UTILIZING CONJECTURE MAPPING TO DESIGN A DIGITAL TASK FOR DEVELOPING PRODUCTIVE GRAPHING MEANINGS

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In this preliminary theoretical paper, we describe our use of conjecture mapping (Sandoval, 2014) to guide the design of a digital task sequence to support 6th graders' meanings for points as a simultaneous representation of the amount-ness of two quantities. The conjecture map ultimately serves as a theoretical framework with testable conjectures about how the design of the digital task sequence might promote the intended learning outcomes.

Keywords: Design Experiments, Technology, Mathematical Representations

Graph construction and interpretation are critical skills for advanced mathematics coursework and consumption of popular media (Glazer, 2011). Despite the importance of graphs in K-12 mathematics and science curricula (CCSS; NGSS), research indicates that students struggle with graph construction and interpretation well into their post-secondary studies (e.g., Carlson et al., 2002; Glazer, 2011). One explanation for students' challenges with graphing is that they develop meanings for graphs that are useful initially but are limited as they advance through the mathematics curriculum (e.g., understanding points as a set of directions for how far to move over and up from the origin; Frank, 2016). We posit that one way to address this challenge is to support students in developing more productive meanings for graphs when they first encounter graphs in the curriculum. A promising approach to understanding graphs is emergent graphical shape thinking (EGST; Moore & Thompson, 2015). In this theoretical report, we describe our effort to design a task sequence that supports 6th grade students (11-12 years old) in developing EGST. This task design effort was undertaken between rounds of a design-based research study (Cobb et al., 2003), and we utilized conjecture mapping (Sandoval, 2014) to guide our design toward the dual goals of developing theory about how the design of the learning environment functions and about how the development of productive graphing meanings occurs.

Background

Within a multi-year design-based research study (Cobb et al., 2003), we have been working to develop an instructional sequence that supports 6th grade students in developing EGST. We conducted multiple rounds of small group teaching experiments (Steffe & Thompson, 2000) in which pairs of students worked through our task sequences. The ultimate learning goal we intended to support was students' development of EGST.

EGST entails conceiving of a graph as a record of covarying quantities (Moore & Thompson, 2015) which can be created by imagining the trace of a point moving through the coordinate plane such that the motion of the point is constrained by the relationship between situational quantities. As such, developing EGST requires attention to students' quantitative reasoning within situations and graphs. Quantities are conceptual entities grounded in an individual's conception of a situation. "A person is thinking of a quantity when he or she conceives a quality of an object in such a way that this conception entails the quality's measurability" (Thompson, 1994, p. 7). An attribute of a situation is measurable to an individual if they can conceive of a process for measurement that results in an amount and a unit (or the anticipation of a unit). The situation in which a person constructs quantities can be a graph (e.g., constructing vertical distance above the horizontal axis as an attribute of a point) or some experientially real context (Gravemeijer & Doorman, 1999; e.g., constructing weight and habitat temperature for animals at the zoo). Although engaging in EGST entails conceiving of varying quantities, in this report we detail an effort to support students in what we conjecture is a prerequisite meaning for graphs as representing relationships between static quantities.

Informed by Paoletti et al.'s (2023) LIT for developing EGST with advanced 8th graders, our high-level conjecture is that repeated occasions to draw explicit connections between meanings for situational quantities (situational quantitative reasoning; SQR) and meanings for graphical quantities (graphical quantitative reasoning; GQR) is critical for students' developing EGST. Given space constraints in this report, we focus on the first level of SQR and GQR. Denoted as SQR1 below, students must first construct quantities in a situation and conceive of the quantities as being able to take particular amounts in the situation. GQR1 entails considering the length of a magnitude bar as representing a static amount (i.e., constructing the quantity of length). Bridging SQR1 and GQR1 (SQR1↔GQR1) entails considering a magnitude bar as representing the static amount-ness (Stevens & Moore, 2017) of a situational quantity.

We developed the Zoo Task sequence to provide students with opportunities to reason about points as a simultaneous representation of the amount-ness of two situational quantities. However, retrospective analysis from our first two rounds of teaching experiments that used the task indicated that students needed more (or different) opportunities to bridge SQR and GQR. We decided to redesign the Zoo Task sequence to meet this need and took up conjecture mapping (Sandoval, 2014) as a strategy for redesigning the task in ways that would enable us to test and refine our conjectures about the task design and process of developing SQR and GQR. Conjecture mapping attends to the dual goals of design-based research by differentiating between theories about the design of the learning environment and theories about the process of learning. Conjecture maps depict the ways researchers anticipate the design of the learning environment supporting learners in engaging in observable processes as well as conjectures about how engagement with those observable processes results in the desired learning outcomes. Our initial conjecture map is in Figure 1.

