Enhancing teacher collaboration: Leveraging technology for knowledge expansion through lesson exchange and annotation

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Abstract: This paper explores the potential of online lesson visualization and annotation tools in fostering international lesson-centered teacher collaboration. In an era where teachers face diverse challenges and limited opportunities for peer-to-peer collaboration, leveraging digital tools for asynchronous exchanges emerges as a promising avenue for professional development. This paper will illustrate the potential of emerging technologies for supporting cross-cultural exchanges in which teachers can share insights, perspectives, and innovative practices in durable and archivable forms, thereby enriching the collective knowledge base for teaching. We share data from an ongoing project focused on engaging groups of secondary mathematics teachers in collectively refining a single storyboarded lesson representation. Through collaborative lesson development and iterative refinement, we illustrate how these tools transcend temporal and spatial constraints by sharing data gathered from three different groups involved in cross-cultural exchange (one situated in the western part of the U.S, one situated in the eastern part of the U.S., and one situated in Bulgaria) centered on storyboarded representation of a lesson. In this way, we provide insights on how the lean graphics of the storyboard and the asynchronous nature of annotation can foster a culture of continuous improvement and mutual support among mathematics teachers spread over large geographic distances. Ultimately, we advocate for the widespread adoption of online multimedia authoring tools as integral components of contemporary approaches to cross-cultural collaboration on lessons for facilitating meaningful exchanges and promoting excellence in teaching and learning on a global scale.

Keywords: Instructional Strategies, Educational Technologies, Teacher Collaboration, Emerging Technologies, Professional Development

Introduction

In contemporary mathematics education, the need for innovative approaches for supporting teachers' professional collaboration and growth extends beyond the confines of individual classrooms, schools, and districts. In fact, we would argue teachers' professional collaboration can and for some purposes should transcend geographical boundaries such as states, regions, nations, and even continents. The findings from large scale studies of mathematics teaching (such as TIMSS[1], Stigler & Hiebert, 1999) along with the increasing globalization has impacted the landscape of professional development for mathematics teachers (Zhao, 2010; Mullis et al., 2016). Globalization has heightened teachers' awareness of cultural competence and diversity in mathematics education, leading to an increased emphasis on incorporating culturally relevant content and instructional strategies to meet the needs of diverse student populations (Ernest, 2008). This shift is reflected in professional development initiatives, which now often include training on culturally responsive teaching practices and equity-centered pedagogies (Gay, 2010). Furthermore, the landscape of professional development across the last twenty years has evolved to recognize the pivotal role of collaboration in refining teaching methodologies and enhancing student learning outcomes (Jaworski et al., 2017). These two changes, taken together, suggest a need for the community of mathematics education to embrace the emerging digital innovations for supporting education, many of which have become increasingly prevalent across the globe in the wake of the COVID-19 pandemic (Dede, 2022;

[1] Trends in International Mathematics and Science Study (Institute of Education Sciences, n.d.)

Ferdig et al., 2020). Online platforms offer unprecedented opportunities for international collaboration among secondary mathematics teachers.

In this paper, we explore the fundamental research question: How do online lesson visualization and annotation tools facilitate international collaboration among secondary mathematics teachers, and how can these tools contribute to the collective refinement and improvement of lesson plans across diverse geographic locations? This inquiry not only underscores the significance of leveraging technology for professional growth but also explores the transformative potential of asynchronous collaborative exchanges in the field of mathematics education. The allure of digital tools lies in their capacity to transcend temporal and spatial constraints, enabling mathematics teachers to engage in meaningful exchanges irrespective of geographical distances. By harnessing the power of online lesson visualization and annotation tools, teachers can share insights, perspectives, and innovative practices in durable and archivable forms. This paper aims to illustrate the potential of such technological tools and their implications for fostering international collaboration in the realm of secondary mathematics education.

The significance of our main research question reverberates within the context of contemporary mathematics education, where educators face diverse challenges with limited opportunities for peer-to-peer collaboration beyond their local contexts. With rapid technological advancements and ever-increasing globalization, the need for teachers to adapt their pedagogical practices to suit the evolving needs of students has never been more pronounced (Dede, 2014). By elucidating the mechanisms through which online tools facilitate international collaboration, this study examines the roles of digital tools in paving the way for a more interconnected and collaborative approach to lesson planning and instructional design. Throughout this paper, we

will explore the nuances of online collaboration, drawing upon empirical evidence from an ongoing NSF-funded project. Furthermore, we will feature examples illustrating the potential use of digital tools and platforms for supporting ongoing collaboration between teachers spread across different geographical regions and countries. By highlighting the efficacy of online lesson visualization and annotation tools, we aim to support mathematics teacher educators and researchers in embracing technology as a catalyst for professional growth and collective learning. This article contributes to the ongoing research on designing professional experiences that actively fosters collaborative and inclusive exchanges on lessons among teachers.

Literature review

Trends in Teacher Collaboration

The importance of teacher collaboration for professional development in mathematics education has gained significant traction in educational research and practice. One seminal work that contributed to this interest is the TIMSS classroom video study, which shed light on the potential of teacher collaboration through a focus on Lesson Study practices in Japan (Stigler et al., 1999). This spotlight on collaboration has served to underscore its pivotal role in fostering teachers' professional growth and improving teaching practices. From Lesson Study in Japan to collaborative teacher projects in England and learning communities in Norway, various models of collaboration have emerged, reflecting distinct cultural contexts and educational landscapes (Robutti et al., 2016).

In a survey commissioned for the 13th International Congress on Mathematical Education (ICME-13), Robutti and colleagues (2016) examine teachers' collaborative working, exploring the diverse forms that collaboration has taken on across different educational systems.

They define collaboration as more than mere cooperation; entailing a relational system characterized by negotiation, joint decision-making, and mutual learning. The survey identifies 316 papers published between 2005 and 2015 which showcased teacher collaboration as an explicit part of the research design. The researchers find diverse contexts in which mathematics teachers collaborate and learn together. Notably, only a small percentage of collaborations involve multiple countries or continents. Also, there is a concentration of studies from European, Australian, and North American regions. Surprisingly, particularly regarding Japanese Lesson Study, the researchers report a scarcity of studies from Asia, and limited representation from Africa, with those studies that did emerge from Africa being primarily from South Africa.

Collaborations often involve tailored professional development initiatives driven by various stakeholders, addressing a range of aims from enhancing teachers' awareness of student learning trajectories to evaluating professional development program effectiveness. Some initiatives have dual aims, with researchers investigating teachers' learning while teachers engage in research on their own practice, such as planning “research lessons” where teachers look for curricular resources that might help students learn a particular mathematical concept (Bocala, 2015). The articulation and sharing of aims among participants vary across collaborations, with some explicitly stating dual aims while others focusing on specific objectives relevant to their contexts. Overall, collaborations in mathematics education encompass a wide range of initiatives aimed at improving teaching practices and student learning outcomes.

