DYNAMIC REPRESENTATION AS A TOOL FOR TEACHERS' CONNECTION MAKING

Chandra Hawley Orrill	Rachael Eriksen Brown	Kun Wang
Rethink Learning Labs	Penn State Abington	Rethink Learning Labs
chandra.orrill@rethinklearning.com	reb37@psu.edu	kun.wang@rethinklearning.com

In this paper, we present Epistemic Network Analysis of 32 teachers knowledge resources used to solve two different tasks. Using Drijver's (2018) framework about ways technology can be used for learning, we argue that using technology supports teachers to activate more knowledge resources and to use them in connected ways. We propose that this may offer insight into the design of professional development aimed at supporting teachers in the development of connected knowledge.

Keywords: Technology, Teacher knowledge, Rational Numbers and Proportional Reasoning

Purpose

In this paper, we address two research questions:

- What knowledge resources do teachers use to solve two proportional reasoning tasks?
- Are those knowledge resources connected in ways that seems to support Skills Practice, Conceptual Understanding, or some combination of both?

We argue that technology provides a different experience than static tasks for teachers as they engage with mathematics, thus allows a different kind of thinking. Further, we suggest that using technology to think about mathematics opens opportunities for teachers to develop more connected understandings of the mathematics they teach. We end with a discussion of the implications of this work.

Perspectives

We draw from two perspectives to make our case for the value of technology in supporting teacher conceptual understanding. First, we use a Knowledge in Pieces (KiP) lens to contemplate cognition and how it can be conceived of in teacher learning. Then, we draw from Drijvers' work on technology in mathematics education to situate the ways in which we use technology.

Knowledge in Pieces

Knowledge in pieces is a theory of conceptual change that considers knowledge to be comprised of fine-grained resources (diSessa, 1988, 2018; diSessa, et al., 2016). In KiP, learning can be viewed as creating new knowledge resources, refining existing knowledge resources, and/or creating connections between and among knowledge resources. For this paper, we are particularly interested in the connections participating teachers were making between their knowledge resources.

To make knowledge resources visible and to focus on interactions between knowledge resources, we rely on Epistemic Network Analysis (ENA; Shaffer et al., 2009). ENA is a quantitative ethnography (Shaffer, 2017) method that, in our case, allows knowledge resources to act as nodes and for lines between nodes to indicate the relative frequency of the co-occurrence of the knowledge resources. A co-occurrence, for us, is any instance in which more than one knowledge resource is used to solve a single task. We consider co-occurrences to be proxies for connections between knowledge resources.

Technology in Mathematics Education

In their conceptualization of the uses of technology in mathematics education, Drijvers and his colleagues (e.g., Drijvers, 2015, 2018) have developed a heuristic for technology use in mathematics education. They posit that the ways in which we think about technology in mathematics education should be driven by the function of the technology rather than the form of the technology. Thus, they conceive of technology as either helping us Do math or Learn math. And, in the case of technology that helps us Learn math, there is technology that helps with Skills Practice and technology that helps with Conceptual Development. It is this kind of technology that is the focus of this paper. By using KiP, we are able to think about knowledge as a series of fine-grained understandings that can be grouped in myriad ways. By using the Drijvers framework, we have a language for characterizing the ways in which the knowledge resources are interacting.

Methods

The data reported here comes from a pair of interviews conducted with a convenience sample 32 middle grades teachers from four states. The first interview was a think aloud protocol in which the participants responded to a set of tasks using a LiveScribe pen to capture and coordinate their voices and inscriptions. The second interview was a face-to-face interview with similar mathematics tasks, however some of the tasks used dynamic sketches on an iPad. The live interviews were recorded with two video cameras: one focused on the participants' faces and one on anything they wrote or at which they pointed.

For this analysis, we focused on two tasks: the Santa Task (Figure 2) and the Bears Task (Figure 3). For the Santa Task, which was based on a task from de Bock, Van Dooren, Janssens, & Verschaffel, 2002), we provided scaffolded prompts to support teachers in thinking about the situation. These included:

- Ms. Yarbrough's class had two favorite answers. About 40% of her class chose 18 ml and about 40% chose 54 ml. What might the students who were wrong been thinking about? Is that something you see commonly with your own students?
- One of Ms. Yarbrough's students drew rectangles around the images. Do you think this is a helpful strategy for a student? Why or why not?

For the Bears Task, we asked the teachers:

- Describe what is happening as you drag the slider. How is the image changing? When we started, the two figures were similar. Where you have ended, are they still similar?
- How would you characterize the growth as you move the slider? Is there anything in the relationship of the bears to each other that stays the same? How would you describe how many times larger the new bear is than the original? How would you describe the scale factor of the new bear to the original?

