documented to be successful {Bang, 2022 #12868;Can, 2020 #10948;Clements, 2021 #10567;Ginsburg, 2019 #11245}. We return to such characteristics in our description of the Bajillions project, but first, we review research reporting negative findings that demand equal attention {Tekin, 2023 #11940}.

Persistent Problems

Unfortunately, much educational technology uses ineffective pedagogies, with problems including "missing instructions, poor feedback … ineffective guidance and modeling … lack of responsiveness" {Sarama, 2020 #7337`, p. 185}. Reviews of mathematics apps report considerable limitations {Ariff, 2022 #11769;Callaghan, 2018 #8299;Guss, 2022 #11295;Konca, 2023 #12392}. We briefly review three important types.

Limited Content Coverage (Goals)

In a review of digital mathematics apps {Guss, 2022 #11295}, most apps focused on counting (11) or number comparison (10); 7 involved geometry, five arithmetic, and three spatial skills, and only one covered each of the five content areas of mathematics identified by national organizations {NGA/CCSSO, 2010 #4143}. Rarely did they address subitizing, composing numbers, measurement, patterns, angles, fractions, volume, or data analysis/classification {e.g.`, \Elia, 2023 #12837}. Many that claim to cover these topics do so in a limited sense, and others provide experiences that are not mathematical, contain mathematical inaccuracies, or may confuse children's mathematical understanding {see \Guss, 2022 #11295`, for specific examples}. Moreover, the apps had limited digital manipulatives, meaning children could not engage with various objects, layouts, and settings {Guss, 2022 #11295}. Thus, an initial problem to understand and address is expanding the content and practices addressed in mathematics education technology for young children and adults in their lives.

Limited Views and Use of Children's Thinking and Learning

In addition to the lack of breadth in the reviewed games, apps, and websites, there is also a lack of depth connected to a rigorous understanding of children's mathematical thinking {e.g.`, instrumental rather than relational understanding`, see \Clements, 2023 #12287} and developmental progressions. That is, although recently, a small number of digital activities were purposefully designed or aligned to the research-validated learning trajectories of early mathematics {Bang, 2022 #12868;Can, 2020 #10948;Clements, 2021 #10567;Ginsburg, 2019 #11245}, most were not. Further, research using these activities contained subgroup analysis suggesting that the skills addressed were more constrained than the thinking children are capable of learning. Some do not address early developing levels of thinking, so children at those levels are frustrated {e.g.`, \Betts, 2020 #11231;Doan, 2023 #12727}. One system was more effective for children with higher baseline skills – indicating that the conceptual "floor" was too high {Betts, 2020 #11231}. Another system was less effective for older children, indicating that the conceptual "ceiling" may have been too low {Bang, 2021 #11229}. Thus, a focus on the wide range of levels of thinking that young children can start from and grow toward is essential in developing digital technology for teaching and learning early mathematics.

Limited or Suboptimal Teaching Practices

Effective early mathematics education is precise and responsive, honoring the discipline of mathematics and young children. Unfortunately, although most educational apps are generally age-appropriate and interesting for preschoolers, instructional feedback is rarely included and is often suboptimal. This squanders the opportunity of digital technology, which can be especially effective because feedback can be automatic, frequent, responsive, and specific {Konca, 2023 #12392;Sarama, 2020 #7337;Tazouti, 2023 #12107}. Also, young children report that frequent verbal encouragements help them enjoy digital experiences {LeSage, 2021 #11336}, but these are usually lacking.

In a related vein, studies show that existing software often does not adapt instructional activities sufficiently. Some do not address early developing levels of thinking, so children at those levels are frustrated {e.g.`, \Betts, 2020 #11231;Doan, 2023 #12727}. Few apps help children who have already attained the target level of thinking to progress further {Bang, 2021 #11229}. When children are frustrated, many apps do not provide adequate scaffolding {Elia, 2023 #12837}.

The remainder of the paper describes how a new research and development project, Bajillions, was grounded in what research has revealed. We demonstrate how we apply positive characteristics and avoid limitations.

