Pulse Shape Estimation for Bio-Magnetic Signals under Low SNR via Translation-Invariant Denoising and Phase Retrieval

Ali Kaiss, Kiryung Lee, and Asimina Kiourti
Dept. of Electrical and Computer Engineering, The Ohio State University, Columbus, OH 43210, USA

***Abstract*—The human body naturally emits magnetic fields whose detection is vital for diagnosing various health conditions. In our lab, we have previously demonstrated a wearable sensor that detects the magnetic field of the heart. Unfortunately, the sensed bio-magnetic signals are usually very weak, resulting in a noisy signal. In this work, we overcome this limitation by proposing a method that denoises and reconstructs a semi-periodic bio-magnetic signal using its analytical model. The approach is based on segmenting the signal into several windows, and then averaging the magnitude of the Discrete Fourier Transform (DFT) of each window. Once that is complete, phase retrieval is done on the signal's derived analytical model to reconstruct the noise-free signal. Simulation results show that the proposed approach achieves an error of less than 5.71% with SNR= -7.45 dB when compared to a signal that is perfectly periodic. The proposed approach is expected to have significant impact on the detection of bio-magnetic signals that can be obtained in a non-contact and non-invasive manner but suffer from noise.**

***Index Terms*—Bio-Magnetic signals, Magnetocardiography (MCG), Windowed Discrete Fourier transform (WDFT).**

I. Introduction

The flow of ionic current through the human body results in naturally emanated bio-magnetic fields radiated by the muscles, heart, brain, nerves, and more [1] [2]. These signals provide a great advantage when it comes to diagnostics as they can be retrieved in a non-contact and non-invasive manner. They also propagate unaltered through the biological tissues that are not magnetic in nature. However, bio-magnetic fields are extremely weak, making their sensing very challenging [3].

For example, in the field of MagnetoCardioGraphy (MCG), state-of-the-art sensors entail Super Conducting Quantum Interference Devices (SQUIDs) and Atomic Magnetometers (AMs). These devices are extremely sensitive and, thus, able to detect weak magnetic fields. That is, they result in a signal with high Signal-to-Noise ratio (SNR) [4]. However, these devices have several limitations: they operate at very low temperatures (achieved by liquid helium cooling systems); are highly expensive, bulky, and sophisticated to fabricate; and/or require shielding [5] [6].

To overcome these limitations, we have recently demonstrated a portable MCG sensor that does not require shielding; is low-cost (in the order of tens of dollars) and simple to fabricate; and does not require shielding. The sensor design is detailed in [2], but, in brief, it consists of an array of 8 coils that capture the heart's magnetic field via Faraday's law. However, when it comes to SNR, our sensor is not as sensitive as SQUIDs or AMs, resulting in a captured signal that is noisy.

In [7] [8], we reported an algorithm that was capable of locating the heartbeats in the MCG signal obtained through our sensor. Although promising results were achieved, the algorithm assumed a constant period between consecutive heartbeats. However, this is not the case as the heartbeats vary in duration due to several physiological factors. In this work, we propose a time-invariant method of detecting the heartbeats in the MCG signal, i.e, the proposed method is unaffected by the varying period between consecutive activations within a signal. Notably, the method is suitable for high-noise environments, such as those encountered in bio-magnetic signal detection. Without loss of generality, the approach applies to any semi-periodic bio-magnetic signal, besides MCG.

II. Proposed Signal Processing Approach

Let $z[l] = z(lT_s)$ represent samples of a (bio-magnetic) periodic signal, defined as:

$$z[l] = \sum_{i=0}^{M} u[T_s l - iT_p] \tag{1}$$

where $u[l] = \sum_{j=1}^{5} A_j \exp\{-(l-\delta_j)^2/\sigma_j^2\}$ for all $j \in [1,\ldots,5]$, T_s is the sampling period, and T_p is the signal's period. It is important to note that $u[l]$ is an example, and the method is not restricted to this particular model. Since $\pm 3\sigma$ includes 99.7% of the signal's power, the sampling period is chosen such that $T_s \ll 1/B := \min_j 6\sigma_j$ where B denotes the effective bandwidth of the signal. In this example, the pulses of Eq. (1) are included at synchronized locations, thus, simple averaging can provide a denoised estimate of the pulse shape. However, this is not the case for some bio-magnetic signals. In this case, the updated signal model is defined as:

$$\hat{z}[l] = \sum_{i=0}^{M} u[T_s l - iT_p - \tau_i] + n[l] \tag{2}$$

where τ_i is the random shift, and $n[l]$ is the noise. In this case, simple averaging would not be beneficial. One would argue that correlation can be used to synchronize the pulses across the segments. However, this will not be effective in low SNR regimes, which is the case for bio-magnetic signals. Therefore, we propose the following approach. The first step in our method is to segment $\hat{z}[l]$ into windows where each window contains one pulse. Then, Discrete Fourier Transform (DFT) is applied to each window to convert the signal into the frequency domain. Once complete, the DFT magnitude