- What is the ratio of the bear's belly to the frame? Does that ratio stay the same as the bear change size?

These tasks were selected for this analysis because they engaged the teachers with similar mathematics, though Santa was done on paper and Bears was an interactive app.

Data were coded using an emergent coding scheme (Weiland et al, 2021) in which each code was a knowledge resource. We used ENA for the data analysis (Shaffer et al., 2009), which meant that each utterance was coded using a binary system (present/not present). In this case, an utterance was the answer to a single task. Once coding was done, the ENA webtool created the maps of participants' use of knowledge resources. For these maps (Figure 2a & b), the nodes indicate the knowledge resources being used by the participants. The size of the node is relative to its frequency in the dataset. The lines connecting knowledge resources indicate that those pairs of knowledge resources occurred together within an utterance. The thickness of the line indicates the relative frequency of the co-occurrence.

Bart is a publicity painter. In the last few days, he had to paint Christmas decorations on several store windows. Yesterday, he painted a 56 cm high Santa on the door of a bakery. He needed 6 ml of paint. Now he is asked to make an enlarged version of the same painting on a supermarket window using the same paint. This copy should be 168 cm high. How much paint will Bart need to do this?

Figure 1. The Santa Task (based on de Bock, Van Dooren, Janssens, & Verschaffel, 2002)

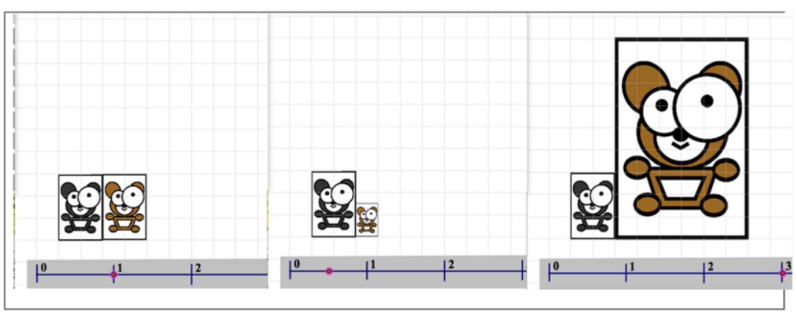


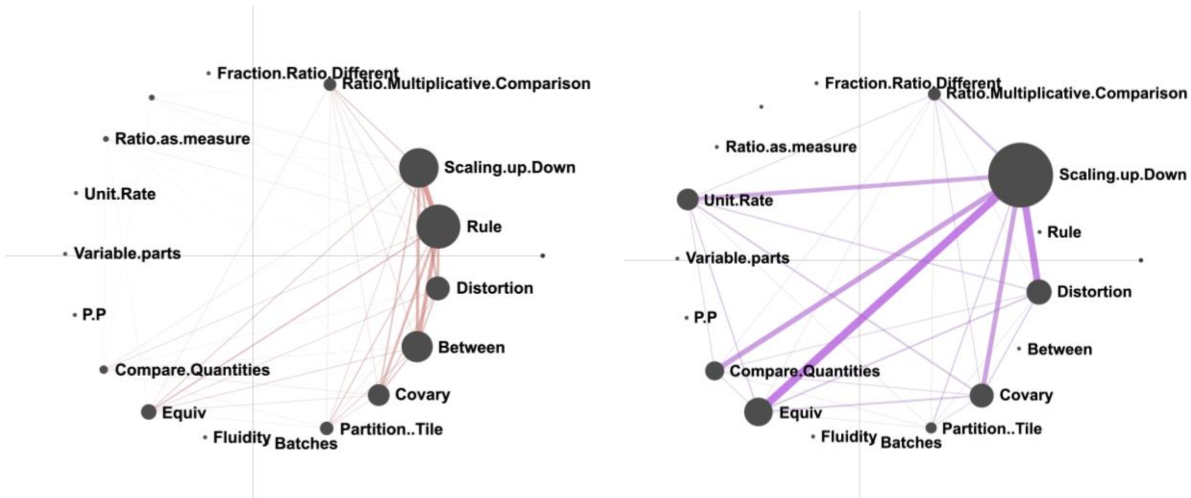
Figure 2. The Bears Task

Results

As shown in the ENA graphs (Figure 3a & b), the teachers relied heavily on Rules (mostly cross multiplication) and Scaling Up and Down to solve the Santa Task. Because they jumped to these two procedures, most of the teachers missed that the relationship between height and amount of paint is not a proportional one. The proportional relationship is between the area of the big Santa and the area of the small Santa (for a full qualitative analysis of the answers given, see

Kosko, K. W., Caniglia, J., Courtney, S., Zolfaghari, M., & Morris, G. A., (2024). *Proceedings of the forty-sixth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Kent State University.

