WHAT DO YOU SEE IN MATHEMATICAL PLAY?

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As part of a longitudinal study focused on mathematical play, we (Melissa, Amy, and Anita) are often faced with questions about what counts as play and what mathematics (and other learning) we see in play, and whose play is most likely to be seen or dismissed. Rather than discuss our findings from classroom videos of kindergarten children engaged in mathematical play, we asked scholars who bring different lenses to research on play, young children, and teaching and learning mathematics to look at some of our data and provide their perspectives. In this session, we will share video and discuss with our panel (Nathaniel, Naomi, and Tran) various ways to interpret that video. This paper provides background on the potential of mathematical play and the details of the study that generated data for analysis. We conclude with a copy of a transcript that is associated with a video we will watch during the plenary with hopes that participants will watch prior to the session and come with their own questions/perspectives.

Keywords: early childhood, identity, mathematics, play

Introduction

As content standards become more rigorous and demanding, and high stakes assessment becomes the norm even early grades, the time for exploration and play is growing increasingly scarce (Miller & Almon, 2009). Even in the earliest grades, curriculum can be tightly scripted, recess strictly timed, and toys are often absent or hidden. While prescriptive activities are efficient solutions to the now-defined work of schooling--moving large numbers of students through a large numbers of topics—this approach rarely supports students in developing robust number sense. Further, the practices that such approaches to teaching mathematics require, such as a focus on efficiency, speed, and memorization, are known to undermine students' enjoyment and deep understanding (Boaler, 2002; Boaler & Staples, 2008). In contrast, classrooms that offer time for exploration, that emphasize reasoning and understanding over accuracy and speed, and which place student identity at the center of instructional design, have been found to support a more productive relationship with the domain of mathematics (Cobb et al., 2009; Gresalfi, 2009). And yet, for a variety of reasons, such classroom practices have been challenging to develop at scale.

One aspect of mathematical exploration that is a topic of great interest, but has received very little empirical focus, is mathematical play. While the role of play in supporting student learning is well-understood and often advocated for very young students, we have little understanding

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about mathematical play for older children. Indeed, once children enter formal schooling, discussions of learning rarely reference play, and work investigating the design of mathematical learning environments has seldom explores whether or how play might be involved.

Researchers who study play across contexts define play as "spontaneous, naturally occurring activities with objects that engage attention and interest" (Lifter & Bloom, 1998)p. 164). Burghardt (2010) argued that play is spontaneous or pleasurable, functional, different from similar serious behaviors, repeated, and initiated in the absence of stress. Increasingly, educators have recognized that play provides both social and cognitive benefits, such as increasing creativity, reducing stress, and promoting problem solving (Elkind, 2007). In this work, we define play as pleasurable activities that allow children to explore, to engage with interesting materials, to make choices, and to possibly engage in social interactions. The play is *mathematical* if the play context can promote a mathematical learning goal.

For many, "mathematics" and "play" are terms that anchor two opposite extremes, with things that look like "math" resembling nothing that looks like "play." But for mathematicians, play is an acknowledged element of engaging the discipline, in part because mathematicians are offered a different version of mathematics with which to engage. Rather than taking as their task the acquisition of other people's knowledge, mathematicians are afforded the opportunity to think with and about mathematics, to inquire into its structure, its limits, and its possibilities. Many scholars have highlighted how play allows one to think beyond oneself, to test and explore the limits of ideas (Gadamer, 1975; Vygotsky, 1978), and such ideas are expressed by mathematicians. For example, as Mathematician Sharon Whitton states:

> Play has a role both in the work of mathematics and in the evolution of mathematics. Although play is not often acknowledged as a major contributing factor in mathematicians' work, their methods of inquiry resemble many of the behaviors of children involved in meaningful play (cited in Bergen (2009), p.421).

We argue that play should not be reserved only for those who are tasked with exploring the frontiers of mathematics, but rather, that play is itself an important vehicle for exploration, understanding, enjoyment, and learning. In play children can encounter and explore mathematical concepts and relationships through their engagement with carefully chosen materials (Ginsburg, 2006; Tudge & Doucet, 2004), such as when they compare quantities of toys, compose and decompose shapes of wooden blocks, and count forward and backward on a gameboard (Seo & Ginsburg, 2004). In lessons where children engage in mathematical play, they have opportunities to solve problems and explore concepts in low-stakes contexts that encourage social engagement, experimentation, and the use of interesting materials (Parks, 2015).