Development of Bajillions Mathematics: Theory, Research, and Principles

Bajillions will be a digital platform that provides mathematics instruction and practice for young children ages 1 to 8 years (note that only adults use the platform for infants and toddlers). The platform will use children’s interactions to provide diagnostic information, guidance to children, and professional development for teachers. However, Bajillions’ digital offerings are just one component: The platform also integrates *LearningTrajectories.org,* our library of more than 700 in-person learning activities across the 20 topics relevant to early mathematics development, thereby honoring and supporting teachers' face-to-face work's value and critical importance. By carefully integrating digital and in-person activities and basing all development on the research corpus, we are designing Bajillions to use "the best of both worlds," an optimal synthesis of research-based teaching practices {Clements, 2023 #12287}.

Ways Bajillions' Integrated Approach is Designed as the "Best of Both Worlds"

Bajillions is built on five empirically-supported design principles to achieve the "best of both worlds" by synthesizing approaches for platforms not usually combined and sometimes considered incompatible.

Learning Trajectories + Just-in-Time Support

LTs are a particularly effective foundation for teaching and learning early mathematics. However, they can be challenging to understand and use without the support of often expensive curriculum and professional development {e.g.`, \Clements, 2011 #4177;Clements, 2015 #6812;Sarama, 2016 #8119}. The Bajillions system will encourage, guide, and support everyone– teachers, childcare practitioners, and families (as well as researchers)– to use the vast knowledge embedded in LTs to support children's mathematical learning and development, *providing guidance as they need it,* and inviting them to learn more as they wish. Bajillions will also support children directly through *adaptive* digital activities {Jung, 2015 #11815}. These activities will estimate children's level of thinking, allowing them to work at the most developmentally appropriate, efficacious LT level on digital and in-person activities by providing diagnostic information and specific personal, face-to-face activities to teachers. Further, the levels of thinking in the LTs are comprehensive, including concepts, processes, skills, facts, multiple representations, and strategies (relational mathematics).

Two supports are linked through the system. The first is carefully focused and sequenced daily hands-on, face-to-face lessons, including specific directions, research tips, and videos of teachers and children engaged in the playful activities. These are complemented and enriched by the second support, research-based digital activities (for preschoolers and older children—the technology provides guidance on environments, centers, and activities for infants and toddlers), which serve as models of best practice as they are matched to developmental levels. The platform will link the research of LTs to teachers' practices by providing these resources "just-in-time." The mechanism of these linkages is the invaluable assessment information described in the next section.

High + Low Tech; Digital + Personal (Consistent Formative Assessment)

Research and expert practice indicate two guidelines for facilitating young children's mathematics learning. First, *synchronize personal and digital activities* {e.g.`, \Dahshan, 2024 #12935;Galen, 1997 #1655;Hutinger, 2000 #13036;Sarama, 2020 #9896}. Second, for both these types, *engage children at their developmental level* for each mathematics topic and *monitor and support their progress using formative assessment* {e.g.`, \Jung, 2015 #11815;Konca, 2023 #12392;Tazouti, 2023 #12107;Thai, 2022 #12877}. Further, both components of formative assessment, the *assessment* and the linked *adaptive instruction*, benefit from a combination of high- and low-tech approaches. The Bajillions system will coordinate and combine *low-tech instructional activities of personal, face-to-face experiences* with *high-tech digital activities* targeted through embedded assessment grounded in validated pedagogy {Clements, 2023 #12287}. Such ongoing and deliberate use of formative assessments to monitor progress and adjust instruction helps all children learn {Black, 1998 #2843;Connor, 2018 #10194;Gallego, 2018 #9318;National Mathematics Advisory Panel, 2008 #3480;Penuel, 2016 #6050;Shepard, 2018 #8673;Thomson, 2005 #3251}. Unfortunately, most early childhood teachers lack the time and training to assess mathematics accurately and link results to instruction {Kilday, 2012 #6962}. Also, unfortunately, although digital technologies could reduce the assessment burden on teachers, most apps do not use formative assessment, especially along research-validated developmental progressions {Callaghan, 2018 #8299}.

The Bajillions system will address this problem because it conducts formative assessments of each child's level of thinking within key LTs and provides detailed information about specific topics that need particular. Further, teachers' observations of children playing the activities and reflections on the resulting reports will support teachers' use of low-tech curriculum-embedded assessments built into every personal activity.