Three-quarters of the studies provided enough information to aggregate them based on the number of teachers and duration, with a predominance of smaller-scale (less than 100 participants) and shorter-duration collaborations (less than 1 year). Teacher collaboration involved various stakeholders, including teachers, teacher educator/researchers, school

principals, and community leaders. Facilitation of these groups is often led by teacher educators or researchers. The dual role of facilitator and researcher can be problematic if one aims to establish equal authority amongst those collaborating, because the group authority can inadvertently shift towards the researchers over time as individuals settle into their expected roles (Besmusca & Drijvers, 2013; Nachlieli, 2011). Crucially while the study by Robutti and colleagues (2016) demonstrates a wide diversity of professional development efforts that center on teacher professional development, few of them support collaboration across multiple countries, involve teachers for more than a few weeks, engage large numbers of teachers, or are led by individuals different than the researchers of the reported studies.

Role of Technology in Facilitating International Collaboration between Teachers

To begin, the technology needed for designing and studying interventions for fostering international teacher collaboration (or any collaboration across a distance) requires a dual focus: enabling effective collaboration among teachers on mathematics instruction practices and supporting researchers in facilitating, observing, and studying such collaboration. The first activity calls for technologies that allow teachers to communicate with colleagues in a context where the common language is not sufficient for communicating about crucial details about practice. The practice of teaching mathematics in classrooms, just like any teaching, lacks a standardized technical vocabulary for enabling teachers to communicate. While some have advocated for the development of such a lexicon (Clark et al., 2017; Grossman, 2020), others caution against its oversimplification (Horn & Kane, 2019). Specifically, researchers have asserted that teaching relies on collective tacit knowledge (e.g., knowledge of instructional norms, Herbst & Kosko, 2014) and is therefore inherently resistant to linguistic representation. To bridge this gap, multimodal representations like videos, or animations and storyboards have

proven invaluable (Herbst et al., 2011). Processes such as those featured in Story*Circles* (Herbst & Milewski, 2018) enable educators to collaboratively script lessons, facilitating knowledge exchange (also see Zazkis & Herbst, 2018).

For international collaboration to happen at scale, technological mediation becomes indispensable. Versatile technology capable of handling diverse media types for authoring and reading representations of practice, alongside facilitating collective annotation of such representations enables teachers to engage their tacit knowledge, which researchers can then document. In our own experiences as designers of professional development centered on teachers working at a distance to collectively author representations of lessons, lean graphic characters and online storyboarding software have been essential. These resources offer teachers the ability to communicate in ways akin to written language while retaining the expressive capacity to prompt tacit knowledge just as video does. From these experiences, we argue that the requisite infrastructure for supporting collaboration across distance and time should remain flexible and open, encouraging teachers to engage in meaningful authoring, interpretations of, and interaction with representations of practice and one another.

Technology should not only facilitate the sharing of teachers' tacit knowledge but also establish a social infrastructure for discourse and exchange on representations of practice. Again, drawing on our own experiences, tools like Anotemos, an online social and multimodal media annotation tool, can play a pivotal role in facilitating collaborative annotation and editing of media, fostering rich exchanges among teachers, even when they are temporally and/or geographically distant (meeting at different times and seasons or different parts of the world). The social infrastructure we envision for supporting teachers' collaboration on representations of

practice should support multimodal interactions, allowing users to engage graphically, aurally, and textually.

While off-the-shelf software for annotating and representing lessons may suffice initially, integrating them into long-term research agendas, exemplified by platforms like Lesson*Sketch* (Chieu & Herbst, 2012), is crucial for sustained data collection supporting research on teacher collaboration. That is, if the technology is to support the iterative design and improvement of innovative forms of professional development, the technology should not only foster teacher collaboration but also streamline data collection for research. Though the Lesson*Sketch* platform (Herbst et al., 2013) is no longer operational, attempts to replicate usage with other tools have underscored the need for robust integration of tools for enabling researchers to curate experiences for practitioners, collect data, and analyze engagement and feedback. Ensuring seamless integration of collaborative tools with data collection mechanisms is crucial for advancing research on teacher collaboration about practice effectively. Furthermore, the continuous evolution and improvement of technology for studying teacher collaboration necessitates ongoing attention from researchers. This entails refining existing tools and exploring innovative approaches to meet evolving needs. Collaborative efforts among technology developers, educators, and researchers are paramount in creating user-friendly, effective, and adaptable technology solutions that bolster teacher collaboration and support robust research endeavors in mathematics education.

Background

Story*Circles* is a model for professional development wherein teachers collaboratively create a lesson by participating in the process of scripting, visualizing, and arguing by utilizing multimedia tools (Herbst & Milewski, 2018). Using online storyboarding and annotation tools,

teachers script the progression of a lesson in a given instructional course, starting from a particular mathematical task to an associated mathematical goal. To support teachers to move beyond generic description of practice, we employ a depictor or storyboarder who uses an online storyboarding tool to visually represent teachers' shared lesson ideas on a storyboard, prompting participants to transform vague contributions (e.g., “Next, we should help the students understand”) into specific instructional actions. By visualizing ideas as a sequence of storyboard frames, participants get a sense of what the lesson looks like moment-to-moment, annotating their impressions, and as they compare the impressions from that visualization they realize the need to reach consensus on nuanced details of the lesson (e.g., what a teacher might say or do in response to an unexpected student contribution).

As the lesson takes shape visually, practitioners’ diverse perspectives, shaped by their varied experiences, enrich discussions on crucial contingencies and instructional strategies. This diversity fosters argumentation, where alternative ideas are scrutinized to converge on a unified representation of the lesson. Participants have the opportunity to evaluate decisions on the basis of their affordances and risks, fostering reflective dialogue that is grounded in teachers’ practical rationality (Fenstemacher & Richardson, 1993). Such reflection enables teachers to gain insights into their practices and those of their peers, promoting professional growth (Herbst et al., 2020; Milewski et al., 2018). Once the cycle of scripting, visualizing, and arguing about a given moment is complete (see Figure 1), the cycle continues with the participants having the opportunity to either select one of the original ideas scripted by a participant, or craft a new alternative that none of the teachers originally offered. The interaction continues this way as participants come to consensus about how the lesson should go.

