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Fostering Set-Based Reasoning for Mathematical Proofs: Student Interaction and Challenges in an Inquiry-Oriented, Transition-to-Proof Course

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This poster aims to provide salient insights into an inquiry-oriented transition-to-proof course designed to cultivate undergraduate students' set-based reasoning for mathematical proofs. By set-based reasoning, we refer to a person's reasoning with predicates and relationships between predicates and their truth-sets (Dawkins, 2017). The course emphasized learning fundamental logical principles – such as non-contradiction, contrapositive equivalence, and converse independence – via set-based reasoning. Opportunities were given to students to actively engage with their peers and the instructor team, including the primary instructor for whole-class discussions and research assistants responsible for facilitating small-group discussions. These interactions occurred in the physical classroom and extended to virtual discussion forums.

We examined several aspects of student engagement: peer-to-peer interaction, interaction with the instructor team, assignments and exams, and online discussion forum posts. Our focus was on four key areas: (1) students' participation in building the learning community's acknowledgment of mathematical logic, (2) instructional deliveries in the utilization of set operations and set relationships, (3) formative assessment for students' main takeaways from classroom activities, and (4) student challenges.

Our analysis reveals that (1) students' active participation in sharing their reasoning and reflection on their learning with their peers contributed to developing their collective understanding of logic for mathematical statements and proofs. In addition, (2) the incorporation of mathematical representations (e.g., Euler diagram) and analogies from everyday language (e.g., an empty Hermès Bag for an empty set, fried eggs for proper subset relationships, and Mastercard for non-disjoint relationships) served as a valuable pedagogical tool. These aids fostered student engagement in set-based reasoning for logic. Furthermore, (3) encouraging students to share their main takeaways with their peers in the last five minutes of class and through online discussion forums emerged as an effective formative assessment strategy in gauging students' comprehension and progress. We attended to (4) the challenges faced by students when conveying their reasoning using set-builder notation and Euler diagram, especially those who had prior experience with truth tables for logic and resisted using these new tools introduced in the course for set-based reasoning. Addressing these challenges is an ongoing focus as we refine the course and support student learning in the inquiry-oriented transition-to-proof course.

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References

Dawkins, P.C. (2017). On the importance of set-based meanings for categories and connectives in mathematical logic. *International Journal of Research in Undergraduate Mathematics Education,* 3, 496–522. https://doi.org/10.1007/s40753-017-0055-4

Fostering Set-Based Reasoning for Mathematical Proofs: Student Interaction and Challenges in an Inquiry-Oriented, Transition-to-Proof Course

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Gotta run? **Scan here** for a copy of our poster!

Objective

We provide salient insights into an **inquiry-oriented** transition-to-proof (**TTP**) course designed to cultivate undergraduate students' **set-based reasoning** for mathematical proofs.

Aspects of Student Engagement

- Peer-to-peer interaction
- Interaction with the instructor team
- Assignments and exams
- Online discussion forum posts

Curriculum Implementation

Our curriculum was implemented in an in-person TTP course in an inquiry-oriented classroom during the Fall 2023 semester at a large public southwestern university in the United States.

- Unit 1: To cultivate students' set-based reasoning about logic for mathematical proofs (Dawkins, 2017)
- Unit 2: To cultivate proof reading comprehension through apprenticeship
- Unit 3: To humanize mathematics through engaging with proof authors' stories

Analytic Foci

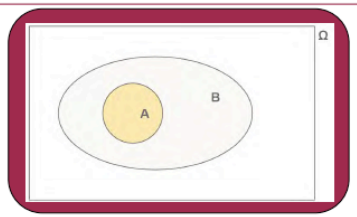
A(1) Students' participation in building the learning community's acknowledgment of mathematical logic

A(2) Instructional deliveries in the utilization of set operations and set relationships

A(3) Formative assessment for students' main takeaways from classroom activities

A(4) Student challenges

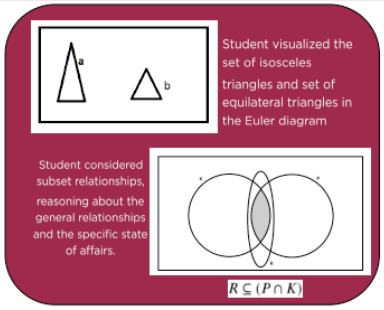
Normative Set Visualizations



Refer to A as the "yolk" of a **fried egg** and B as the "egg white".

Instructor used this description and students adopted these analogies in their verbal and written peer discussions.

Nonnormative Adoption of Set Visualizations



Multimodal **peer interactions**, **set-based reasoning** tools, and active **reflections** help students make meaning in **transition to proof classroom.**

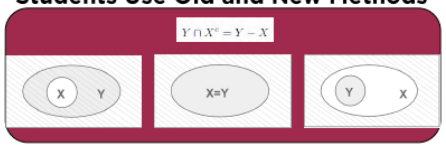
Student Reflections

"Truth sets which equal the empty set have complements that are the universal set and that set can show what proves the original statement!"

"I've found myself using set notation, structure, and different proof approaches in my *linear algebra* class. It's been very helpful using a structured argument rather than winging it!"

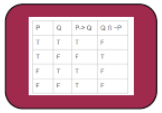
"I see sets all the time in my linear algebra class...subsets also come up a lot when talking about vector spaces."

Students Use Old and New Methods



Student used the Euler visualizations, guided by instructional assistant, **to verify a tautology** using set-based reasoning

Some students resisted the use of set-based reasoning, instead using previously acquired tools such as truth tables to evaluate claims about sets throughout the course.



All images are researcher generated reproductions of observed student work and are not authentic student generations

Conclusions

C(1) Students' **active participation** in sharing their reasoning and reflection on their learning with their peers.

C(2) Incorporation of **mathematical representations** (e.g., Euler diagram) and **analogies** from everyday language (e.g., fried eggs for proper subset relationships) served as a valuable **pedagogical tool**.

C(3) **Students shared their reflections** in class and in the online discussion forum, **revealing their understanding** of key ideas and application of these tools to other contexts.

C(4) Initial **student challenges** included using new tools in a nonnormative manner or using old tools such as truth tables.

Overall, students exhibited progress and gained ownership in adopting the set-based reasoning tools normatively across various contexts.

Arizona State University

TEXAS STATE UNIVERSITY

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