Optimization of Electric Vehicle Fast Charging and Charging Station Quality of Service: A Hardware-in-the-loop Study

Mohammad Hossein Abbasi, Saroj Paudel, Dillip Kumar Mishra, Jiangfeng Zhang, and Venkat Krovi
Department of Automotive Engineering
Clemson University
Greenville, SC, USA
jiangfz@clemson.edu

***Abstract*—This paper optimizes power allocation among charging electric vehicles (EVs) at a fast charging station (FCS) when the available power is limited. Lyapunov optimization (LO) is used to optimally distribute the limited power among EVs such that the quality of service (QOS) is maximized, where maximizing the QOS maximizes charging speed. Three case studies are explored and verified via simulation and hardware-in-the-loop tests. The first case study focuses on uniformly optimizing the QOS for each customer using Jain's fairness index. Subsequently, the second case study integrates prioritization where EVs, who pay more, are given precedence over others and can be charged faster. Finally, the third case study optimizes the EV fast charging profile in addition to optimizing power allocation at the FCS. In this case, two LO problems work together to find the optimal allocated power and charging profile in real-time. Each EV uses the outcome of the allocated power to update its charging profile, while the updated charging profile is used to re-update the allocated power and so on. The results show that the proposed approach reduces the increment in charging time by over 30%.**

***Index Terms*—EV fast charging, Fast charging station, Lyapunov optimization, Quality of service, Hardware in the loop.**

I. Introduction

Electric vehicles (EVs) powered by renewable generation are viewed as the primary solution to the emission mitigation of the transportation sector [1]. However, some of the main challenges hindering EV development stem from the limitations of fast charging stations (FCSs), ranging from accessibility to power restrictions [2]. The literature tackles EV fast charging challenges in two aspects. Quite a few papers have addressed the EV fast charging problem, focusing on charging speed and degradation reduction. On the other hand, researchers work on FCS management, minimizing FCS operation costs and its strain on the power grid while maximizing the quality of service (QOS). However, concurrently addressing both aspects is challenging, considering their mutual impact on each other.

EV fast charging optimization approaches can be categorized into model-based and model-free strategies, where model-free techniques use deep machine learning algorithms. Further, the model-based methods use either battery electrochemical models or equivalent circuit models (ECMs). Ref. [1] exploits model-free deep reinforcement learning (DRL) to fast charge an EV battery pack as rapidly as possible while maintaining a minimal aging. The authors in [3] explore transfer learning DRL to fast charge EVs without needing to retrain the RL agent from scratch when some environmental factors change. A DRL agent is trained in [4] to fast charge a single cell within 15 minutes and keep the battery temperature under 70°C. In [5], a personalized charging strategy is formulated that incorporates user behavior. The problem is then solved via DRL, adding 1.5 years to battery life.

Electrochemical battery models are precise but sophisticated and are not currently used in practice due to their slow computation [1]. In addition, ECMs boast straightforward implementation but are bereft of internal battery states [6]. With the help of an ECM, Ref. [7] exploits a material-agnostic approach based on asymmetric temperature modulation to achieve a 70%-75% state of charge within 12 minutes. In [8], a novel fast charging method is proposed where supercapacitors assist the Li-ion battery while charging. The proposed approach models the battery using an ECM and yields 40% longer battery life. A multi-stage constant current charging protocol is designed using an electrochemical battery model in [9]. The proposed model is then solved by a nonlinear model predictive control algorithm, which reduces charging time by 11.7% and enhances capacity loss by 59.4%.

DC FCS optimization is necessary to hedge against range anxiety and enable long-mile travel [10]. The FCS optimization encompasses minimization of operation costs as well as maximization of the QOS experienced by users. In the scope of FCSs, QOS can be defined as delivering the required energy to the customers, charging time, variations of charging power, etc. [11]. In [2], FCS QOS is maximized when power demand exceeds the station's power limit by dynamically distributing the available power among charging EVs. The optimal power allocation minimizes the increase of charging time. Dynamic pricing is used to mitigate FCS operation costs in [10]. The research in [12] factors the battery charging rate and user decision in the FCS optimization problem. An FCS aggregator's participation in electricity markets is optimized in [13] considering numerous FCSs with renewable energy sources (RESs). The results show a reduction in operating costs and RES curtailment.

Simultaneous optimization of EV fast charging and FCS QOS is challenging, considering their intertwined interaction. Therefore, this paper first formulates the power allocation among charging EVs at an FCS considering the power cap at the station. Subsequently, it fuses the FCS power allocation with EV fast charging optimization and solves both problems.

