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# Using Teaching Simulations to Launch Work on Eliciting and Interpreting Student Thinking

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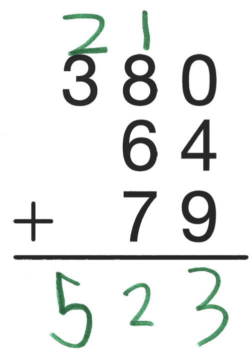
## Introduction

Hannah, a college junior, is taking an elementary mathematics methods course. During the first week of the semester, in lieu of a regular class, she participates in a series of activities, designed to let her try out teaching practices that will be the focus of the course and to explore the specialized knowledge of content needed to teach mathematics. For one activity, Hannah schedules a 30-minute appointment with her course instructor.

In advance, Hannah is given a sample of student work (see Figure 1) with the instructions that she should prepare to talk with this student, with a goal of learning the student's process for solving the problem and the student's understanding of the process or mathematical ideas that underlie the work. In examining the piece, you might notice that the answer is correct and that it appears that the student is using the U.S. traditional algorithm for addition. You might wonder what the student understands about the 2 that is recorded above the 3 or the 1 that is recorded above the 8 (i.e., Does the student understand that the 2 represent 2 hundreds?). You might wonder what the student understands about "when" to carry and why it is necessary to carry. You might also wonder about the strategy that the student uses to add within the tens column (e.g. top to bottom, near doubles etc.). Hannah is instructed to take ten minutes to engage in this sort of analysis and then craft a set of questions to ask the student. Hannah is told that her goal is to learn about the student's current thinking. In this way, the activity is bound to the work of eliciting and interpreting student thinking, removing momentarily, the work that teachers need to do to support the further development of a student's thinking.

**Figure 1**

*A fourth grader's work on an addition problem*



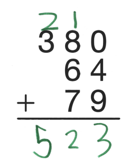
Hannah arrives at her designated time and the teacher educator (TE) prepares her for working with the student. The TE explains that the student is a simulated student and that they will be taking on this role. In taking on role of an elementary school student, the TE is following a carefully scripted protocol that is grounded in research on student thinking, analysis of mathematical algorithms, and anticipation of teacher candidates' (TCs') thought and action that is grounded in the "wisdom of practice" of TEs (Shulman, 1986). The protocol lays out the student's process, understandings, and ways of talking about mathematics (Shaughnessy & Boerst, 2018). The TE reminds Hannah of the purpose of the interaction and how to get started and end the interaction. The TE says:

> You should ask questions to learn about what the student did to produce the answer and what the student understands about the process used and the mathematical ideas that underly that process. By the end of the interaction. you should be able to anticipate what the student would do on a similar problem and what the student would understand about the process used. You do not need to teach the student anything new. You can just begin asking the student about the work right away. You do not need to have a conversation to break the ice with the student to get started. You have five minutes for your interaction with the student.

You can stop when you feel that you have been able to elicit the student's process and the student's understanding of that process. Once you have elicited this information, you can say something like: "I'm finished."

The TE asks Hannah if she has any questions and then starts the video recording.

| | | |
|---|---|---|
| 1 | Hannah: | So looking at his problem, can you tell me what your first ideas was of how to add these together. |
| 2 | Student | Well, I started over here by adding these numbers to make thirteen. [The student points to the right column as he talks.] |
| 3 | Hannah: | Okay, and then what did you do after that |
| 4 | Student: | I recorded three here [pointing to the 3 in the 523] and I had to put the one up top [pointing to the top of the tens column]. |
| 5 | Hannah: | Okay, so why did you think that you should move the one over to the top? |
| 6 | Student: | There's only room for one number down here [pointing to the ones column], so you usually, I would put the three here [pointing to the ones column] and then you take the one and put it over there [pointing to above the tens column]. |
| 7 | Hannah: | Okay, and then you did the same thing in the middle row that you did on the right |
| 8 | Student: | Uh-huh. I added 'em up |
| 9 | Hannah: | All right. And you got twenty-two so you moved your two |
| 10 | Student: | Yup, that's right |
| 11 | Hannah: | And then you added them together? |



12 Student: Yup. That's right

13 Hannah: Okay. So what do you know what this one [pointing to the 1 that has been recorded at the top of the tens column] represents? Is this one? Is it ten? Is it one hundred?