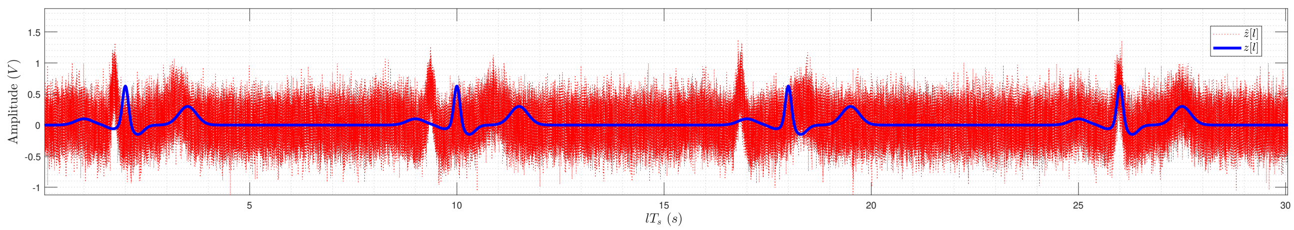

Fig. 1. Pulses of constant T_p ($z[l]$ in blue) and varying T_p with noise ($\hat{z}[l]$ in red).

of each window is taken and averaging is finally performed over all windows to obtain $\hat{Z}[k]$. This process ensures that noise gets averaged out across the signal despite the varying shifts $\{\tau_i\}_{i=0}^{M}$. Given the denoised $\hat{Z}[k]$, multiple algorithms can then be applied to retrieve the phase and reconstruct the original noiseless signal.

III. Results

Fig. 1 shows example signal models for Eq. (1) and Eq. (2), representing MCG signals. That is, $\hat{z}[l]$ in Fig. 1 (in red) has random shifts between consecutive pulses, whereas $z[l]$ (in blue) has a constant duration (i.e., T_p) between consecutive pulses. To obtain the denoised DFT version of the signal, as discussed in Section II, we define the unit amplitude window $w[l]$ supported on $l \in \{0, 1, \ldots, N-1\}$, and calculate the windowed DFT (WDFT) as $\hat{Z}[k, \Delta] = \sum_{l=0}^{N-1} \hat{z}[l] w[l - \Delta] \exp\{-i2\pi kl/N\}$, for $k \in \{0, \ldots, N-1\}$, for a window of size $N = T_p/T_s$ starting at Δ. In turn, the average Fourier transform is:

$$\hat{Z}[k] = \frac{1}{M} \sum_{i=0}^{M} \left| \hat{Z}[k, iT_p] \right| \tag{3}$$

The same process can be used to define $Z[k]$ from $z[l]$. The reasoning behind using Eq. (3) is two fold. The DFT magnitude is invariant under unknown time shifts and averaging helps reduce the noise level. The resulting $Z[k]$ and $\hat{Z}[k]$ signals are shown in Fig. 2. It is evident that even though $\hat{z}[l]$ had much higher noise levels than that of $z[l]$, the obtained average magnitude of the DFT of the segments of both signals overlap. To check the percentage of error between the obtained results, the normalized euclidean distance, obtained according to

$$\text{Err} = \frac{\left\| \left[Z[0] - \hat{Z}[0], \ldots, Z[\lceil 2NB/F_s \rceil] - \hat{Z}[\lceil 2NB/F_s \rceil] \right] \right\|_2}{\| [Z[0], \ldots, Z[\lceil 2NB/F_s \rceil]] \|_2}, \tag{4}$$

resulted in an error less than 5.71% when SNR$= -7.45$ dB.

IV. Conclusion

In this paper, we proposed an algorithm capable of denoising and reconstructing noisy, semi-periodic bio-magnetic signals (such as MCG signals) using an analytical model. A simulation study showed that the approach achieves a percentage error of less than 5.71% when compared to a perfectly periodic signal. In our future work, we will be applying this

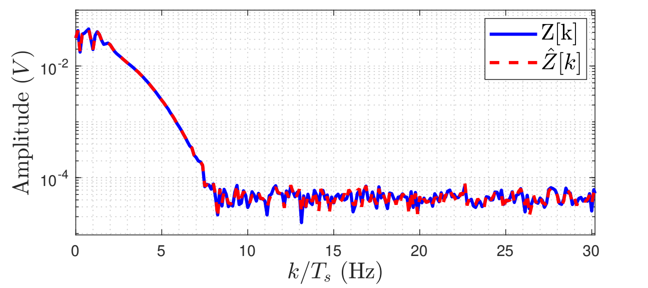

Fig. 2. Average magnitude of the DFT of the segments of $z[l]$ denoted by $Z[k]$ in blue and $\hat{z}[l]$ denoted by $\hat{Z}[k]$ in red.

approach on the analytical model of the MCG signal obtained through a portable sensor reported in our previous work. Once this process is complete, a parametric model-based phase retrieval method can be applied to reconstruct the noise-free signal by retrieving the pulse parameters (amplitudes, time shifts, and dilation values).

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