II. Problem Formulation

The present work formulates the power distribution among charging EVs at an FCS considering the power cap at the station. The power cap may change throughout the day for various reasons, such as electricity prices. The power allocation is done based on the required charging power of each connected EV, which follows the EV's charging profile. It is not possible to know the charging profile of EVs in advance. However, it is possible to know their current power requirement through built-in communication means at charging ports (CPs). Consequently, LO is chosen to tackle the problem. LO decomposes the problem into single-step subproblems and solves each subproblem greedily. Yet, we prove the greedy or local optimum solutions accumulate to a suboptimal solution close to the global optimal solution [10]. This characteristic of LO makes it suitable for FCS management problems that are heavily influenced by user behavior randomness. Thus, the problem is solved for each time step separately. At each time step, connected EVs communicate their requested charging power with the FCS, which is sufficient information for the designed LO algorithm to optimize the problem.

The proposed LO algorithm distributes power among charging EVs by maximizing all customers' QOS uniformly or prioritizing those who pay more. Prioritization can also be used to charge first responder vehicles quickly. Lastly, the devised LO approach is fused with EV charging profile optimization. The latter problem is also cast into the LO framework so that two LO problems work together. At each time step, EVs compute their optimal charging power using the second LO algorithm and communicate it to the FCS. The station then updates the power distribution among EVs via the first LO algorithm. Then, the loop repeats indefinitely.

A. Case 1: Power Allocation Without Prioritization

Firstly, power allocation without prioritization is formulated. In this regard, system time ratio (STR) is exploited to measure QOS as follows [14],

$$\mathscr{X}_t^i = \frac{\mathscr{T}_t^l}{\mathscr{T}_t} = \frac{E_t^i/P_t^i}{E_t^i/P_t^b} = \frac{P_t^b}{P_t^i}, \tag{1}$$

$$\mathcal{S}_t = \frac{\left(\sum_{i=1}^N \mathscr{X}_t^i\right)^2}{N\sum_{i=1}^N \left(\mathscr{X}_t^i\right)^2}, \tag{2}$$

where $\mathscr{X}_t^i$ indicates the STR at the i-th CP, $\mathscr{T}_t^l$ and $\mathscr{T}_t$ denote the remaining charging time with and without station power cap, respectively, E_t^i states the remaining energy needed to finish charging, P_t^i and P_t^b express the charging power with and without FCS power limit, respectively, $\mathcal{S}_t$ is QOS, and N shows the total number of CPs. Note that $P_t^i \leq P_t^b$, $0 \leq \mathcal{S}_t \leq 1$, and $\mathcal{S}_t = 1$ if all $\mathscr{X}_t^i$s are equal.

Accordingly, the optimal power allocation problem is [2],

$$\min_{P_t^i} \qquad -\frac{1}{T}\sum_{t=1}^T \mathbb{E}\left\{\mathcal{S}_t\right\}, \tag{3a}$$

subject to

$$\sum_{i=1}^N P_t^i \leq P_t^s, \quad \forall t, \tag{3b}$$

$$0 \leq P_t^i \leq P_t^b, \quad \forall t, \tag{3c}$$

$$\bar{Q} < \infty, \tag{3d}$$

where P_t^s is the FCS's power limit, and $\bar{Q} = \frac{1}{T}\sum_{t=1}^T \mathbb{E}\left\{Q_t\right\}$ represents the time average of Q_t defined as,

$$Q_{t+1} = \max\left[Q_t - e_t, 0\right] + a_t, \tag{4}$$

where Q_t is the total unmet demand of EVs at the FCS, e_t presents the total energy delivered to EVs at t, and a_t shows the total energy demand added at t.

Define a Lyapunov function as $L[Q_t] \triangleq \frac{1}{2}Q_t^2$, which is zero if all EVs are charged [2]. Next, define the Lyapunov drift as $\Delta[Q_t] \triangleq \mathbb{E}\left\{L[Q_{t+1}] - L[Q_t]|Q_t\right\}$, which shows the expected change of Lyapunov function over one time step [10]. Finally, the drift-plus-penalty term is $\Delta[Q_t] - V\mathbb{E}\left\{\mathcal{S}_t|Q_t\right\}$ [15]. Weight $V > 0$ provides a trade-off between satisfied demand and QOS.

According to the LO theory, the stability constraint (3d) is met if an upper bound exists for the Lyapunov drift $\Delta[Q_t]$ at each time step. Besides, optimizing the penalty term minimizes the time average of QOS, equivalent to optimizing (3a). Theorem 1 derives the upper bound for the Lyapunov drift.

