

Healthcare facility capacity optimization during COVID-19 pandemic using epidemiological models and mathematical modeling

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Abstract

This research proposes a mathematical model to assign COVID-19 patients to healthcare facilities. The study aims to minimize unmet demand and the number of infected people, while accounting for capacity constraints of the healthcare facilities. The number of infected COVID-19 patients is estimated using epidemiological models (i.e., Susceptible-Infectious-Recovered model). The study focuses on 67 counties in Florida, where the number of infected people and healthcare capacity vary between counties. The model takes into account the movement of patients between counties, as patients may need to move to a different county for treatment. The movement of infected individuals between counties further spreads the COVID-19 virus, and this effect is also incorporated into the SIR model.

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1. Introduction

The first Covid-19 cases were identified in December 2019 in Wuhan, China, and by March 2020 it was declared as a global pandemic by World Health Organization (WHO) [1]. Since then, Covid-19 has been a major health problem globally because it is highly contagious and can cause severe illness or death. According to the reports and statistics, as of January 2023, more than 673 million individuals have been infected and about 7 million of positive cases have lost their lives [2]. Yet, there is no effective or definitive treatment for Covid-19 patients, so healthcare facilities such as hospitals faced serious challenges during the pandemic. Healthcare facilities are built over time in different areas and have limited capacity. However, during the pandemic, allocating patients to healthcare facilities and utilizing resources effectively has been a critical issue due to rapid spread of Covid-19 and its transmission rate [3]. For instance, in Italy and Spain, the demand for intensive care unit (ICU) exceeds the available capacity in several hospitals and put healthcare facilities in these countries under heavy strain [4]. The prevalence of the pandemic is not uniform in different regions (i.e., provinces, counties, etc.). There are several influential factors that may affect how pandemic evolves in a region such as, population density, vulnerability, climatology parameters [5]. Besides that, healthcare capacities are not distributed equally in different regions as well, so allocating patients to hospitals located in other regions could be a promising way to circumvent the capacity constraints. Due to the variety of the demand and healthcare capacity, it is possible to adjust patient admissions so that excessive demand is sent to locations with extra capacity. A spontaneous initiative that happened in Madrid (Spain) in early April 2020, when the Spanish capital was seeing a major rise of Covid19 cases, highlighted the clinical necessity for such a system [6].

During large scale pandemics like Covid-19, one of the major challenges is the ability of local health care systems to treat the patients. For example, many rural counties in the U.S. may not have sufficient staffed bed capacity for treating severely ill patients, and it may become necessary for the patients to travel to neighboring counties. Optimal assignment of patients to healthcare facilities located in other regions could be a challenging task when dealing with infectious diseases since the movement of patients can further spread the virus, causing more positive cases. To address the problem, we focus on assigning Covid-19 patients to healthcare facilities to minimize unmet demand and number of infected people. Mathematical models are powerful tools which could be applied to calculate the size, timing, and impact of pandemics. Epidemiological models are one of the models that have been widely used for understanding disease transmission. In this study, we incorporated the SIR model (Susceptible, Infected, Recovered) in a mathematical model as an optimization problem. We used daily Covid-19 case data from Florida at the county level for 150 days to estimate the number susceptible, infected, and recovered individuals over time. After calculating the number of infected people, the mathematical model assigns patients to the healthcare facilities in order to satisfy patients' demand and at the same time minimize number of newly infected individuals due to their movement between counties.

2. Literature Review

Epidemiological disease spread models and optimization models are two research categories that are relevant to this study. Epidemiological models have been extensively studied for the estimation of the dynamics of disease spread and impact of intervention measures on populations with complex social and spatial structures [7]. In particular, the Susceptible-Infected-Recovered (SIR) compartmental model is the most popular model in epidemiology and disease control [8]. Cooper et al., [9] evaluated the efficiency of several modeling techniques for the Covid-19 pandemic. They suggested the SIR model for analyzing the disease's propagation within a community. For example, the disease incubation period can be included in the SIR model by adding the exposed compartment. He et al., [10] proposed control strategies for the Covid-19 pandemic based on a SEIR model. The effect of management strategies such as social distancing and people movement have been investigated in several research papers. Goel et al., [11] examined the impact of lockdown in managing the pandemic by developing two distinct versions of mobility-based SIR model, which included demographic distribution and global connectivity of different locations. In another study, Mo et al., [12] uncovered how regional differences and cross-regional movement of the people drive the spread of epidemics.