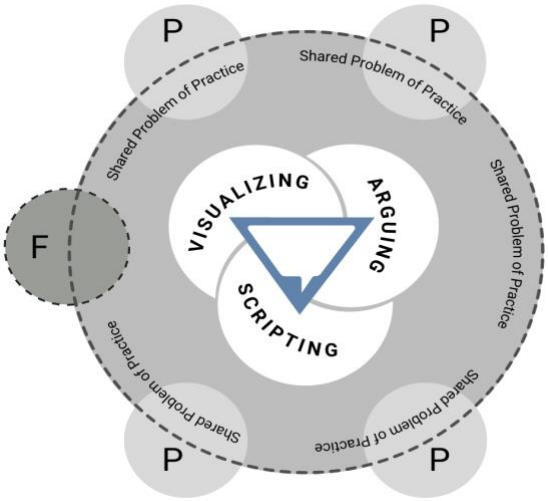

Figure 1. A representation of Story*Circles*

At the heart of Story*Circles*, teachers have the opportunity to collaborate on lessons, with support from the facilitator who orients participants towards one another and towards building consensus. Unlike traditional models of professional development, facilitators do not prescribe solutions, but instead encourage teachers to address their own instructional dilemmas, fostering authentic sharing of teachers' stories about practice. Because the product is a hypothetical lesson that participants may or may not elect to implement, participants do not have to feel threatened by others' suggestions. Instead, the storyboard provides them a virtual space to engage in professional experimentation in which they have the opportunity to try something out that they might not otherwise be willing to try if they were acting in a real classroom with real students (Milewski et al., 2018). Through this experimentation, teachers have the opportunity to reflect on and authorize their own and others' ideas about practice; and in this way have the freedom to come to their own understanding of what it means to act productively within specific lessons. This kind of individual agency frees the group up to move beyond engagement in longstanding debates on teaching about discovery versus direct instruction. Furthermore, teachers can focus on

collective meaning-making about what it might look like to productively facilitate students' mathematical learning in the context of a problem-based lesson, where they may notice and make use of students' various mathematical contributions (Herbst et al., 2023).

One challenge we've noticed in the Story*Circles* approach is the need for teachers to embrace professional growth through constructive argumentation. Specifically, the model relies on teachers' willingness to express dissenting views and explore alternative ideas. However, when groups lack diversity, the available perspectives may not surface enough alternatives organically. To address this, we've implemented strategies focused on integrating new materials sourced from teachers' classrooms or research on teaching into Story*Circles*, resulting in valuable innovations (Brown & Herbst, 2023).

In recent efforts, we've concentrated on broadening the Story*Circles* group by incorporating new voices of practitioners through annotations on lesson storyboards (Figure 2).

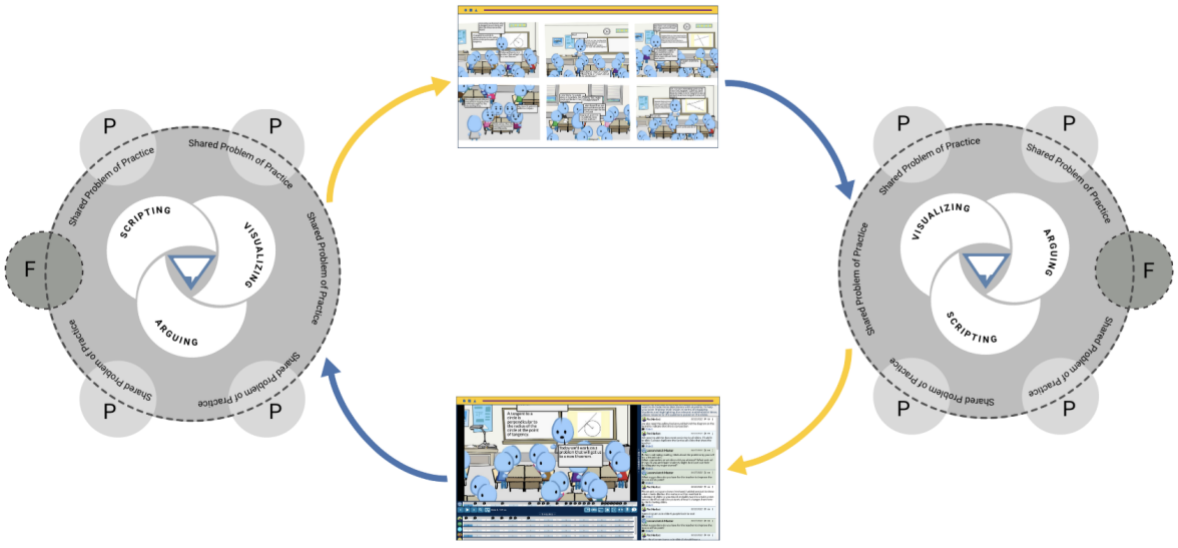

Figure 2. A representation of a newly adapted model for implementing Story*Circles* with cross-cultural collaborations

Initially, groups of secondary mathematics teachers from the U.S. (shown on the left in Figure 2; composing groups hereafter) are organized into algebra or geometry groups to draft a

storyboarded lesson focused on a given task and aimed at a particular instructional goal. Across a six-week time period, through a series of asynchronous and synchronous activities, a draft of the storyboarded lesson is produced.

Subsequently, the storyboarded lesson is shared with two new groups of secondary mathematics teachers recruited from both the U.S. and four partner countries (shown on the right in Figure 2; annotation groups hereafter). Over a three-week period, these annotation groups engage in asynchronous and synchronous activities to review and comment on the lesson, aiming to produce a collective commentary for improving the lesson. Similar to the composing Story*Circles* groups, the annotation groups use the processes of scripting, visualizing, and arguing to develop their commentary—but they start by visualizing a lesson where the composing groups start by scripting a lesson. Each commentary is then shared with both the original composing groups and one additional annotation group (who have not seen the lesson storyboard). In a second six-week cycle, the composing groups utilize the received commentaries (one from the U.S. and one from a partner country) to inform the production of two new drafts of the storyboarded lesson. Simultaneously, the annotation groups are tasked with reading their newly assigned commentary and offering additional perspectives for improvement over a three-week period.

In summary, using this new iteration of Story*Circles*, a single storyboarded lesson becomes the object of cross-cultural exchange amongst three groups of teachers from diverse backgrounds. Over 18 weeks, these four groups collectively produce an initial draft of the storyboarded lesson, three distinct commentaries, and two second drafts of the storyboarded lesson (each of which are informed by a commentary). Throughout this process, online

storyboarding and annotation tools facilitate collaborative work across different locations, time zones, and seasons, offering flexibility for cross-cultural engagement and exchange.