Theorem 1. *At any time slot t, the Lyapunov drift-plus-penalty function has the following upper bound,*

$$\Delta[Q_t] - V\mathbb{E}\left\{\mathcal{S}_t|Q_t\right\} \leq B + Q_t\mathbb{E}\left\{a_t - e_t|Q_t\right\} - V\mathbb{E}\left\{\mathcal{S}_t|Q_t\right\}, \tag{5}$$

where $B = \frac{1}{2}\left(e_{\max}^2 + a_{\max}^2\right)$.

Proof. Note that, the maximum possible value for e_t is $e_{\max} = NP_{\text{CP}}^{\max}\Delta t$, where $P_{\text{CP}}^{\max}$ is a CP's rating power. Similarly, maximum of a_t is $a_{\max} = NC_b^{\max}$, where $C_b^{\max}$ is the maximum EV battery capacity. It can be shown that $Q_{t+1}^2 \leq Q_t^2 + e_t^2 + a_t^2 + 2Q_t\left(a_t - e_t\right)$ [2]. Hence,

$$L[Q_{t+1}] - L[Q_t] \leq \frac{1}{2}\left(e_{\max}^2 + a_{\max}^2\right) + Q_t\left(a_t - e_t\right).$$

By taking conditional expectations w.r.t $Q(t)$, we obtain

$$\Delta[Q_t] \leq \frac{1}{2}\left(e_{\max}^2 + a_{\max}^2\right) + Q_t\mathbb{E}\{a_t - e_t|Q_t\}.$$

By adding the penalty term to both hand sides of the inequality,

$$\Delta[Q_t] - V\mathbb{E}\{\mathcal{S}_t|Q_t\} \leq B\ + Q_t\mathbb{E}\{a_t - e_t|Q_t\} - V\mathbb{E}\{\mathcal{S}_t|Q_t\}.$$

■

Algorithm 1 QOS Optimization Algorithm

Inputs: V, P_t^s, Δt, a_t

Outputs: P_t^i

Initialization: $Q_1 \leftarrow \sum_{i=1}^{N} E_1^i$

for $t \in [1, T]$ **do**

  find P_t^i by solving problem (6)

  $e_t = \Delta t \sum_{i=1}^{N} P_t^i$

  update a_t

  $Q_t \leftarrow \max[Q_t - e_t, 0] + a_t$

end for

The upper bound of (5) can be iteratively minimized for each time slot utilizing the following,

$$\min_{P_t^i} \qquad Q_t\left(a_t - e_t\right) - V\mathcal{S}_t, \tag{6a}$$

subject to

$$\sum_{i=1}^{N} P_t^i \le P_t^s, \tag{6b}$$

$$0 \le P_t^i \le P_t^b. \tag{6c}$$

The iterative process of minimizing the upper bound of (5) is presented in Algorithm 1. Theorem 2 proves that Algorithm 1 yields a suboptimal solution for (3), which is close to the true optimal solution.

Theorem 2. *For all $T > 0$,* (3a) *has the following upper limit,*

$$-\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\{\mathcal{S}_t\} \le \frac{B}{V} - p^* + \frac{1}{TV}\mathbb{E}\{L[Q_1]\}, \tag{7}$$

where p^* is the maximum value of $\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\{\mathcal{S}_t\}$.

Proof. Given enough time, the time-average of $a(t) - e(t)$ is zero. Assume that all EVs at each CP come and finish charging during the period $[1, T]$, i.e., no EV will come before $t = 1$ or finish charging after $t = T$. Since the total amount of energy charged to the EVs is equal to the total amount of energy required by the EVs at each CP, then for any k-th CP, with $k \in \{1, 2, \cdots, N\}$, $\sum_{t=1}^{T}[a_t^k - e_t^k] = 0$.

By taking expectations from both sides of the inequality (5) and summing over all time slots,

$$\mathbb{E}\{L[Q_T]\} - \mathbb{E}\{L[Q_1]\} - V\sum_{t=1}^{T}\mathbb{E}\{\mathcal{S}_t\} \le TB - VTp^*.$$

Additionally, $L[Q_T]$ is a positive-definite function; thus, we can omit it from the left-hand side. By rearranging and dividing by VT, we obtain

$$-\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\{\mathcal{S}_t\} \le \frac{B}{V} - p^* + \frac{1}{TV}\mathbb{E}\{L[Q_1]\}.$$

■

Theorems 1 and 2 prove that instead of solving problem (3), one can minimize the upper bound of the drift-plus-penalty term for each time slot, yielding a suboptimal solution with at most $\mathcal{O}(1/V)$ deviation from the global optimal point.