The importance of directly linking planning, instruction, formative assessment, and feedback cannot be overstated {Jung, 2015 #11815}. Therefore, teachers and parents are provided with specific low-tech personal activities based on the assessments. These activities will be drawn from the Learning and Teaching with Learning Trajectories (aka LTLT or $[LT]^2$) online tool, a resource with 20 learning trajectories (topics).

The result of over two decades of successful work starting with the NSF-funded Building Blocks project and continued in a series of IES large-scale evaluations, $[LT]^2$ is a substantial extension of a website used in those projects, providing complete learning trajectories for 20 topics in early childhood mathematics, birth through grade 2. Initially developed as a professional development tool for those research projects, $[LT]^2$ was funded to be a resource for teachers and researchers. Used alone, $[LT]^2$ remains best used under the guidance of those providing professional development. This is the main reason Bajillions provides extensive guidance and support for teachers.

Clear, specific feedback is a high-yield instructional technique {Bower, 2020 #10135;Gersten, 2015 #7451}. As noted, only some apps for young children include feedback, which is often not optimal {Callaghan, 2018 #8299}. Bajillions' feedback offers reinforcing instruction ("You matched the *square*."), encouragement ("Way to go!"), effort {Sarama, 2020 #9896}, and persistence ("Keep up the great work!"). Further, feedback in Bajillions is:

- layered {prompts or general strategy questions`, then specific hints if needed`, only then modeling`, \Clements, 1987 #5659;Kramarski, 2007 #7072;Sarama, 2020 #7337};
- specific and informative {Sarama, 2020 #9896;Van der Kleij, 2015 #7772};
- immediate {as appropriate`, children often build or problem solve for periods before any evaluation`, as appropriate`, e.g.`, \Fyfe, 2017 #10740};
- adaptive (at the micro and macro levels), and
- intrinsic to the task {i.e.`, not a "reward" external to the mathematics`, \Laurillard, 2016 #8932}.

[LT]^2's high-tech activities for children have been empirically validated over two decades {Clements, 2007 #2091;Clements, 2008 #2785;Clements, 2011 #4177;Clements, 2013 #4346;Sarama, 2004 #2068;Sarama, 2008 #3324;Sarama, 2012 #4002}, providing significant, complementary, and unique high-tech learning experiences {Clements, 1985 #5794;Foster, 2018 #8351;Sarama, 2020 #7337`, see also the previous section`, "Educational Technology and Early Childhood Mathematics Education"}. They all develop along the LTs, of course. They also use characteristics that promote experiences that complement hands-on activities {Sarama, 2016 #8031}. For example, when children match shapes to outlines physically, they may rotate and flip without reflection. In contrast, when matching shapes with virtual manipulatives, they intentionally choose not only whether to turn or flip the shape but also how, or by how much, to rotate it {Sarama, 2016 #8031}. Such "slowing down" and explicating action and thinking can build important knowledge about shapes and spatial reasoning,

Play and Intentional Instruction; Home and School

A pernicious false dichotomy in early education is "Play vs. academics." Children should play at home and school and learn through intentional activities in both contexts {Tazouti, 2023 #12107}. They should also learn to play with mathematics {Sarama, 2009 #3700}.

Children naturally engage in mathematics in free play, at home, and at school {Seo, 2004 #1714;van Oers, 1996 #2348}; however, if not guided to mathematize (reflect on, give language to) their early "theorems in action" {Vergnaud, 1978 #3073}, the ideas do not become concepts in thought. Therefore, the Bajillions approach incorporates child choice of play and center activities—both personal, hands-on, and software playground activities that promote autonomy and creativity with mathematical objects and processes {Lohnmann, 2018 #9172}, enhanced by the explication of the mathematics in each, alongside playful, intentional activities.

Providing stories and context can bring joy and meaning to learning. For example, non-digital instructional activities that provide context, such as a storyline, are more effective at improving children's mathematics learning than those without context, particularly for children with lower executive function {Veraksa, 2020 #10609}.