Theoretical framework

Narrative analysis serves as a methodological framework enabling individuals to delve into the human experience through storytelling (Bruner, 1987; Connelly & Clandinin, 1990; MacIntyre, 2013). At its core, narrative analysis suggests that humans are natural storytellers, both shaping and being shaped by the narratives they and others tell about their lives (Connelly & Clandinin, 1990; Sfard & Prusak, 2005). Over time, narrative analysis has become a valuable approach in educational research for helping us understand how teachers' professional narratives—both individual and collective—take form and has been used to study teachers' experiences, knowledge (Ball & Goodson, 1985; Connelly & Clandinin, 1990, 1995; Sikes et al., 1985), and professional identities (Li & Niyozov, 2008; Sfard & Prusak, 2005).

In tandem with narrative analysis, narrative inquiry has emerged as a means of "honoring lived experience as a source of important knowledge and understanding." (p. 7, Clandinin, 2022). This approach, as exemplified by scholars like Clandinin and Connelly (2004), has been used for self-study and to provide sociological insights into the communal construction of narratives (Carr, 1986). In education, narrative inquiry sheds light on teaching practices, teacher education, and professional ways of knowing (Goodson, 1988; Lyons & LaBoskey, 2002; Schön, 1995). When paired with a community of practice, narrative inquiry creates "safe, storytelling places where educators narrate the rawness of their experiences, negotiate meaning, and authorize their own and others' interpretations of situations" (p. 116, Craig & Olson, 2002). Unlike traditional professional development paradigms that seek to impose external narratives as a means to reform educational practice, narrative inquiry empowers teachers to share personal stories and engage in

collaborative storytelling, fostering personal growth and reshaping professional identities (Clandinin, 2022).

As an analytical method, narrative inquiry aims to understand the genesis of stories by synthesizing field texts into cohesive narratives, drawing on temporal, social, and place-based dimensions (Connelly & Clandinin, 2006). ***Temporality*** considers how past, present, and future experiences shape a narrative, while ***sociality*** encompasses personal and social conditions such as emotions and cultural contexts. ***Place*** refers to the physical settings or sequence of locations shaping a narrative. In teacher development, these dimensions prompt deeper understanding, recognition of socio-cultural influences, and contextualization of experiences. Our methodological approach utilizes Story*Circles* to foster collaborative exchanges among teachers, effectively integrating with narrative analysis and inquiry. Similar to narrative analysis, Story*Circles* centers on teachers' experiences, purposely avoiding ''strategies, tactics, rules and techniques that flow out of other considerations'' (p. 188, Clandinin & Connelly, 2000). By privileging teachers' practical knowledge and positioning them as knowledgeable practitioners, Story*Circles* fosters shared understanding and transformative learning experiences. Moreover, by supporting teachers in refining their narratives through reflection (Schön, 1995) and engagement in a community of practice (Little, 2002), Story*Circles* sources learning from practice and collaboration (Herbst & Milewski, 2020).

Lesson visualization for supporting teacher inquiry and illuminating teacher rationality

Lesson visualization plays a pivotal role in supporting teacher inquiry and eliciting teachers' practical rationality within the Story*Circles* framework. Through collectively crafted lesson visualizations, Story*Circles* presents unique opportunities for delving into teachers' practical rationality. Specifically, the emphasis on creating lesson visualizations prompts

teachers to narrate stories that transcend surface-level discussions, delving into substantive details regarding the ***temporality***, ***sociality***, and contextual nuances (***place***) of teaching mathematics.

The representation of these aspects in the form of a lesson storyboard is instrumental in fostering collaborative inquiry into mathematics teaching practices among teachers. Firstly, the visual nature of the constructed lesson storyboards compels teachers to share narratives grounded in specific settings (***place***). These storyboards depict frames portraying a mathematics classroom environment, showcasing lessons targeting specific mathematical goals within particular courses of study. This encourages teachers to reflect on how the context shapes their instructional decisions, such as whether to prioritize symbolic equations or alternative algebraic representations like tables or graphs in different lesson contexts Secondly, the storyboard medium, comprising a sequence of comics-like frames outlining the beginning, middle, and end of a lesson, prompts teachers to scrutinize the temporal aspects influencing their instructional choices. Teachers are prompted to consider the appropriateness of their decisions based on the timing within a lesson, fostering a deeper understanding of the nuanced ***temporal*** dynamics inherent in classroom instruction. Thirdly, the lean, nondescript, customizable set of graphics employed in the lesson representations facilitate inquiries into the ***social*** dimensions of teaching. By depicting a diverse set of characters, including students and a teacher, along with their interactions, the storyboards encourage teachers to contemplate the social dynamics at play within the classroom. Whether customizing characters to reflect individuality (e.g., giving students names, different facial expressions, differently designed vests, skin tones) or using more generic provided representations of the characters (see Figure 3), participants are prompted to

reflect on the social implications of their instructional decisions, such as accommodating the needs of diverse students during classroom interactions.

Figure 3. Characters from Lesson*Depict* displaying different levels of customization

In summary, the lesson visualization approach used within Story*Circles* serves as a catalyst for collaborative inquiry among teachers, encouraging them to delve into the complex interplay of temporal, social, and contextual factors shaping their instructional practices. Through reflective engagement with lesson visualizations, teachers have the opportunity to gain insights into their practice, fostering professional growth and thereby enhancing their effectiveness in the classroom.

Lesson annotation for supporting and understanding teachers' professional growth

The annotations of lesson storyboards, produced by both individuals and groups of teachers toward the representation of a lesson storyboard, holds significant potential for nurturing and inquiring into teachers' professional growth within the Story*Circles* approach. The focus on constructing and refining annotations to enhance lesson storyboards drives teachers towards engaging in professional argumentation, where multiple alternative narratives for the same moment are produced, accompanied by justifications to discern their merits. Through this process, teachers have the opportunity to tell and retell their narratives of their lived experiences within a community of practitioners (who might agree or disagree in subtle or less than subtle

ways), gaining profound insights into their practical knowledge and moral frameworks through social interactions among peers.

In these exchanges, it becomes apparent that the construction of the lesson storyboard is not solely under the control of any one individual or group. Teachers grapple with unexpected developments, adapting their stories to accommodate new perspectives and insights. This lack of complete control underscores the dynamic nature of storytelling within collaborative environments, enabling educators to uncover and critically reflect on their own teaching practices and challenge the underlying assumptions, which gives them stability (Smith, 1996) and can lead to transformative shifts in their professional perspectives and approaches.

These kinds of interactions around improving lesson storyboards, occurring synchronously and asynchronously, offer a new second perspective on the concepts of ***place***, ***sociality***, and ***temporality*** in our work. This time, however, the terms are not used in reference to the narrative embedded in the representation of a lesson, but rather the narrative of how the lesson storyboard came to exist across many iterations and with the influence of many annotations that guided the development of the lesson visualization. Considering that Story*Circles* does not occur in a single "place" but rather through online digital spaces (which bring together people from different physical and temporal places) that provide structures for supporting teachers' engagement, we choose to replace the term "place" with "***structure***" in this section to prevent confusion.