Brown et al., 2020). There were few connections made between knowledge resources, meaning that teachers often went straight to an algorithm and did not invoke other proportional knowledge resources. Most interestingly, most of the resources they did use were focused on answer finding, which is more related to Skills Practice than Conceptual Understanding (Drijvers, 2018). We argue that the only structures they attended to in connected ways were Covariation (e.g., the numerator and denominator change together) and the Between Measure Space relationship (e.g., attending to the relationship of one quantity, such as height, to the other, such as width), both of which were used less than the two skills.



(a) (b)

Figure 3. ENA results for the static Santa Task (a) and dynamic Bears Task (b)

In contrast, on the Bears Task, these same teachers did not rely on Rules at all. Further, there is much more interaction between knowledge resources., overall. While we still see a reliance on procedures with most of these interactions, we do see more variety in approaches. This suggests that there is something about the dynamic representation that both cues more knowledge resources and promotes more interaction between those resources.

Discussion and Conclusions

Given that learning in the KiP framework can happen through development of new resources, refinement of existing resources, or making new connections between resources, we posit that using technology-based tasks is a way to engage teachers in learning. Further, we suggest that doing this may support the development of connections between knowledge resources, which is potentially useful for supporting teacher learning. Consistent with Drijvers (2018) framework, the technology-based task seemed to offer more opportunity for Conceptual Understanding than did the static task. To this end, we propose that developing professional development that is designed to take advantage of technology in ways that supports the development of conceptual understanding may be fruitful for support teachers in better connecting their already-existing mathematics knowledge resources.

The implications of this are pragmatic. Too often, professional development is focused on building new knowledge without regard to the knowledge teachers already have. Because they are adult learners, often with degrees beyond a bachelors, teachers need professional development that caters to them. As shown in Figure 3, the teachers in our sample all had knowledge resources important for understanding proportional relationships. However, they needed the Bears task to activate some of them and to activate more than one in an utterance.

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References

Brown, R. E., Orrill, C. H., & Park, J. (2020). Differences in knowledge used by practicing teachers in a dynamic versus static proportional task. *Mathematics Education Research Journal*. https://doi.org/10.1007/s13394-020-00350-x

de Bock, D., Van Dooren, W., Janssens, D., & Verschaffel, L. (2002). Improper use of linear reasoning: An in-depth study of the nature and the irresistibility of secondary school students' errors. *Educational Studies in Mathematics, 50*(3), 311–334.

diSessa, A. A. (1988). Knowledge in pieces. In G. Forman & P. Pufall (Eds.), *Constructivism in the computer age* (pp. 49–70). Lawrence Erlbaum Associates, Inc.

diSessa A.A. (2018). A friendly introduction to "Knowledge in Pieces": Modeling types of knowledge and their roles in Learning. In G. Kaiser, H. Forgasz, M. Graven, A. Kuzniak, E. Simmt, B. Xu (Eds.), *Invited Lectures from the 13th International Congress on Mathematical Education. ICME-13 Monographs*. (pp. 65–84). Springer International Publishing. https://doi.org/10.1007/978-3-319-72170-5_5

diSessa, A. A., Sherin, B. L., & Levin, M. (2016). Knowledge analysis: An introduction. In A. A. diSessa, B. L. Sherin, & M. Levin (Eds.), *Knowledge and interaction: a synthetic agenda for the learning sciences* (pp. 30–71). New York: Routledge.

Drijvers, P. (2015). Digital technology in mathematics education: Why it works (or doesn't). In S. J. Cho (Ed.), *Selected regular lectures from the 12th International Congress on Mathematics Education* (pp. 135–151). Springer. https://doi.org/10.1007/978-3-319-17187-6_8

Drijvers, P. (2018). Tools and taxonomies: A response to Hoyles. *Research in Mathematics Education, 20*(3), 229–235. https://10.1080/14794802.2018.1522269

Shaffer, D. W. (2017). *Quantitative ethnography.* Cathcart Press.

Shaffer, D. W., Hatfield, D., Svarovsky, G. N., Nash, P., Nulty, A., Bagley, E., ... Mislevy, R. (2009). Epistemic Network Analysis: A prototype for 21st-century assessment of learning. *International Journal of Learning and Media, 1*(2), 33–53. https://doi.org/10.1162/ijlm.2009.0013

Weiland, T., Orrill, C. H., Nagar, G. G., Brown, R. E., & Burke, J. (2021). Framing a robust understanding of proportional reasoning for teachers. *Journal of Mathematics Teacher Education, 24*, 179–202. https://doi.org/10.1007/s10857-019-09453-0