Also, studies suggest that an important feature of educational technology is autonomy. When children control what happens in a digital environment, it can improve their engagement and learning {Hirsh-Pasek, 2015 #11813;Laranjeiro, 2021 #11735;LeSage, 2021 #11336;Otterborn, 2020 #10646}. One avenue of autonomy and active learning {Hirsh-Pasek, 2015 #11813} is creative tasks; studies have emphasized the value of children creating within digital environments {e.g.`, \Elia, 2023 #12837;Larkin, 2022 #11657}. In their systematic review of research on technology with young children, Mantilla and Edwards {, 2019 #11814} found that creating in digital environments is like a launchpad, inspiring children to create in real life. Therefore, Bajillions provides children the autonomy to choose and control mathematics playgrounds for each major topic. These will be available following a Daily Quest that is leveled for the child, as research indicates that instruction and guidance followed by free choice is more effective than free choice alone {Birklein, 2024 #12596}.

Intelligent Algorithms + Learning Trajectories (Micro and Macro Adaptations)

Learning trajectories support children's mathematical learning and development *if* adults understand where to start and how to progress, *and* digital instruction is similarly matched with children's developmental level. Although technology can provide invaluable assistance in managing and presenting information, algorithms are more intelligent and effective if they are productively constrained and guided by the research. The Bajillions management system dynamically assigns and adjusts, for each child, the levels of thinking of each LT (macro adaptations) and the scaffolding of individual tasks within each level (micro adaptations, such as number size or representations).

The management system integrates three information sources. First, the activities' designs provide children with varying opportunities to act on objects in ways that develop and show their mathematical understanding. Second, the feedback provided to children following incorrect answers changes based on research-based assumptions of the type of error the child made and how often an error is made given the same problem. Third, children's correct and incorrect answers determine the number and structure of problems children are then given. The adaptive nature of these procedures does not use the type of predictive machine learning of recent innovations in artificial intelligence (AI). Research does not yet exist that allows AI to adapt to all situations or even across the three aforementioned dimensions of content, scaffolds, and task structure. However, the complexity of adaptations of the Bajillions system and the massive amount of data it will collect may generate a set of training data for future generations of early mathematics digital technology, including Bajillions.

In summary, the Bajillions system will be the first to comprehensively assess children's learning along the developmental progressions of early mathematics and dynamically support children's learning within (micro adaptations) and over (macro adaptations) all levels of the early mathematics learning trajectories.

Curriculum + PD

Bajillions provides a seamless curriculum path from birth through 8 years of age. As mentioned, compared to most existing apps or curriculum packages, coverage is comprehensive, including all 20 important mathematics topics (see [LT]^2). Further, the system retains and enhances [LT]^2's effectiveness as a professional development tool. Educators are encouraged and guided (but not required) to simultaneously learn about their children's development through the activities' reports. Thus, they develop a more profound understanding of their students' developmental progressions to inform their instructional decisions, becoming more able to take charge of the children's learning.

The Path guides children's experience, using each child's interaction with the activities and monitoring their advancement (ages 3+; teachers and caregivers of infants and toddlers observe and provide this information). Children's location on the Path is continuously updated with data from children's interactions with Bajillions' suites of digital activities to guide and support further progression through the curriculum. An extensive integration of all learning trajectories in $[LT]^2$ built on two decades of curriculum development and evaluation that has illuminated the most beneficial intertwining of these topics through a leveling matrix that determines how children move across game levels based on performance.

The Path will provide a continuous, comprehensive, integrated curriculum structure and, in concert with the other four integrated approaches, will support teachers in choosing goals and instructional activities. In addition, the system retains and enhances $[LT]^2$'s effectiveness as a professional development tool while simultaneously curating its content for individual teachers and parents. Bajillions guides educators to learn about children's development on LearningTrajectories.org whenever they receive a new personal activity. For example, they are invited to see a child displaying just the level of thinking a new activity is designed to teach (and, as they wish, the earlier and later levels). Further, they learn about their children's development through the reports of children's interactions with the activities. Similarly, teachers, childcare staff, and parents can *see* the LTs' linked instruction by viewing video clips of classroom instruction using the activity.

In summary, Bajillions is being developed based on these five design principles above and other guidelines from research {e.g.`, \Ruiz, 2014 #6929}, complementing the pedagogical and mathematical expertise of the co-PIs and their research-validated and efficacious learning trajectories, collaborators' educational gaming expertise, and project staff's and advisors' measurement and equity expertise. This will ensure the integration of opportunities for high-quality mathematics learning –in-person and online – through adaptive leveling algorithms that inform instructional decisions in playful, engaging ways.