Structure directs our attention to the characteristics of the virtual settings where Story*Circles* professional development unfolds, shaping participants' understandings, motivations, and experiences. It plays a pivotal role in organizing teachers' collective work and guiding engagement and contributions in collaborative activities. One way in which structure

becomes apparent is through the organization of teachers' collective work in professional development, particularly in designing or annotating lesson storyboards. This includes the nature of digital platforms and tools, as well as the sets of frames and instructions provided for activities. Specifically, variations in activity structures—such as whether teachers annotate lessons for personal improvement or produce commentaries for other groups—can result in different levels of engagement and contributions. This diversity in activity structures raises questions into the effectiveness of annotation methods within collaborative lesson inquiries.

The ***social*** nature of Story*Circles* highlights the fact that both personal (beliefs, knowledge, disposition) and social factors (societal norms, professional norms, professional obligations) play a role in shaping teachers' experiences. One of the ways that the social nature of Story*Circles* is made evident is through the interactions between participants, as observed in the annotations or exchanges that shape the lesson storyboard. These annotations provide insights into participants' underlying assumptions and interpretations of instructional practices as well as their influence on the development of the storyboard. Additionally, when these interactions are happening across cultural boundaries, they can shed light on the interpretability of various instructional practices or suggestions across cultural boundaries, fostering a deeper understanding of cultural influences on teaching practices.

The ***temporal*** nature of Story*Circles* encourages us to look across all three of these facets and try to understand how they intersect dynamically to shape the lesson storyboard that teachers construct as they navigate through time and space, integrating new stories from a wider audience and gaining fresh insights along the way. The temporal nature of Story*Circles* becomes apparent through the evolution of the lesson and teachers' annotations on the lesson over time, as evident by the dates when the annotations were provided. By aligning these artifacts, one can trace the

progression of the lesson storyboard. Similar to the lesson visualization itself, this narrative of multiple lesson visualizations and annotations on those visualizations unfolds with a beginning, middle, and end—commencing with teachers' initial encounters with the lesson and concluding with their final reflections—providing clues about how teachers grew across their collaborations. In summary, our use of collectively developed lesson visualizations and annotations on those lessons integrates temporal, social, and structural dimensions of narrative inquiry. In doing so, the approach provides a comprehensive framework for inquiring on teachers' practical rationality and fostering teachers' collaboration in ways that are conducive to transformative professional growth.

To illustrate the utility of this framework, we reformulate our research question into the following two related questions: (1) How do the type of annotations elicited from groups of secondary mathematics teachers differ when they are operating under different activity structures, and what might this say about the influence of annotation structures on shaping teachers' collaborative inquiry on a lesson? (Structure), (2) How do the type of annotations elicited from groups of secondary mathematics teachers differ when they are gathered from different cultures, and what might this indicate about differences in the cultural norms for collaboration amongst these groups? (Sociality). The ensuing sections illustrate some of the ways that lesson visualization and annotation tools can foster international collaboration among secondary mathematics teachers and enhance lesson plan refinement by addressing the first two questions. Subsequently, we engage in a discussion on future work that will address the third question.

Methodology

This study is a part of the large project in which twelve groups of secondary mathematics teachers–six groups focused on geometry and six on algebra–participated in separate iterations of Story*Circles*. Among these twelve groups of secondary mathematics teachers, four groups of teachers were recruited from the U.S. to serve as composing groups (see background). The lesson storyboards were then disseminated to eight groups of secondary mathematics teachers recruited to serve as annotation groups (see background). Four of the annotation groups were recruited from the U.S. and the remaining four were recruited from four different partner countries: Bulgaria, India, Philippines, and South Africa. In Fall of 2023, the four composing groups were to draft their first version of the lesson storyboard and the eight annotating groups were to draft a commentary for sending back to their composing groups. In Spring of 2024, the composing groups were to produce two revised versions of the lesson storyboard, informed by two separate commentaries and the eight annotating groups were to produce a commentary of a new storyboarded lesson after reviewing the commentary produced by another team.

All but one of the composing groups were able to get their start in early Fall of 2023. Extenuating circumstances made it such that one of the geometry facilitators could not start until Winter of 2024. This created a design problem for us as we had two annotation groups that needed a storyboarded lesson to review. We resolved this problem by making two crucial adaptations. The first adaptation was that two annotation groups started with the task of annotating a storyboarded lesson that we (rather than a group of teachers) had previously created for a different purpose (e.g., supporting composing group in getting to know the lesson)—though we did not call attention to this difference. The second adaptation was that the composing group

began with a slightly different task as well—improving a storyboarded lesson provided to them rather than improving a storyboarded lesson that they themselves had previously developed.

Crucially, the storyboarded representation in both of these adaptations depicted a version of the Tangent Segments Lesson (see Figure 4) that we have described in other contexts as less desirable (Brown et al., 2024) because it depicts a teacher using a teacher-centered approach to demonstrate (with minimal student input) how solving a rich mathematical problem (see Figure 4a) leads to a new theorem (see Figure 4b). In total, the Tangent Segments Lesson is made up of a total of 20 frames with 38 teacher dialogue bubbles and 0 student dialogue bubbles. To be clear, the approach of the teacher portrayed in this lesson is contrary to the intended goal of including classroom discussions in problem-based instruction. Yet, in our prior work (Brown et al., 2024), we have observed how the less desirable version of the Tangent Segments Lesson has the capacity for serving as a kind of "low-floor, high ceiling" (Resnick et al., 2009) activity for enhancing teachers' engagement making them feel comfortable sharing ideas about how to refine the lesson.

