

## A CASE OF HOW AN ELEMENTARY MATH TEACHER ATTENDED REFERENT UNIT THROUGH PROFESSIONAL DEVELOPMENT

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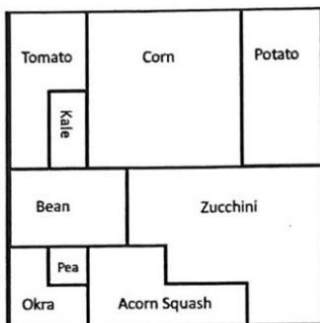
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Referent units in fractions are often overlooked by elementary and secondary math teachers. Research highlights the challenges teachers face in presenting fractions effectively (Copur-Gencturk & Ölmez, 2022; Izsák et al., 2019; Wang et al., 2023). This study, grounded in Knowledge in Pieces (KiP) (diSessa, 2016, 2018), aims to provide insights into PD initiatives designed to enhance teachers' attending of referent units, crucial for future fraction teaching.

This intrinsic case study (Stake, 1995) focuses on Marcus, a middle school teacher with six years of experience. He participated in a weeklong PD including solving fraction problems and discussing referent units. Analysis centered on Marcus's intuitive (pre-PD interview data) and developed understanding of referent units (post-PD reflection data).

Before the PD, Marcus intuitively partition a garden to rows and did not realize he used a row as his referent unit. Post-PD, he began connecting these intuitive methods with a clearer understanding of the referent unit. For example, in the Mathtopia problem, Marcus initially misinterpreted the referent unit (e.g., "Okra of  $3/16$ "), but adjusted his thinking with facilitator guidance, leading to a better attending of the where the whole appears (e.g., "Okra is  $3/64$ ").



**Figure: Mathtopia problem used in PD**

Our study aligns with existing research, revealing a notable issue among teachers struggling to keep track of referent units in fraction operations. This highlights the necessity for targeted support to help teachers like Marcus establish stronger connections between partitioning strategies and a consistent awareness of the referent unit.

Kosko, K. W., Caniglia, J., Courtney, S., Zolfaghari, M., & Morris, G. A., (2024). *Proceedings of the forty-sixth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Kent State University.

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### References

- Copur-Gencturk, Y., & Ölmez, İ. B. (2022). Teachers' attention to and flexibility with referent units. *International Journal of Science and Mathematics Education*, 20(6), 1123–1139.
- Izsák, A., Kulow, T., Beckmann, S., Stevenson, D., & Ölmez, İ. B. (2019). Using coordinated measurement with future teachers to connect multiplication, division, and proportional relationships. *Mathematics Teacher Educator*, 8(1), 49–75.
- diSessa, A. A., Sherin, B. L., & Levin, M. (2016). Knowledge analysis: An introduction. In A. A. diSessa, M. Levin, & J. S. Brown (Eds.), *Knowledge and interaction: A synthetic agenda for the learning sciences*, pp. 30–71. New York: Routledge.
- DiSessa, A. A. (2018). A friendly introduction to “knowledge in pieces”: Modeling types of knowledge and their roles in learning. In *Invited lectures from the 13th international congress on mathematical education* (pp. 65–84). Springer International Publishing.
- Stake, R. E. (1995). *The art of case study research*. Sage.
- Wang, K., Orrill, C. H., & Brown, R. (2023). Recognizing referent unit in fraction multiplication problems: is the whole always the same? In Lamberg, T., & Moss, D. (Eds), *Proceedings of the 45th annual meeting of the North American Chapter of the International Group for Psychology in Education* (Vol.1, pp. 72–76). University of Nevada, Reno.

Kosko, K. W., Caniglia, J., Courtney, S., Zolfaghari, M., & Morris, G. A., (2024). *Proceedings of the forty-sixth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Kent State University.