14 Student: It's one.

15 Hannah: So what does the two represent [pointing to the 2 that has been recorded at the top of the hundreds column]? Just two?

16 Student: Yeah.

The interaction continues with Hannah asking about the value of the 3 in 380. Eventually, Hannah says "I'm done." The interaction takes about three minutes.

Then, the TE switches out of the role of the student and follows up with a series of questions focused on what Hannah has learned about the student's process and understanding, including the extent to which Hannah's interpretations are evidence-based. Hannah is asked about the student's understanding of the value of the carried digit and the student's strategy for adding digits within a column. Further she is asked to use the student's process to solve a similar problem (see Figure 2).

**Figure 2**

*A Second Addition Problem*

$$\begin{array}{r} 1\,6\,4 \\ 4\,7\,9 \\ +\;3\,0\,8 \\ \hline \end{array}$$

The session ends with a feedback conversation. The TE begins by asking whether Hannah has any questions.

17 Hannah: I didn't think to ask about the order he was actually adding, like the individual lines because when they are individual digits, like my own person way of doing it is to do it however it looks easiest.

18 TE: So, when you were adding these together [pointing to the ones column in Figure 2], you sort of instantly can't see the answer to eight, nine and four, so you have to pick and choose how to proceed and you might have different reasons for different situations

19 Hannah: Right.

20 TE: If you would have asked, the student would have talked about adding the greatest numbers first. And that makes it so that the rest of the numbers could be more easily added onto that total

21 Hannah: [Nods in agreement]

22 TE: But some students might have a handy thing where they see a ten and they do the ten first. So, it's a little bit of an opportunity to see into a student's strategy. In this problem (see Figure 1), there isn't much to ask about sequence in the ones column because there are just two digits and there is not much to ask about sequence in the hundreds place because there are just two digits, so there is only one sweet spot, the tens column, to find out something about the student's strategies for adding multiple digits

23 Hannah: Yep.

24 TE: Your prompt was more along the lines of, 'did you do the same thing in the next column?' so then you don't get a chance to hear

more about what the student was thinking. So instead of, ‘did you do the same thing?’, you could ask ‘what did you do when you added this second column?’ to gather more information. It’s subtle but it could make a difference.

25 Hannah: [Nods in agreement]

26 TE: Your questions really zoomed in on something that mattered, namely that you were really trying to find out what the student understands about the carried digit. As you continue in the program and you discuss different methods for addition, there are points in different approaches where it is particularly important to ask about a student’s understanding because children typically have differing degrees of understanding there or it is something that isn’t very apparent from the way that things are written. We care both about students being able to proficiently compute and understanding of their processes. If they have understanding and they forget a step, they have something to fall back on to figure out the step. You were persistent in your questioning about it.

The conversation continues. By the end, the TE has described and reinforced two aspects of eliciting student thinking that were demonstrated (posing questions that zoom into key parts of the process or understandings and persisting with and modifying questions to find out about particular understandings) and described two potentially generative moves to try and why those are important (asking about components of the process that are not revealed through the and asking questions in ways that invite students to share thinking). The TE has also unpacked big ideas around the mathematics featured, including the meaning of the carried digit and situational strategies for adding. Of course, the conversation has not addressed all facets of the teaching that was enacted. For instance, toward the end of her interaction with the student, Hannah seeks agreement/disagreement with a series of hypotheses about the next steps in the process. This is importantly different than asking about them in terms of securing insight into the student's thinking. The feedback space has limitations, including time and the amount and type of information that is reasonable to expect TCs to process, that require TEs to make choices about what to discuss and how to discuss it. Such choices are influenced by a multitude of factors making the process of providing feedback both nuanced and challenging.

All of Hannah's classmates engage in the same activity. The next week in class, Hannah and her classmates collectively view and discuss other TCs' performances in the same activity. The videos come from a prior year and are deliberately selected to surface moves that are productive as well as moves that might be less productive (moves very similar to the ones Hannah and her classmates had enacted). The TE focuses the discussion on (1) what information was gathered about the student's process and understanding, how it was gathered, and why it was/was not important to gather it, (2) missed opportunities, and (3) questions and/or other observations. Transcripts are used to support the discussion.