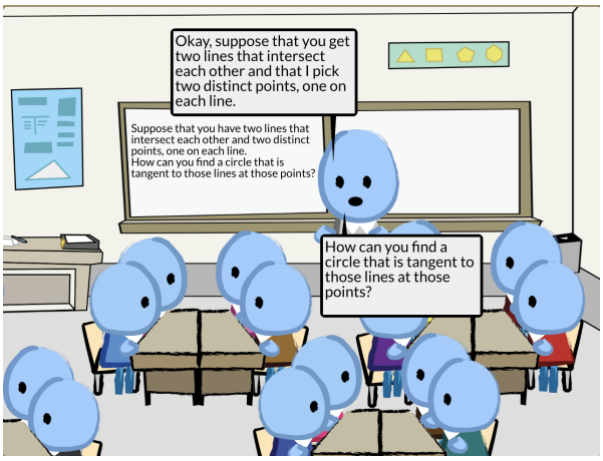

Figure 4a. One of the initial frames showing the problem posed in the Tangent Segments Lesson

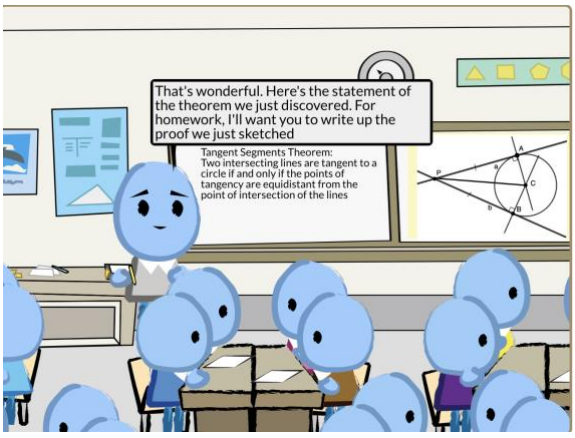

Figure 4b. Final frame showing the goal of the Tangent Segments Lesson

While these adaptations across these three groups were initially unplanned, they created a unique opportunity for this study. All three groups were working on the task of improving the less desirable version of the Tangent Segments Lesson but were operating under different structures (toward differing purposes), working at different time frames of their choice, and gathered from different locations. We leverage this opportunity by examining the annotations that resulted from the collaborations within these three groups.

Description of groups and corpus

Groups 1 and 2, consisting of 8 participants each, were designated as annotating groups and convened for a total of 3 weeks. Group 1 comprised 8 teachers from 7 different secondary schools in Bulgaria and convened in January 2024. Group 2 comprised teachers from 1 secondary school in the Northwest region of the U.S. and convened from February to March 2024. Over the course of their three-week cycle, participants in both groups individually annotated the less desirable version of the Tangent Segments Lesson in their asynchronous work before and in-between group sessions. During the three group sessions, both groups focused on producing a single set of annotations on the same lesson that they developed collectively by reviewing and discussing the individual annotations. They were informed that their collective annotations would be shared with another group whose members would further refine the lesson based on their collective feedback.

Group 3, on the other hand, served as a composing group and met over a 9-week period. This group consisted of 3 teachers from 3 different secondary schools in the Northeast region of the U.S. and convened from February to April 2024. During the initial three weeks of their 9-week cycle, Group 3 individually annotated the less desirable version of the Tangent Segments Lesson and subsequently engaged in collective revision of the lesson based on their annotations.

All three of these groups represented teachers with a range of experience teaching geometry—with each group having at least one inexperienced teacher (having 1 year or less) while the majority of teachers in the group had more experience (ranging from 3 to 39 years with the groups' average years of experience ranging from 13 to 18 years). This study's analysis focuses on the individual annotations generated across all three groups.

Analysis methods

To address both the first and second research questions, we initially coded the complete set of annotations resulting from the three groups (n=92, excluding 52 annotations lacking a verbal statement, such as participants merely placing colored pins on the timeline or making screen markings) using a coding scheme adapted from Chieu and colleagues (2015). This coding scheme, rooted in systemic functional linguistics, categorized participant comments into three linguistic categories: ***evaluation***, ***reflection***, and ***alternativity***.

The evaluation code encompassed comments containing linguistic markers of appreciation (e.g., ''I disagree that the teacher could characterize it as an 'exploration' and claim that 'we just discovered' a theorem''), or judgment (e.g. "[The students in the comics] don't look very involved''). The reflection code captured instances of comparison, causal reasoning, or conditional statements (e.g., "I think the students could have come up with this *if* you gave them time/opportunity.") The alternativity code included instances where participants proposed actions using modal verbs (e.g., "Teacher should ask students to suggest useful knowledge instead of volunteering it") or expressed uncertainty about proposed actions using the imperative mood (e.g., Ask: "Is this circle the solution?").

Since each annotation contribution was treated as a unit of analysis, some annotations received multiple codes. For example, the following contribution received two codes: "Could the

students have small whiteboards at their tables, or a shared white board, and try to draw the starting image [alternativity], instead of the teacher drawing it? [reflection]". After categorizing the entire corpus, we aggregated the data to identify patterns that could shed light on the influence of activity structure or group composition. To answer research question 1, exploring the relationship between annotation types and differing activity structures, we compared the distribution of annotation types elicited from Groups 1 and 2 with that from Group 3. Similarly, to answer research question 2, investigating how annotation types might reflect differences in the cultures of the three groups, we analyzed the distribution of annotation types across all three groups.

Findings

Our findings are presented in two main sections, each aligned with operationalized research questions. In the first section, we address research question 1 (Structure): How do the type of annotations elicited from groups of secondary mathematics teachers differ when they are operating under different activity structures, and what might this say about the influence of annotation structures on shaping teachers' collaborative inquiry on a lesson? We approach this by delineating between the two distinct activities in which participants were involved and examining the differences that can be observed across the teachers' annotations of the same lesson representation. Subsequently, in the second section, we address research question 2 (Sociality): How do the type of annotations elicited from groups of secondary mathematics teachers differ when they are gathered from different cultures, and what might this indicate about differences in the cultural norms for collaboration amongst these groups? We do this by describing the differences in the composition of the groups and examining the differences that can be observed across the teachers' annotations of the same lesson representation.

*The **Structure** of the Activities in StoryCircles*

Overall, 19 participants contributed 92 comments to annotate the storyboard of the less desirable lesson. When looking across the entire corpus, it became apparent that participants' annotations were seldom characterized by markers of appraisal (16%, n=15) but more frequently contained markers of reflection (45%, n=41), especially when addressing students' thinking. Additionally, participants' contributions frequently offered suggestions for alternative actions (65%[2], n=60), particularly concerning the depicted teacher's actions.

When we inquire about this corpus through the lens of participants' activity structures, notable differences emerge. Despite all participants annotating the same lesson storyboard, namely the less desirable representation of the Tangent Circle Lesson, they engaged in the annotation of that lesson under two distinct activity structures. Groups 1 and 2 were defined as annotation groups (as illustrated on the right-hand side of Figure 2), where individual annotations fueled collaborative efforts toward producing a collectively-developed annotation. This collective annotation was created to be shared with another group in order to aid in their efforts to improve the lesson storyboard, characterizing **Activity Structure 1**. In contrast, Group 3 was defined as a composing group (as illustrated on the left-hand side of Figure 2), where individual annotations contributed to collaborative efforts to revise the storyboard. This revised storyboard aimed to enhance the initial less desirable version of the lesson, characterizing **Activity Structure 2**.