Various optimization techniques have been proposed by researchers to address the distribution of patients and healthcare resources, including the scheduling of operating rooms, the allocation of emergency patients, and the distribution of healthcare facilities. Li et al., [13] introduced a multi-objective model for allocation of beds in hospitals. Oddoye et al., [14] proposed a goal programming method for resource allocation in medical centers. In [15], Mitropoulos et al., developed a bi-objective mathematical model for minimizing travel distance between patients and healthcare facilities, and equitable distribution of healthcare facilities among people. Blake and Carter [16] employed goal programming approach to allocate resources in healthcare facilities. With the help of their model, decision-makers can regulate the case mix and volume in hospitals. Sun et al., [17] studied the allocation of patients and resources across healthcare facilities when demand for constrained resources increases due to the influenza pandemic. Tsai et al., [18] utilized different mathematical models to reduce the total distance all patients had to travel during a dengue fever outbreak while allocating patients and additional resources optimally. Lacasa et al., [19] created an algorithm that offers the optimal re-routing alternatives for patients who need to be transferred to Intensive Care Units (ICU) or ventilators, subject to transferability constraints. However, a major gap exists for seamless integration of SIR models with mathematical optimization techniques for decision making in pandemic intervention.

3. Methodology

This research proposes a new mathematical model for optimal assignment of COVID-19 patients to healthcare facilities by utilizing epidemiological disease spread models. The method aims to minimize unmet demand and the number of infected people while accounting for capacity constraints of the healthcare facilities. The number of infected COVID-19 patients is estimated using epidemiological models (i.e., Susceptible-Infectious-Recovered model). In particular, we propose new nonlinear mathematical model formulation that allows optimal assignment of patients to healthcare facilities to minimize unmet demand during a pandemic. Also, due to demographic variables and healthcare capacity differences, Covid-19 patients might need to seek medical treatment in other counties. Therefore, we incorporated the movement of the patients in the proposed model and tried to minimize the number of people getting infected. The notations used to formulate the mathematical model are as follows:

Sets	
i, j	Set of counties $i \in \{1, 2, \dots, m\}, j \in \{1, 2, \dots, m\}$
t	Set of days $t \in \{t_B, t_B + 1, \dots, t_E\}$
Parameters	
N_i, C_i	Population and healthcare capacity in county i
$d_{i,j}$	Distance between county i and j
$S_{i,tB}, I_{i,tB}, R_{i,tB}$	Initial number of susceptible, infected, and recovered individuals in county i
m	Number of counties
β_i, γ_i	Infection and recovery rate for county i
M	Penalty cost of unmet demand
t_B	Beginning time of study period

Decision
variables

$u_{i,t}$	Number of unmet demands in county i in day t
$S_{i,t}, I_{i,t}, R_{i,t}$	Number of susceptible, infected and recovered individuals in county i in day t
$Z_{i,j,t}$	Number of patients moved from county i to county j in day t
$x_{i,t}$	Number of satisfied demands in county i in day t

$$\min_{u_{i,t}, Z_{i,j,t}, I_{i,t}} \sum_{i,t} M u_{i,t} + \sum_i R_{i,t} + \sum_{i,j,t} Z_{i,j,t} d_{i,j} \quad (1)$$

$$x_{i,t} + u_{i,t} = \alpha I_{i,t} + \sum_j Z_{j,i,t} - \sum_j Z_{i,j,t} \quad \forall i, t \quad (2)$$

$$x_{i,t} \leq \frac{C_i}{14} \quad \forall i, t \quad (3)$$

$$S_{i,t+1} = S_{i,t} - \frac{\beta_i S_{i,t} I_{i,t}}{N_i} \quad \forall i, t \quad (4)$$

$$I_{i,t+1} = I_{i,t} + \frac{\beta_i S_{i,t} I_{i,t}}{N_i} - \gamma I_{i,t} + \sum_j Z_{j,i,t} - \sum_j Z_{i,j,t} \quad \forall i, t \quad (5)$$