Prior research suggests that designing spaces that allow students to explore mathematical ideas, encounter parameters and structures, develop a sense of mathematical aesthetic, and engage in cycles of revision and justification would be a productive use of class time (Sinclair, 2004, 2026). Likewise, research has documented what young children learn in mathematics through play, such as through developing spatial reasoning through block building (Casey et al., 2008) or magnitude through linear board games (Siegler & Ramani, 2008). However, little research has sought to link mathematical learning in schools to playful experiences in mathematics for children in the primary grades (Wager & Parks, 2014), or has addressed how such play could be supported.

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Why Study Play

Bringing play into the context of mathematics requires recognizing the mathematical content embedded in spontaneous play and the possibilities for designing mathematical engagement that embody a spirit of playfulness. For example, work on embodied cognition by Lakoff and Nunez (2000) has demonstrated how spontaneous play with containers can build concepts that support set thinking about numbers while play on balance beams can create an embodied sense of the meaning of equality. At the same time, research on mathematical play that has been orchestrated by adults, such as work with linear board games or puzzles, has been shown to develop mathematical understandings while still retaining many of the features of play (Kamii et al., 2004; Siegler & Ramani, 2008).

The Role of Play in Learning and Identity

There is growing evidence that play can support students' mathematical competence and the development of productive mathematical identities (Wager & Parks, 2014). Mathematical play offers opportunities for children to engage in all of the standards for mathematical practice, for example by making sense of problems and persevering in solving them, or exploring the structure of tools, symbols, and numbers. Play with particular materials, such as blocks (Wolfgang et al., 2003), linear games (Siegler & Ramani, 2008), and puzzles (Clements et al., 2012) has been shown in a range of empirical research to impact young children's mathematical learning of both number and geometry.

With respect to identity, bringing play into mathematics also provides opportunities for children to come to see themselves as "mathematics people" (Parks, 2015, p. 83). When teachers create opportunities for children to engage in mathematical play, and when they label children's play with mathematics vocabulary, they help children see themselves as people who enjoy mathematics (Esmonde, 2009). In addition, unlike formal lessons, play contexts often allow for children and teachers to use language more fluidly and to enact different kinds of social relationships, which may be more likely to mirror the ways that children talk and play at home. This can be particularly meaningful for children from low-income or marginalized families who may experience conflicts between the performance of school mathematics in formal lessons and the ways they engage at home ((Enyedy & Mukhopadhyay, 2007). Reshaping of mathematics norms and practices serves to position students differently with respect to the discipline, broadening not only the kinds of mathematical experiences that students might have, but also, how those experiences lead them to reach different conclusions about their abilities and preferences (Gresalfi & Hand, 2019).

Supporting Play in the Mathematics Classroom

Our approach to design begins with the starting point that the classroom is an ecosystem (Greeno, 1991; Gresalfi et al., 2012): changing one element of the system can influence others, but not always directly in a simple causal relationship. Play is an excellent example of an activity that cuts across multiple elements of a system, as it involves attending to the tools and physical resources available in a classroom, but also the concomitant norms and practices that make space for such toys and tools to become objects of inquiry. Likewise, while students' beliefs about themselves and about mathematics influence how they might play in math class, those beliefs develop through their participation with these classroom norms (Gresalfi, 2009; Hand, 2010). Thus, in thinking about how to support mathematical play in elementary school classrooms, we articulate the different elements of the system, and how those elements of this ecosystem interact.

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The Role of Tools in Attuning Students to Mathematical Ideas

Within early childhood, Montessori (Montessori, 1917) was one of the earliest advocates for supporting children's mathematical learning through their independent engagement with material objects. Current studies demonstrate that young children who experience Montessori programs (even when randomly assigned) perform more strongly on mathematics assessments than children who do not (Laski et al., 2015; Lillard & Else-Quest, 2006). Research suggests that a wide variety of toys and tools can be productive in promoting mathematics learning, but that these tools are most productive when children have multiple opportunities to explore the same materials over time in a variety of contexts, when adults support children in assigning mathematical meaning to the materials, and when materials make mathematics (rather than the everyday world) salient (Laski et al, 2015). Likewise, research on children's mathematical engagement in museums has demonstrated that allowing children agency in their interactions with well-designed materials supports their engagement with mathematical concepts (Kelton et al, 2018) and that observing children's physical engagements can provide insight into their developing understandings (Nemirovsky et al., 2012).

The Role of Classroom Culture

Classrooms that support learning through mathematical play offer a different experience for students than traditional math classrooms. This shift requires an environment in which students have agency to explore, make their own decisions, and feel comfortable making mistakes - an environment that invites students to participate (Gutiérrez, 2012) and develop their identities as learners and doers of mathematics. Agency, the opportunity to take action with regard to one's own learning (Gresalfi, Martin, Hand, & Greeno, 2009), enables students to participate in mathematics in ways that are meaningful and sensible to them (Gresalfi & Cobb, 2006; Nasir & Hand, 2006). Classroom environments that provide opportunities to participate exercise agency can "transform how students see themselves as mathematical thinkers, how they see the discipline, and ultimately, the mathematics they learn" (Turner et al., 2013), p. 229).