Bajillions*: Implementation and Translation*

The Bajillions system is well into production but, of course, a complex enterprise. In addition to the design principles that conceptualized it, the research team has identified and addressed the complexity of translations needed to implement them. Two dimensions of translation have provided particularly challenging obstacles, including translating from research to practice, from in-person to digital, and from concepts to programming.

Research to Practice

Translating extant research to practice is a core aspect of all Bajillions' design principles. Each activity's mathematics is intrinsic to the goal, providing meaningful feedback rather than unrelated rewards for "getting through" the tasks {Laurillard, 2016 #8932}. We compiled a list of actions-on-objects that would translate to the digital space {Sarama, 2009 #3380} and synthesized research on curricular activities and instruction {Clements, 2021 #10567;Clements, 2023 #12287} to suggest tasks and teaching strategies.

Clear, specific feedback is a high-yield instructional technique {Bower, 2020 #10135;Gersten, 2015 #7451}. As previously described, we include multiple types of feedback{Sarama, 2020 #9896}{Clements, 1987 #5659;Kramarski, 2007 #7072;Sarama, 2020 #7337}{Sarama, 2020 #9896;Van der Kleij, 2015 #7772}. Extensive time and thought must go into planning the essential aspects of research that must be included and how they will be included in practice.

In-Person to Digital

A second dimension of translation is considering how in-person teaching decisions can be implemented digitally. Young children process information differently than adults, so we simplify verbal directions and improve visual guides. This adaptation should support children with hearing impairments and respond to teacher feedback that classroom noise sometimes interferes with children hearing the directions. Additionally, more careful consideration of universal design for learning (UDL) has been added to the projects' planning processes. Other UDL capacities of the system will include keyboard control options and color choices that meet color contrast standards. To ensure in-person teaching moves are replicated in the digital platform, we have partnered with local schools to engage in iterative rapid-cycle playtesting using low-cost prototypes. Effective interactions with children from diverse backgrounds using physical materials and manipulative actions are scripted for a digital prototype. This is used to check children's understanding and progression across each game, revised as necessary, and built into the fully digitized product designed to guide children with minimal frustration.

Bajillions' design embeds culturally responsive instruction. The personal activities on [LT]^2 support teachers' seeking input from families about everyday experiences, contexts, representations, and routines that can be leveraged in teaching practices to empower children's feelings of belonging in STEM {e.g.`, \Clements, 2023 #12287;Gay, 2018 #8100}. The Advisory Board, comprised of early childhood researchers and expert practitioners who focus on equity, has suggested multiple ways to be intentionally inclusive and promote the mathematical development of historically marginalized groups in both personal and digital activities. Children from many groups see themselves in the activities and positive representations of people who are not necessarily like them.In summary, across collaborative processes, we seek to apply an anti-racist, anti-ableist lens by consistently asking how we resist negative messages about any group of people, how we may design to center their experiences, and how we lift up narratives about the potential of historically marginalized people.

What is Next: Research on If, How, and for Whom Bajillions Works

Although Bajillions has a firm basis in research, much remains to be determined. As an engineering science {Clements, 2007 #2632}, the design and implementation of particular content, structure, and processes of components can increase or debilitate the effectiveness of any curriculum, even if based on previously successful work. Therefore, the efficacy and effectiveness of the research phases are critical {Clements, 2007 #2632}. The Curriculum Research Framework (CRF) structures this work by iterating the development in small-scale pilots, scaling up to extensive evaluations {Clements, 2007 #2632;Sarama, 2019 #8129;Sarama, 2019 #8601}.

The world of early childhood educational technology and media is crowded. However, there remains a need for high-quality, intentionally designed products that incorporate what we know about how children learn generally and digitally. With teachers, families, children, and other research, we aim to work toward a future where educational technology is used strategically, effectively, and with respect for the rich in-person experiences that teachers and other caregivers provide. With learning trajectories at the core, Bajillions is grounded in research on children's cognition and development and includes connected, validated instructional activities and strategies; learning trajectories enable and direct formative assessment for targeted instruction.

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Declaration of Interest

The Learning and Teaching with Learning Trajectories ($[LT]^2$) standalone website is free. The authors are funded to develop the Bajillions project, which will coordinate with $[LT]^2$ as a separate project. It is free at present but may have equitable access charges to sustain updating and development in the future.