$$R_{i,t+1} = R_{i,t} + \gamma I_{i,t} \quad \forall i, t \quad (6)$$

$$u_{i,t}, S_{i,t}, I_{i,t}, x_{i,t} \geq 0 \quad \forall i, t \quad (7)$$

$$R_{i,t} \geq 0 \quad \forall i \quad (8)$$

$$Z_{i,j,t} \geq 0 \quad \forall i, j, t \quad (9)$$

The objective function (1) minimizes unmet demand, number of recovered individuals and distance traveled by patients to healthcare facilities. Constraint (2) balances the satisfied demand and unmet demand in each county for every day. In other words, the summation of satisfied demand and unmet demand is equal to number of infected individuals plus incoming patients from other counties or minus patients leaving the county. Constraint (3) demonstrates that satisfied demand cannot exceed the capacity for every county. Constraints (4), (5) and (6) calculate the number of susceptible, infected, and recovered individuals in each county every day. Constraints (7), (8) and (9) indicate the eligible domain of decision variables.

4. Results

The proposed methodology is applied to a case study involving healthcare facilities in 67 counties of Florida. The COVID-19 daily infection counts reported by the Florida Department of Health at the county level for the period between July 22 and October 18, 2020 (150 days) are used to estimate the β and γ parameters of the SIR model through a maximum likelihood-based approach to estimate the number of susceptible, infected, and recovered individuals. After estimating the model, we use the GAMS software with Baron to solve the proposed mathematical model for the days between $t_B = 150$ and $t_E = 300$.

Figure 1 shows the infected patients assigned to other counties on day 40 and 130, respectively. The thickness of the edges is proportional to patient flow ($Z_{i,j,t}$), and the size of the nodes is proportional to the number of available beds (C_i) in each county. As depicted in Figure 1, Brevard county is one of the main destinations for patients due to its relatively high capacity and location. It is also evident from Figure 1 that counties with large healthcare capacities do not receive patients from other counties because they are highly populated, and the number of infected individuals is high.

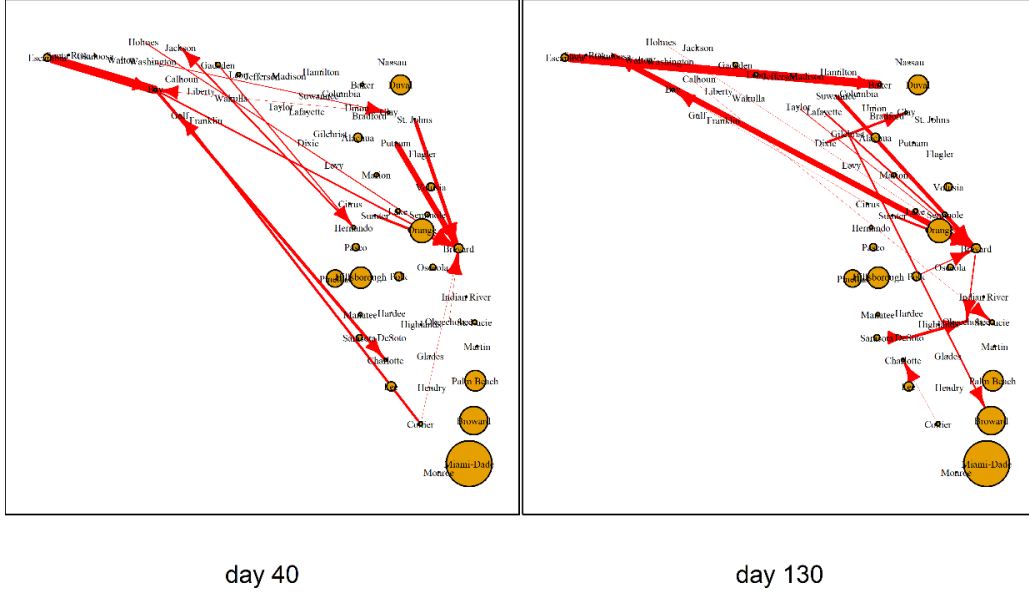


Figure 1. Assignment of the patients to other counties in day 40 and 130.