The Role of Teachers in Supporting Mathematical Play

The potential of play to support students' mathematical engagement depends largely on the teacher. Although students are naturally experts at play, they are less likely, on their own, to play in ways that easily translate to rich mathematical thinking. When adults make conscious choices in constructing play environments and choose to intentionally engage with children during play and other informal activities, student mathematical engagement is enhanced (Trawick-Smith et al., 2016; Van Oers, 2010) In particular, teachers' use of 'math talk' during formal and informal lessons has been shown in a variety of studies to have a significant impact on children's later learning (Levine et al., 2010; Wiebe Berry & Kim, 2008). More broadly, there is growing evidence that providing materials that support mathematical play and intervening in play to deepen thinking and extend vocabulary can promote more significant early mathematics learning (e.g., van Oers, 2010; Trawick-Smith, Swaminathan & Liu, 2015; Wager, 2013). Trawick-Smith, Swaminathan, and Liu (2015) found that teachers' asking of open-ended questions during play and providing of appropriate levels of guidance during play (not too much or too little) had positive relationships with mathematics learning in a pretest/posttest study. However, despite the benefits of appropriate adult interactions in mathematical play, research has found that teachers sometimes fail to lift up the mathematics in children's play because they are not skilled at recognizing the mathematics (Moseley, 2005), lack the time to observe play (Seo & Ginsburg, 2004), or are constrained by curricula and other instructional demands (Parks & Bridges-Rhoads, 2012).

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The Playful Learning Project and the Challenges of Analysis

This four-year longitudinal study was designed to investigate play in early elementary math education by developing in-depth accounts of: 1) how kindergarten teachers learn to integrate play into their instruction over multiple years, and how their teaching changes over time; 2) how the task of integrating play changes with different demands of mathematics curricula over the grades (kindergarten through second grade); and 3) how the relationship that students develop with mathematics might be transformed by experiencing playful mathematics learning over their early elementary careers (kindergarten through second grade).These fairly straightforward goals have become much more complex as we have embarked upon the study, and new challenges have emerged. Specifically, and the he focus of this paper and our plenary, is to explore and problematize mathematical play by considering what mathematical thinking looks like when it is transformed through play, what counts as play, and how some children's play is visible, suppressed, or otherwise rendered invisible by broader structures and biases, from researchers, teachers, and other children.

Over the first year of the project, the team worked with six kindergarten teachers who taught in teams in three classrooms at a racially and economically diverse public school in the Southern United States. The teachers and the project team co-designed four weeks of instruction with the goal of introducing playful tasks into the lessons of the mandated curriculum. During the weeks these lessons were taught, the project team video recorded children's engagement in small groups using GoPro 360 cameras, which captured all four students per table simultaneously. We also videoed the teacher during whole-class and small group instruction. From the three classrooms across the four units, we captured 580 videos of student mathematical play, ranging from two to twelve minutes in duration.

Because the GoPro cameras record in 360 degrees, analyzing the videos presents several challenges in terms of attention. Often four children were engaged in four separate activities, although sometimes the whole group would come together or partners would team up. The video could be "unwrapped" to see all four children at once or could be watched so the viewer could control the focus of attention. As a research group, we found that where we directed attention in the videos and the sense we made of interactions during play about mathematics was shaped by on our professional backgrounds (e.g., former early childhood teachers and former high school mathematics teachers often attended to different aspects of the mathematics), our identities (e.g., our racial identities shaped our focus and interpretations) and our academic histories (e.g., being immersed in learning sciences, early childhood education, or teacher education framed interactions in different ways). This became even more complex when teachers were included in watching video, as their perspectives brought in additional differences.

Conversations across the group about these differences reminded us of a decades old special issue in the *Journal of Learning Sciences* (Sfard & McClain, 2002) where a group of researchers analyzed the same video through a variety of lenses to provide insight on the role of symbolic tools in shaping mathematical thinking. At the time, we remembered being struck by the different understandings of the same video that each theoretical perspective allowed, but in looking back we noticed instead how similar each of the socio-cultural perspectives taken really were. This raised some questions for us. We wondered about what perspectives we were missing even within the diversity of our research group. We were especially concerned with perspectives that were not represented in our research group and that we would be unlikely to encounter at mathematics education research conferences, but yet felt highly relevant to play.