The Zoo Task

We describe the opening sequence (screens 2-5) of the Zoo Task to ground descriptions of the conjecture map in the next section. Due to space constraints, we only report on the opening sequence of the task which we designed to support students' SQR1, GQR1, and SQR1↔GQR1. To support the reader, we provide a link to the opening activity sequence so that the digital interactions we describe here can be experienced as we designed them (https://bit.ly/ZooTaskPMENA). On Screen 2 (Figure 1a), students are prompted to weigh three mystery animals at the zoo and record their weight. Students can weigh each animal by dragging it to a scale and pressing the 'Weigh It' button. In response to those actions, the scale depresses with a bounce (imagine a heavy object being placed on a spring-loaded plate) and the weight of the animal is represented with numbers and a magnitude bar. Students then record the animal's weight by dragging a point to construct a vertical magnitude bar with a numeric readout. On Screen 3 (Figure 1b), students are prompted to order five animals from lightest to heaviest given five vertical magnitude bars. Three of the magnitude bars are copied over from their construction on Screen 2 and include a numeric readout, but the other two magnitude bars represent the weight of new animals and do not have numeric readouts. When students select an order and press the ‘Check It’ button, the vertical magnitude bars are dynamically rearranged to reflect the student’s selection and evaluative feedback (a green checkmark) appears next to the animal names in the list if they are in the correct location within the list. Screens 4 and 5 follow a similar design but with opportunities to measure habitat temperature by dragging a temperature probe into each enclosure and then recording values on horizontal magnitude bars.

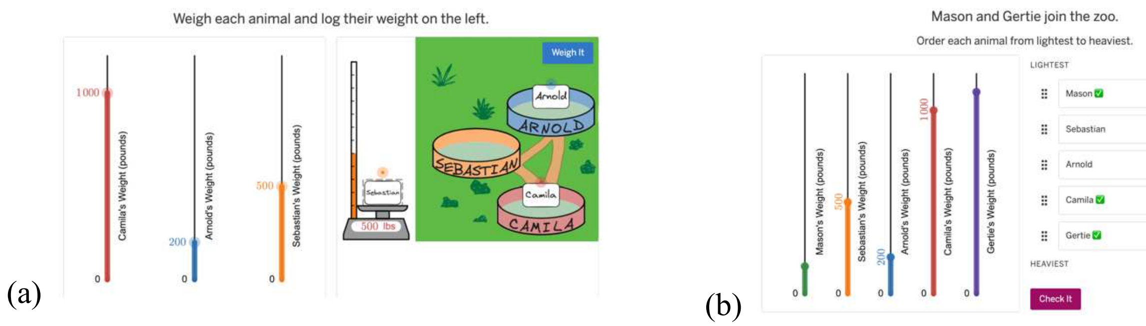

Figure 1: (a) Screen 2, and (b) Screen 3 of the redesigned Zoo Task.

An Initial Conjecture Map for the Redesigned Zoo Task

Recall, our high-level conjecture is that repeated occasions to draw explicit connections between meanings for situational quantities (SQR) and meanings for graphical quantities (GQR) is critical for development toward EGST. To test that conjecture, we need students to (1) develop SQR, (2) develop GQR, and (3) bridge SQR and GQR meanings. Due to space constraints, we report on the opening sequence of the zoo task that we intend to support students in developing the first level of SQR, the first level of GQR, and bridging between those two meanings.