After class, Hannah and her classmates each watch video of their own performance, analyze it, and set goals for future work. A portion of Hannah's analysis is shown in Table 1.

**Table 1**

*Excerpt from Hannah's Analysis*

| | |
|---|---|
| Productive Move | I asked the question: "could you tell me what your first idea was of how to add these numbers together?" - The reason I think this is a productive move is |

| | |
|---|---|
| | because I did not put any ideas on him and I did not just ask him to tell me how he solved it. I used a specific enough question that I knew I would get to learn his thought process starting from the very beginning, but an open enough question that he did not think I was looking for something in particular. |
| Move in Need of Revision | I asked him "Do you know what this one represents? Is it 1, 10, 100 ...?" - This was a really leading question that could have stopped him from telling me what he thought it really was. It also may have prompted him to stop and think that maybe its not just a 1, but something else since I made the suggestion that it could be. This is in need of revision because again, I did a little bit of filling in student thinking rather than letting him tell me his idea. To revise, I would not ask "do you know...", but instead would ask "what does this 1 represent?" (while pointing to the number I am talking about). |
| Goals | 1. To avoid filling in student thinking when asking questions that are meant to elicit their thinking. I think I do a nice job of asking a good initial question, but then using that information, I fill in the rest of their thinking instead of asking more open-ended questions.<br>2. Become more comfortable with silence and not knowing what to ask next right off the top of my head. I think some of the reason behind why I filled in thinking was because I did not know what to ask next yet, and I thought I had a good idea of their current understanding. In the future, I want to be able to better recognize when I know enough, and not to keep asking unhelpful questions for the sake of it. This, and just being patient with |

| | |
|---|---|
| | myself in the moment and give a second to come up with a truly beneficial question. |

**Framing Approaches to Enhance Teacher Candidates' Capabilities with Eliciting and Interpreting Student Thinking**

The National Council of Teachers of Mathematics Principles to Action (2014) explicitly names *elicit and use evidence of thinking* as one of the eight research-supported teaching practices that must be consistently utilized to support student learning. This includes the work that teachers do to learn about and make sense of student thinking, which we refer to these practices as "eliciting and interpreting practices." Teachers need to be able to ask questions that encourage children to share their thinking about content, listen to what children are saying, ask follow up questions linked to important disciplinary ideas that build on children's ideas, and make sense of children's thinking. These practices are central to everyday classroom teaching (AMTE, 2017). The ways in which teachers elicit and interpret student thinking supports the development of students' mathematical identities, including broadening students' senses of what it means to be mathematically competent, their learning of mathematics itself, and their flourishing in school (NCTM, 2014). They are core to a more equitable and just teaching practice in which teachers seek out, value, and utilize the ideas and resources that students bring to instruction. By surfacing the resources that students bring, eliciting student thinking provides a mechanism to disrupt patterns of oppression and inequity. In contrast, less skilled eliciting and interpreting can lead to the marginalization of students by failing to recognize and leverage students' mathematical resources.

Eliciting and interpreting student thinking has long been a focus of work in teacher preparation. Common approaches include using video to notice a teacher's questions (i.e., the work of eliciting student thinking) and to practice making claims about a student's understanding (i.e., interpreting student thinking). This sort of approach is often coupled with field-based interviews (see Shaughnessy et al., 2019 for a discussion of the limitations of field-based interviews). Since 2011, we have aimed to develop and study teaching simulations as a usable complement to – or if the situation dictates, an alternative to – assessment of engagement in field contexts or through other pedagogies of enactment.