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We were interested in what scholars who centered young children's perspectives and the social context of schooling might see in the children's play. This wondering led to this plenary. We choose a video from the first year of our study, which took place in November, during a unit on counting to twenty. The video shows four children playing at a center invited children to design a zoo with plastic animals by putting a specified number of animals in each "pen." We chose this video because it revealed rich and diverse social interactions among the children and because we felt that all the mathematics was not immediately apparent. The points of ambiguity, both social and mathematical in nature, makes it a good candidate for rich analysis, particularly analysis that draws from different perspectives.

The children pictured in this video are called Quentin, King, and Kiera (pseudonyms). Quentin is a black boy who was moved into the classroom about two weeks prior to the episode, in an effort to balance the children with "challenging behaviors" across the three kindergarten classes. In his previous classroom, Quentin was frequently removed from activities due to the teacher's perception that he was being disruptive to the class. King is a black boy who tended to be quiet in class but paid close attention to instructions in general and math in particular and was almost always on task. Kiera is a white girl who also tended to be quiet in class and was reluctant to offer answers to questions in whole-class settings. There is a fourth child in the video from whose parents did not give consent to participate, and therefore we do not include his data (he spoke very little in this interaction).

We invited three scholars who center young children in their work to share their insights as part of the plenary. They are Dr. Tran Templeton, whose work draws on critical childhood studies to explore how young children make sense of their own lives, Dr. Nathaniel Bryan, whose work draws on Critical Race Theory and PlayCrit theories to understand the experiences of young Black boys and their teachers, and Dr. Naomi Jessup, whose work draws on Critical Race Theory to reimagine mathematics education in humanizing ways, with particular attention to Black children.

Rather than share these scholars' perspectives in this paper, we invite readers to prepare for the plenary by engaging on their own with the transcript of the video and to form their own questions and (admittedly) partial understandings to bring to the conversation. The episode shown in the transcript below shows the children engaging in the zoo activity described above, along with occasional interruptions from the kindergarten teacher, Ms. Lane.

In reviewing the transcript, we encourage readers to think about the following prompts:

- What questions about mathematics and play do the interactions in the video raise for you?
- What theoretical frames do you think would illuminate the analysis?
- Which moments in the video do you find most engaging? Why?

Table 1: Transcript of Zoo Video

Time	Quentin	King	Kiera	Ms. Lane
0:00	[Sticks tongue out at camera and giggles]			
0:15	King you have two yellow ones. I have two orange ones. I know.. [unclear]			

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	Two yellow ones.			
0:26		A lion. That's what I want.		
0:28	Can I have that orange one cause I have this orange one right here and I want this?			
0:35	Can I have that orange one? I just want the orange one.			
0:45			Look these look the exact same. [showing animal to Quentin]	
0:50	Yeah. I can find them. ROAR!!			
0:55		Stop		
0:56	Let's make this a shoe store			
1:00		Hey		
1:09	[Gasp!] I found another one. Let's put them all together.			
1:15	These together [pairs up matching animals]. I got to get another giraffe.			
1:29		No. We can't put that together.		
1:38		What's that?		
1:40	[Gasp] Why do you put the stuff up?			
1:42		Oh my goodness. This is something like a.. that's a dinosaur, but I don't even know what it's called.		
1:47	You're messing up our game. We're trying to put two of each on the board.			
1:53				xxx, are you playing?
1:56				Okay, try to do that one right there. [pointing to the box labeled 7 in the zoo in front of xxx]
1:59	We're doing this one.			
2:03				After you finish you have to make sure that you count to make sure that the amount of animals matches this number

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				[points to the "13" on the stickynote in the box that Quentin and King are working on].
2:11	We're not done. We got to put a ri... [Adds elephant to animals that King is counting]	[pointing and counting each animal] 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, thir...		
2:25	Well, we can knock all of them down [knocks about half of the animals down and gathers them together] and count to..Let put. Let's grab. I'm going to grab a different animal [grabs animal from bin] So yeah….	[knocks down and grabs other half of the animals to put back into the bin]		
2:37		I'm going to count while I put them on there.		
2:41	[place hippo in 13 square] Ooooooneeee			
2:42		[places elephant in 13 square] One		
2:43	[touches elephant] No that will be two. [picks up and places hippo again] One			
2:45		One [touches elephant]		
2:46	[touches elephant] Two			
2:47	[adds dinosaur] Three	No. No. [picks up elephant. One. [picks up hippo] Two. [adds giraffe] Three.		
2:53	[adds gorilla] Four	[points to hippo, giraffe, elephant, dinosaur, then gorilla each in turn counting] One, two, three, four, five.		
2:56	[adds duck] Six			
3:00		[adds rhinoceros] Seven.		
3:04	[adds squirrel] Eight. Hey you're copying me.	[adds chetah] Eight.		
3:08		[adds deer] Nine.		
3:10	[adds lion] Ten.			
3:12		[adds giraffe] Ten		
2:13	[adds hippo] Eleven. [adds buffalo] Twelve.[adds kangaroo] Thirteen.			