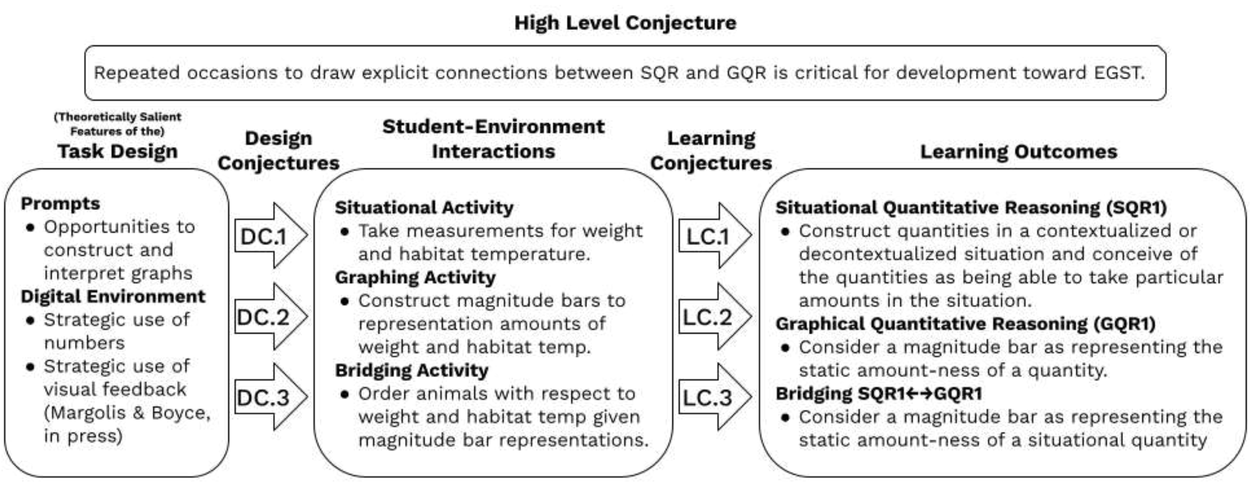

Figure 2: An initial conjecture map to guide the redesign of the Zoo Task.

To support SQR1, we theorized that directly measuring weight and habitat temperature for several animals supports students in conceiving of an attribute of the animals (i.e., heaviness;

hotness) as being measurable (i.e., having an amount and a unit) (LC1; see Figure 2). To support GQR1, we theorized that constructing magnitude bar representations to record particular values would support students in understanding that the length of a magnitude bar represents an amount-ness (e.g., longer bar = larger amount) (LC2). Furthermore, using the magnitude bars to represent particular amounts of weight and habitat temperature derived through direct measurement can help students bridge SQR1⟷GQR1 (LC2). Lastly, we theorized that ordering animals with respect to a situational quantity (weight or habitat temperature) when provided with information about those quantities via magnitude bar representations would support SQR1⟷GQR1 because students would have to set a goal related to the situation (e.g., determine which animal weighs the least) and then use information from the graphical representation to achieve that goal (e.g., which magnitude bar is the shortest) (LC3).

The observable interactions between student and digital environment that are necessary for testing these learning conjectures are listed in the middle column of the conjecture map. Students need to (1) directly measure the weight and habitat temperature of several animals (toward LC1), (2) construct magnitude bar representations of particular weight and habitat temperature amounts (toward LC2), and (3) interpret magnitude bar representations to order animals with respect to weight and habitat temperature (toward LC3).

Next, we developed design conjectures that link the theoretically salient aspects of the task design to the production of desired student-environment interactions. Our goal was for this activity to be a stand-alone digital activity, so we wanted students to be able to directly measure the weight and habitat temperature of zoo animals *within* the digital environment. We theorized that we could design student-environment interactions that emulate direct measurement by coordinating available actions (i.e., drag to the scale and click 'Weigh It') and visual feedback (i.e., "bouncing" on the scale) (DC1). To support students in constructing magnitude bar representations of particular amounts of weight and habitat temperature, we theorized that strategic use of numbers could support students in linking the result of direct measurement in the situation with their understanding of magnitude bars as representing amounts (DC2). The reification of this design conjecture can be seen in Figure 1a where the 500 pounds can be seen as the dynamic label on the magnitude bar representing Sebastian's weight as well as the result of measuring Sebastian's weight on the scale. Our final design conjecture is that strategic use of numbers and strategic use of visual feedback (Margolis & Boyce, in press) can support students in utilizing magnitude bar representations to order animals with respect to weight and habitat temperature (D3). Specifically, when we prompt students to order animals (Screens 3 and 5; Figure 1b), they can view the magnitude bars with numeric readouts for the three animals that they measured on Screens 2 and 4 but are not provided with numbers for the two new animals. We anticipate that this strategic use of numbers will result in students' reasoning about the bars' lengths rather than reasoning about the relationship between values. After students select an order and press the 'Check It' button, the magnitude bars dynamically rearrange to reflect the order of their list. When a student has the animals out of order (as in Figure 1b), we anticipate that the reordered magnitude bars will be useful for reasoning about the necessary adjustments.

Discussion & Future Work

We posit that conjecture mapping is a useful tool for studying the complex links between task design and mathematics learning. Our initial conjecture map serves as a preliminary theoretical framework with testable conjectures about how to design digital tasks that support 6th grade students in developing SQR1, GQR1, and SQR1↔GQR1. Future work can focus on empirically verifying the design and learning conjectures from this conjecture map. Additional work can focus on whether and how the development of SQR, GQR, and SQR↔GQR support the development of EGST. Such research could lead to the development of curricular materials that alleviate student struggles with graph construction and interpretation important for their future in advanced coursework and as critical citizens.

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