B. Case 2: Power Allocation With Prioritization

In order to prioritize EVs who pay more, a weighting factor is added to (1), where $\omega_t^i \propto \frac{1}{\text{price}}$, i.e., ω_t^i is inversely proportional to the purchasing price.

$$\mathscr{X}_t^i = \frac{P_t^b}{\omega_t^i P_t^i}. \tag{8}$$

Algorithm 1 is employed for this case as well. However, it applies (8).

C. Case 3: Power Allocation And EV Charging Optimization

In this subsection, EV fast charging is formulated to be solved by LO. We ultimately introduce an algorithm similar to Algorithm 1 to optimize the EV charging profile iteratively. The EV charging problem is as follows,

$$\min_{I_t} \qquad \frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\left\{\alpha_1\left(z_t - z^d\right)^2 + \alpha_2\Delta\zeta_t\right\}\Delta t, \tag{9a}$$

subject to

$$I^{\min} \le I_t \le I^{\max}, \tag{9b}$$

$$V_t I_t \le P_t^i, \tag{9c}$$

$$\mathscr{Q}_t < \infty, \tag{9d}$$

where I_t is EV's charging current, α_1 and α_2 are weights, z_t denotes battery state of charge (SOC), z^d is the desired SOC, $\Delta\zeta_t$ shows battery degradation, and V_t indicates battery's terminal voltage. Eq. (9c) imposes an FCS power limit on the EV where P_t^i presents the available power at the CP to which the EV is connected. Moreover, $\mathscr{Q}_t \triangleq z^d - z_t$ is a virtual queue that becomes zero when the battery is fully charged. Since, $z_{t+1} = z_t + \frac{I_t}{C_b}\Delta t$, then, $\mathscr{Q}_{t+1} = \mathscr{Q}_t - \frac{I_t}{C_b}\Delta t \triangleq \mathscr{Q}_t + \phi_t$, where C_b is battery capacity. Further, $\Delta\zeta_t$ is defined as [1],

$$\Delta\zeta_t = \beta \exp\left\{-\frac{E_a}{RT_c}\right\} A^{0.55},$$

where $E_a = -31700 + 370.3$C-rate is activation energy, R is the universal gas constant, T_c states battery core temperature, and A represents battery ampere-hour throughput.

Define a Lyapunov function as $\mathscr{L}[\mathscr{Q}_t] \triangleq \frac{1}{2}\mathscr{Q}_t^2$. Then, define the Lyapunov drift as $\Delta[\mathscr{Q}_t] \triangleq \mathbb{E}\left\{\mathscr{L}[\mathscr{Q}_{t+1}] - \mathscr{L}[\mathscr{Q}_t] | \mathscr{Q}_t\right\}$. Lastly, the drift-plus-penalty term is $\Delta[\mathscr{Q}_t] - \mathcal{V}\mathbb{E}\left\{u_t | \mathscr{Q}_t\right\}$, where $u_t = \left(\alpha_1\left(z_t - z^d\right)^2 + \alpha_2\Delta\zeta_t\right)\Delta t$.

Analogous to subsection II-A, two theorems are derived to show the upper bound of the drift term and the objective in (9a), respectively.

Theorem 3. *At any time slot t, the EV Lyapunov drift-plus-penalty function has the following upper bound,*

$$\Delta[\mathscr{Q}_t] + \mathcal{V}\mathbb{E}\left\{u_t | Q_t\right\} \le \frac{1}{2} + \mathscr{Q}_t\mathbb{E}\left\{\phi_t | \mathscr{Q}_t\right\} + \mathcal{V}\mathbb{E}\left\{u_t | \mathscr{Q}_t\right\}. \tag{10}$$

Proof. Similar to Theorem 1. ■

Algorithm 2 EV Charging Profile Optimization

Inputs: $\mathcal{V}$, z^d, C_b, Δt
Outputs: I_t
Initialization: $\mathscr{Q}_1 \leftarrow z^d - z_1$

for $t \in [1, T]$ **do**
  find I_t by solving problem (11)
  $\phi_t = -\frac{I_t}{C_b}\Delta t$
  $\mathscr{Q}_t \leftarrow \mathscr{Q}_t + \phi_t$
end for

Subsequently, the following optimization problem minimizes the upper bound of (10),

$$\min_{I_t} \qquad \phi_t \mathscr{Q}_t + \mathcal{V} u_t, \tag{11a}$$

subject to

$$I^{\min} \leq I_t \leq I^{\max}, \tag{11b}$$
$$V_t I_t \leq P_t^i. \tag{11c}$$

Problem (11) is solved for each time slot separately. Algorithm 2 shows the said iterative process. Note that Algorithm 2 is run for each EV independently. Theorem 4 establishes that Algorithm 2 results in a suboptimal solution for (9), which is close to the global optimal solution.