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3:23		No that's thirteen there. We're done.		
3:25		Now we need to do...		
3:26	16. Let me scoot this over there a little bit [pulls zoo toward him so that the 16 box lies flat on the table.			
3:32		These are a lot of animals		
3:35				
3:39	I'm gonna put a fox on our [unclear]. I want a fox. [add fox to 16 box]. One. King can I go - Can I help you with 16? I'm trying to do 16. [moves around to King's side of the table]. I want to help with 16.			
3:57	[takes a handful of animals from bin]			
4:08		One. [unclear]		
4:12	We doing sixteen. [unclear] Okay.			
4:14		18. Stop! [places lizard, fox, cheetah on 18 box each in turn while counting] One. Two.		
4:18	Okay. I'm gonna do sixteen.	Three.		
4:22	[places panda, tapir, koala, lizard each in turn while counting] One	[places giraffe on 18 box] Four		
4:27	two, three			
4:31		Five		
3:33	Four. Uh, will you [unclear]. Excuse me King! [frustrated]. I'm trying to put this on the table to make sure it works. King, can I get one of these animals?			
4:47	[pointing to each animal in the 16 box] One two, three, four, five.			Y'all don't have to sit down if you don't want to. You can stand up if it's easier. Is it easier?
4:53	Yes			Like what Quentin is doing?
5:03				Okay, we will scoot your chair in. That

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				way you can walk around the table and look at [unclear].
5:08	[scoots his chair in and then goes to scoot King's chair in] King. King. Excuse me. We can push our chair up the teacher said and walk around.			
5:20	[touching each animal in 16 box and counting them in turn] One, two, three, four, five, six.	[touching each animal in 18 box and counting them in turn] One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen.		
5:29		[puts hippo in 18 box] fourteen.		
5:32		Sixteen. Eighteen.	[put 4 new animals in 18 box]. Fifteen, sixteen, seventeen, eighteen.	
5:35	One, two, three, four, five, six. [adds new animal to 16 box] Seven.	What's this? A bull! That's a bull!		
5:44	Kiera, could you give me one of the animals? I need an animal.			
6:02	I know what these are. Are we? Uh, Kiera – there's three of us over here.			
6:07			I know.	
6:10	You're smushing me.			
6:12			[moves around the table to 7 block]	
6:13	What about this guys? We can scoot your chair down here and make it easier.		[moves back to side with King and Quentin]	
6:24		What is this? A monkey?		
6:28	Yeah! Uh, I don't know. Gasp! I know what these are! Um. They're twins. Who wants to match the twin with m–? Let's match the the animals! So animal...			
6:46			Let's match the animals.	
6:49	So this one goes with...			
6:52		Oh, it's not matched yet.		
6:54	I'm matching			
6:55			Me too!	

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6:56	So them go together. The ones that don't go together let's put them over here [touches 16 box]			
7:03	These go together [picking up a pair of matched animals]			
7:04			Here, these go together. [hands Quentin a cheetah that matches a cheetah in his hand]	
7:06	Oh yeah. Uh, it's right...yeah, uh it's another one. Here. [hands her the cheetah]			
7:13			Here. Ha! [hands Quentin a giraffe]	
7:15	No. I don't need that one. These two match. And these match.These two match. I've got one over here that matches. [handing Kiera pairs of animals each time]			
7:34	Is this an elephant?			
7:35			Yep	
7:36	Okay elephant			
7:45	[continues pairing up animals and putting them in front of Kiera]. We're trying to match them!			
7:50			That's you. I'm just standing right here.	
7:54			That's him that's not me. That's him.	
7:57	It's you too. I'm giving them to you and then you're putting them on the thing.		Nuh-uh!	
8:00	We're not doing – We're not doing anything BAD!			
8:05			We just match them.	
8:08	We're just trying to take care of the animals [unclear]			
8:12		We don't need to do that		
8:15			Here, here's this.	
8:22		Look at the timer [to Quentin]		

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8:25			Oh, it's a 6! Clean up. Let's clean up. [all students frantically pick up animals and put them back in bin]	
8:35	[takes bin and put it in front of himself] Fast! Faster! Faster than a - Let's do it faster than a giraffe.			
8:45				Calm it down okay. Go ahead and have a seat.

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