Figure 5 summarizes the percentages of evaluative, reflective, and alternative annotations for each activity structure. When annotating the storyboarded lesson individually, participants in Activity Structure 1 demonstrated greater inclination toward offering comments containing

[2] The percentages do not sum to 100% because some comments received more than one code.

alternatives (58%, n=47) than evaluations (15%, n=12) and reflections (27%, n=22). Notably, as in the larger corpus, the alternatives proposed in Activity Structure 1 predominantly focused on suggesting alternative actions for the teachers, as exemplified by statements such as "*The teacher should ask the students what they need to construct a circle to lead them to the questions about the center and the radius*''. Conversely, participants in Activity Structure 2 displayed a greater tendency toward offering reflective comments (56%, n=19) as opposed to evaluative (9%, n=3) or alternative comments (35%, n=13) when annotating the storyboarded lesson. Furthermore, the reflections provided by participants in Activity Structure 2 closely mirrored those observed in the larger corpus, with a predominant focus on the students. For instance, they frequently made statements such as *"Students might be thinking that they can use the tangent theorem from yesterday, or the bisector theorem or just draw some circles trying to observe something useful''* or *"Students might be thinking 'I'm so lost' at this point, especially if their group did not get this far in their investigation."*

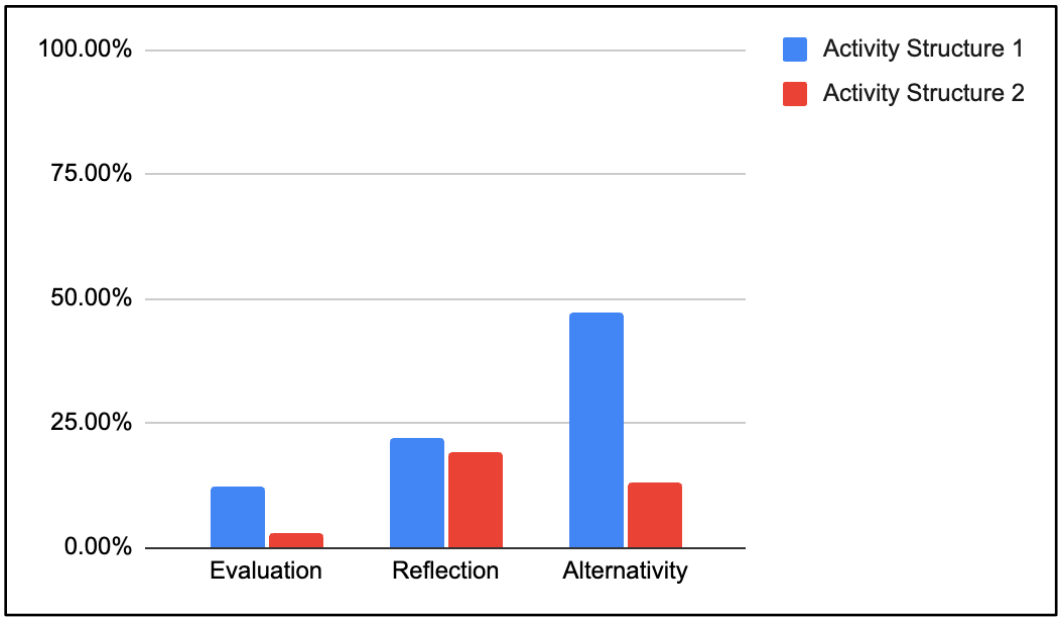

Figure 5. Percentage of comments in terms of evaluation, reflection and alternativity for each activity structure

*The **Sociality** in the Activities of* Story*Circles*

When we inquire about this corpus through the lens of the cultures of the groups, we notice some interesting differences. Recall that Groups 1, 2, and 3 differed from one another in ways that may be meaningful in terms of the group culture. For one, all three were drawn from different regions of the world. Furthermore the groups differed in terms of whether they were recruited from the same school or different schools. Both of these differences could have implications on the groups' norms related to professional exchanges and what would be seen as a reasonable annotation on practice. **Group 1** was composed of 8 participants recruited from 7 different schools across Bulgaria (with schools separated by as much as 242 miles). **Group 2** was composed of 8 participants recruited from 1 school in the Northwestern region of the U.S. **Group 3** was composed of 3 teachers recruited from 3 different schools across the Northeastern region of the U.S. (with schools separated by as much as 276 miles).

Figure 6 illustrates the distribution of evaluative, reflective, and alternative annotations across each group. Despite operating within the same activity structure and utilizing identical annotation types, Group 1 and Group 2 displayed differences in their propensities to evaluate the storyboarded lesson as well as their propensities to suggest alternatives. Interestingly, with regards to evaluation, Group 2 and Group 3 look quite similar, both exhibiting a lower percentage of evaluative comments (5%, n=2 and 9%, n=3 respectively) compared with Group 1 (27%, n=10). In regard to alternativity, it is also interesting to note that Group 1 and Group 3 look quite similar, both exhibiting a lower percentage of alternative comments (38%, n=14 and 37%, n=13 respectively) when compared with Group 2 (75%, n=33). This difference in propensity to offer alternatives might be explained by the fact that Groups 1 and 3 are composed

of teachers who do not know each other well, while Group 2 is composed of teachers drawn from the same school district and with a strong working relationship.

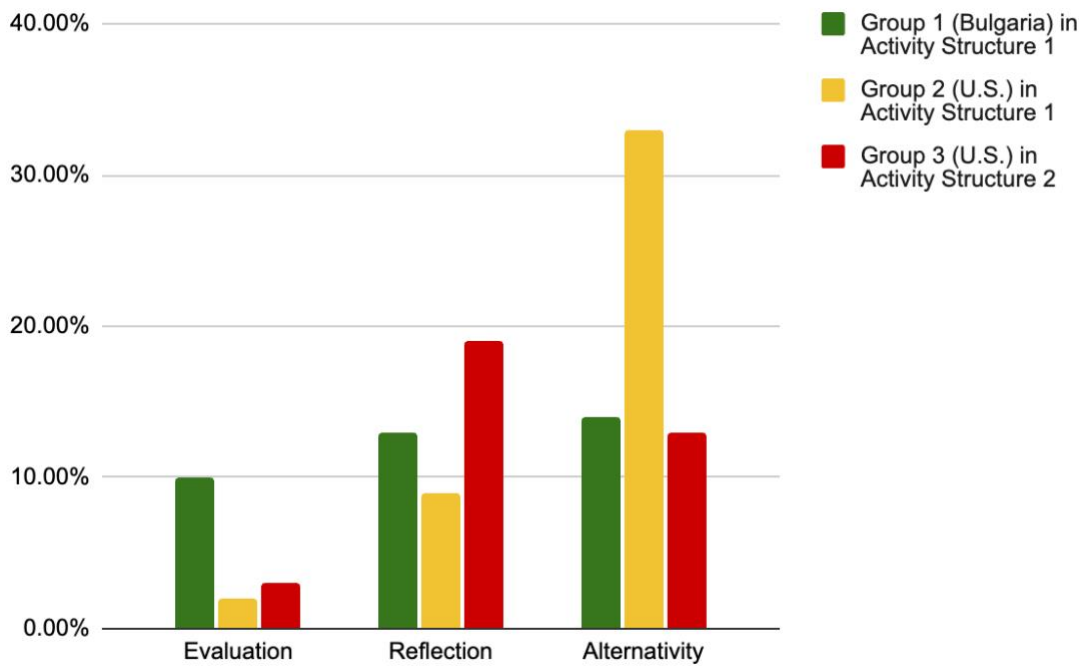

Figure 6. Percentage of comments in terms of evaluation, reflection and alternativity for each group in both Activity Structures