Furthermore, the number of infected individuals and transferring patients between counties reduces over time, highlighting the importance of proper healthcare management in the early stages of a pandemic, which could significantly decrease the number of infected people, as seen in Figure 2. By comparing Figure 2.A and Figure 2.B, we can see that adopting suitable policies such as assigning patients to other counties and managing the healthcare system at a higher level (i.e., state level instead of county level) significantly reduces the number of unmet demands. One interesting finding is that some patients are assigned to healthcare facilities far away, even though the objective function tries to minimize the distance traveled by patients. This is because the penalty cost of unmet demand is significantly high to ensure that all of the available capacity in all healthcare facilities is used every day. For example, the model assigns patients from Collier county to Gulf county or from Bay county to Brevard county on day 40. Figure 2A shows that after 50 days, the unmet demand decreases by about 95% when cross country movement is allowed, whereas when cross country movement is not allowed, the unmet demand only decreases by about 47%.

For a further comparison, we solved the optimization model without assigning patients to other counties as a benchmark case. That is, in this case patients are restricted to stay within their home county for their treatment and we achieved this by modifying the mathematical model to exclude $Z_{i,j,t}$ and $Z_{j,i,t}$ variables. Our analysis of the resulting solution revealed that eliminating these variables had a positive effect on $u_{i,t}$ in the second constraint, as indicated by the proposed mathematical model. The total unmet demand trajectories of both the proposed method and the benchmark case over all 67 counties over the pandemic duration of 150 days is plotted in Figure 2. It can be seen that as the infections wane over time the number of unmet demands decrease with both cross county movement (proposed method) and no cross county movement (benchmark case). However, the rate of decay is much faster with the proposed approach demonstrating the efficacy of the proposed cross county coordination approach during a large-scale health event.

5. Conclusion

This research presented a novel mathematical model that efficiently assigns Covid-19 patients to healthcare facilities in 67 counties of Florida. The model takes into account epidemiological factors, capacity constraints of healthcare facilities, and patient movements between counties. By minimizing unmet demand and the number of infected people, this model can help healthcare authorities to manage the Covid-19 pandemic effectively. Our findings demonstrate that cross county coordination and centralized healthcare management are more effective at reducing unmet demand than separate county-level treatment. The results of this study are relevant for policymakers and healthcare providers who need to allocate resources and control the spread of the virus. Overall, this model can assist in improving the

healthcare system's efficiency and reducing the burden of Covid-19 on society. The findings of this study could provide valuable insights for managing potential pandemics in the future. The model proposed in this paper can be customized by incorporating local factors such as population demographics and healthcare capacity, making it adaptable to other regions or countries. This process of adaptation can facilitate the optimization of patient allocation to healthcare facilities during pandemics and provide a framework for effective management in different settings. In the future, it will be important to explore ways to tune the unmet demand parameter and investigate the different terms included in the objective function of the model.

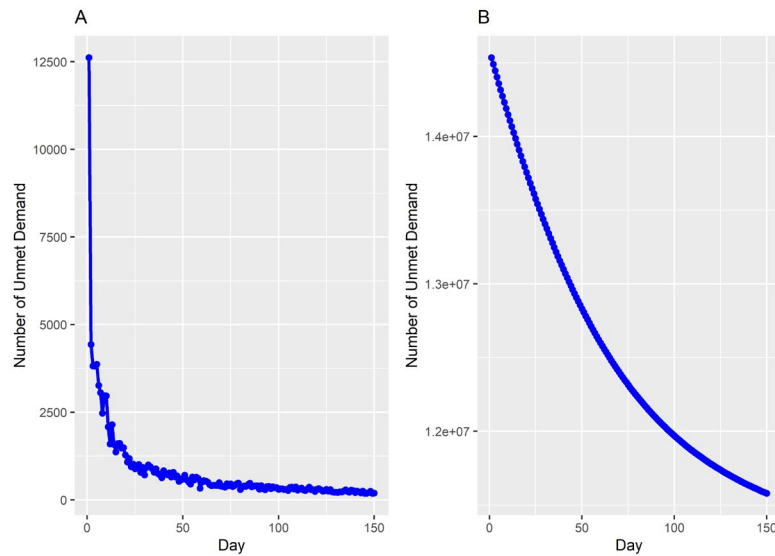


Figure 2. Comparison of number of unmet demands; (A) with cross county movement, (B) without cross county movement.

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