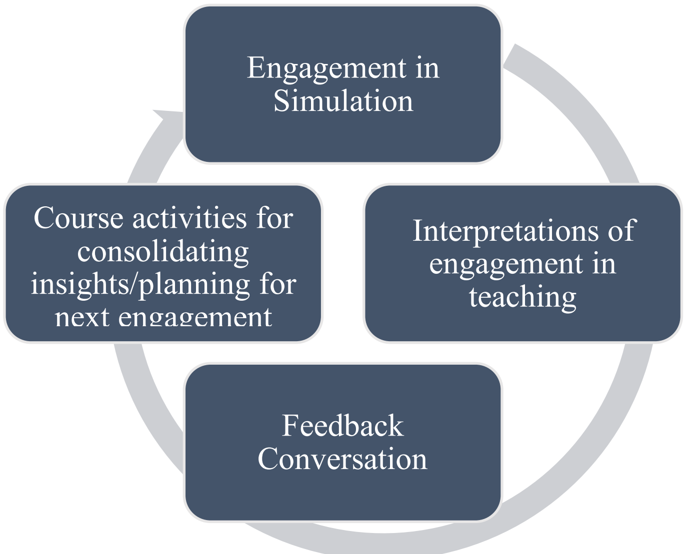
Teaching requires combining instructional techniques and skills together with complex specialized knowledge of the content, insights into students' thinking and development, and the ability to act in light of professional commitments to the learning of all students. Simulations provide a mechanism for customizing teaching situations that strategically juxtapose particular teaching practices, mathematics content, and other contextual factors. By teaching simulation, we mean an approximation of practice that places a TC in a situation that requires authentic engagement in the work of teaching while at the same time standardizing or controlling the context of that teaching in ways that support focus on a particular practice, element of content, or student interaction. Simulations present several advantages for assessing TCs in relation to the questions of when, where, and what of formative assessment. They can focus on teaching without the need to be in a field context. Further, since simulations do not involve TCs directly with students, simulations enable TEs to collect information about the knowledge, skills, commitments, and resources that TCs bring to initial preparation and enable multiple collection points across a program. They can be constructed to embody particular teaching situations that might not be uniformly available. They position TEs close to the action (in terms of time and distance), which makes them amenable to providing feedback and gathering information useful for subsequent learning experiences. Far from being untested, there is much to draw on from the use of simulations in other professional fields. Other practice-intensive professions, such as medicine and law, have invested heavily in human resources, finances, and policies to ensure that clinical simulations are used to enhance professional practice (e.g., Qayumi et al., 2014; Feinman, 1995).

## Recapping the Activity

This activity is designed to build TCs' skills with eliciting and interpreting student thinking in the early stages of a teacher preparation program. As shown in Figure 3, the cycle of work includes four parts. In the first part, each TC engages in a one-on-one teaching simulation in which they are tasked with asking questions to learn about a student's process for solving a mathematics problem and the student's understanding of the process and the mathematical ideas underlying the process. In the second part, each TC is asked a series of questions about what they learned about the student's process and understanding. Third, the TE engages each TC in a feedback conversation. Finally, the TCs participate in a course activity together that is aimed at consolidating insights and planning for subsequent teaching. At the University of Michigan, we have used this activity as a launch to the program. At Boston University, we have used this as a launch activity for the mathematics methods course which occurs after foundation coursework but prior to other methods courses. We have found this to be a powerful means to launch attention in our courses to the ways that teachers ask questions to learn about student thinking, including how teachers position students, and the evidence base that teachers use for making claims about student understanding. This activity serves as springboard into having TCs work on eliciting and interpreting student thinking in the context of their clinical placements.

**Figure 3**

*Activity Structure*




**Discussion Questions**

1. In this chapter, you saw a subset of the feedback that the TE provided to Hannah. Is there other feedback that you would provide? Why?
2. Addition, with a focus on the "US Traditional Algorithm" is the mathematical focus of this teaching. What sort of mathematical focus would be most useful in your context?
3. What resources might you need to implement teaching simulations in your context? What sorts of support might be available to you?

Clinical Practice Activity

In your own context, have TCs engage in a teaching simulation focused on eliciting student thinking and the activities surrounding it. Then, have TCs plan for a student thinking interview in their field placement (Sleep & Boerst, 2012). To support TCs in planning, group

TCs by their clinical placement grade level and provide a set of grade-level appropriate tasks (see Sleep & Boerst, 2012 for examples of such tasks). Support TCs to collaborate to discuss the mathematical tasks that they will pose to a student in their placement, anticipate student thinking, and design questions to elicit and probe student thinking. In their clinical placement, each TC should interview one student using the tasks and video record the enactment. TCs should analyze their video, focused both on their moves to elicit and probe student thinking, and what they learned about the student's thinking. Ideally, TEs watch the videos and provide feedback on eliciting moves and the extent to which interpretations about student thinking match the evidence gathered in the interaction.

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