Discussion

In the preceding section, we explored how the lenses of structure and sociality offer insights into understanding the variations in teachers' annotations concerning a shared lesson representation. Within the structure-focused segment, we shared findings illustrating that participants in Activity Structure 1 predominantly offered annotations which proposed alternative ideas for teacher actions, while participants in Activity Structure 2 tended to offer annotations with reflective comments about student thoughts or emotions within the lesson context. One plausible explanation for this contrast could be the participants' perceptions of their roles within the respective activity structures.

Individuals involved in Activity Structure 1, aimed at assisting other teacher groups in improving a specific lesson visualization, may have interpreted their role as demanding that they help their peers by suggesting alternative pedagogical possibilities that a teacher could use to help lead to a more productive mathematical discussion (as opposed to suggesting more realistic consequences that could result from the pedagogical practices that were already salient in the representation). Conversely, participants in Activity Structure 2, focused on annotating for personal improvement and eventual lesson representation refinement, may view their role as centered around reflecting on students' perspectives during the lesson.

While these explanations are not exhaustive, they offer intriguing insights. They suggest that Activity Structure 1 may implicitly assume that the representation is to be taken as a portrayal of aspirational teaching practices, emphasizing what teachers should ideally do, rather than illustrating the potential complexities that can emerge in more realistic or even less than aspirational classroom scenarios. This ambiguity is particularly relevant as composing groups often grapple with questions about whether to depict lessons realistically or idealistically, and how might they expect an audience to make sense of a decision to intentionally represent some of the less than ideal but perhaps realistic circumstances they know teachers may benefit from considering.

Moreover, the findings from Activity Structure 2 highlights a potential benefit of annotating less desirable lesson representations for motivating teachers' revisions. Such activity structures may foster teachers' curiosity and empathy towards students' experiences, encouraging them to consider the challenges students face in lessons where the teacher does too much of the cognitive work and these considerations can support them in exploring alternative approaches for improving students' learning outcomes.

In the section focusing on sociality, we presented two key findings that highlight the limitations of structure alone in explaining teachers' annotations. Firstly, despite both Groups 1 and 2 participating in similar activity structure, Group 1 exhibited a greater inclination towards offering evaluative comments compared to Group 2. Surprisingly, Groups 2 and 3, despite being involved in differing activity structures, demonstrated similar aversions to evaluation when compared with Group 1. This difference in propensity to evaluate might be explained by the fact that Group 2 and 3 are composed of teachers drawn from the U.S., while Group 1 is composed of teachers drawn from Bulgaria. Prior research has consistently shown that norms of politeness and defensiveness prevail in the U.S., posing challenges in eliciting candid discussions about teaching practices (Wilson & Berne, 1999). To navigate these challenges, many professional development programs in the U.S. have adopted collaborative approaches that prioritize an inquiry stance over an evaluative one (Cohen, 2008), as evident in the frequent use of sentence stems like 'I notice, I wonder' (Dobie & Anderson, 2021).

Secondly, despite both Groups 1 and 2 both being engaged in similar activity structures, there was a notable disparity in their propensity to offer alternatives, with Group 2 being much more inclined to do so. Interestingly, Group 2 also surpassed Group 3 in this aspect, suggesting that the participants' recruitment from within the U.S. might not fully explain these differences. An alternative explanation could be that Group 2 consisted entirely of colleagues from the same school, potentially fostering a supportive environment where individuals feel comfortable taking risks and proposing alternatives without fear of judgment. This cohesion among colleagues may provide them with a unique advantage in embracing vulnerability and innovation in their annotations, rather than merely offering reflections or evaluations on practice.

Conclusion

In this study, we explored the role of lesson visualization and annotation tools in facilitating collaboration among secondary mathematics teachers across cultural boundaries. Our investigation provides insights into the dynamics that shape teachers' collaborative practices and interactions, informing the design of technologically mediated environments for fostering cross-cultural dialogue among educators.

Through the use of lesson visualization and annotation tools, such as those employed in Story*Circles*, teachers can engage in collaborative inquiry on shared lessons, transcending geographical and temporal barriers that once hindered such interactions. By collectively crafting and refining lesson visualizations through the development and exchange of annotations, teachers have the opportunity to engage in reflective dialogue about practice, explore alternative pedagogical approaches, and examine the nuances of classroom instruction. These tools serve as catalysts for collaborative inquiry, encouraging teachers to examine the complexities of teaching practice and co-construct knowledge across international boundaries.

Moreover, our analysis underscores the pivotal role of technological affordances in facilitating cross-cultural collaboration and knowledge exchange among teachers from diverse backgrounds. Lesson visualization and annotation tools offer a shared platform for educators to exchange experiences, challenge assumptions, and innovate instructional strategies. By collaboratively engaging with these digital tools, teachers can create enduring lesson artifacts that contribute to a library of lessons[3] encoding teachers' practical knowledge for teaching.

Our findings suggest the potential of lesson visualization and annotation tools for fostering international collaboration and enhancing teacher professional development. By

[3] A library of digital lesson artifacts (such as storyboards) that would permit others, otherwise uninvolved in the construction of the artifacts, to examine the artifacts, engage with them (e.g., annotating them or even modifying them), and use them for their own purposes (e.g., sharing them with colleagues for continued collaboration.

leveraging technology to facilitate collaborative inquiry and cross-cultural dialogue, educators can cultivate inclusive learning communities that promote continuous growth, innovation, and excellence in teaching practice.

Looking ahead, it is imperative for teacher educators and professional development designers to embrace digital technologies as an integral component of professional development centered on teacher collaboration. Continued research into the use of lesson visualization and annotation tools across diverse educational contexts is essential. Investigating their impact on teacher learning outcomes and professional community building will further enhance our understanding of effective strategies for leveraging technology to enrich teaching and learning in an increasingly interconnected world.

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