Theorem 4. *For all* $T > 0$, (9a) *has the following upper limit,*

$$\frac{1}{T}\sum_{t=1}^{T} \mathbb{E}\{u_t\} \leq \frac{B_1}{\mathcal{V}} + p_1^*, \tag{12}$$

where p_1^* *is the minimum of* (9a) *and* $B_1 = \frac{3}{2} + \frac{1}{T}\mathbb{E}\left\{\mathscr{L}[\mathscr{Q}_1]\right\}$.

Proof. Similar to Theorem 2. ∎

At every time step, each EV calculates its charging power, $V_t I_t$, via Algorithm 2 and announces it to the FCS. The FCS then computes the charging power for each EV, i.e., P_t^i, through Algorithm 1, based on the reported required power of EVs. In other words, in the FCS problem, we have $P_t^b = V_t I_t$. At the next time step, EVs utilize their updated charging power by the FCS to re-update their charging current.

III. Results

This section evaluates the proposed approach in MATLAB Simulink through simulation and HIL tests. The HIL tests are executed on OPAL-RT. Battery specifications from [1] are used for simulations, while three large Li-ion batteries are exploited in the HIL tests, assuming each battery is a connected EV.

Case 1 simulation results, where QOS for all EVs is equivalently maximized, are depicted in Fig. 1. We assume the FCS can accommodate three EVs at a time. The assumption is because we have three large batteries in our lab to run the tests. In over 5000 (s), two sets of three EVs visit the FCS with a period of having no customers in between. The FCS power limit occurs while the first 3 EVs are in the middle of their charging sessions and continues throughout the 5000 (s) horizon. The subplot on the right illustrates how the LO results follow the CC-CV curve with a bit of delay.

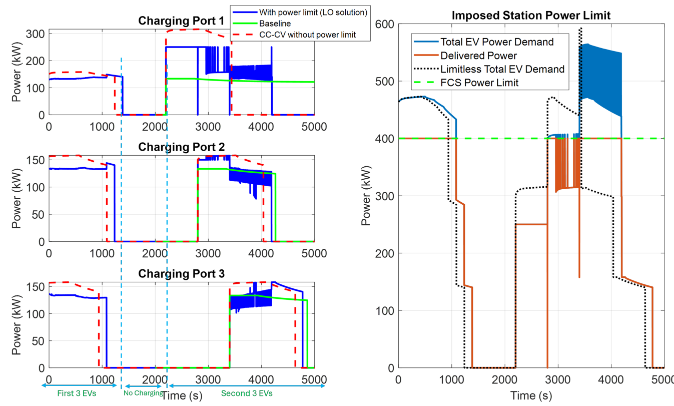

Fig. 1. Case 1 (QOS optimization without prioritization) simulation results.

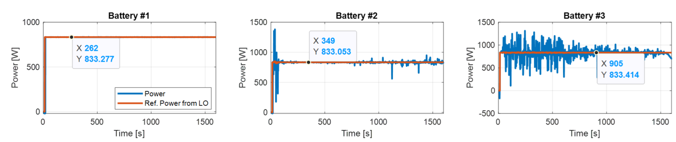

Fig. 2. Case 1 HIL results.

The results in Fig. 1 are compared with a baseline power allocation to demonstrate the effectiveness of the proposed approach. In the baseline method, plotted in green, the power allocation is not optimized, and the available power is equally divided between charging EVs. We only showed the baseline results for the second set of 3 EVs in Fig. 1. The EV in charging port 1 does not finish its charging session by the end of 5000 seconds, extending the charging time by over 800 (s) or 13 minutes. At charging port 2, the EV finishes the charging process at 4037 (s), 4189 (s), and 4264 (s) under CC-CV without power limit, the proposed approach, and the baseline method, respectively. Hence, the proposed approach reduces the increment in charging time by 33% in this case, i.e., 100*(4264-4189)/(4264-4037). Similarly, charging port 3's increment in charging time is reduced by 82 (s) or 36%.

Fig.2 illustrates the HIL results in case 1. The charging profile is constant power for all EVs due to the power limit. All EVs receive identical charging power share because no one is prioritized; they have identical batteries and start their charging session simultaneously and at similar SOCs. The fluctuations

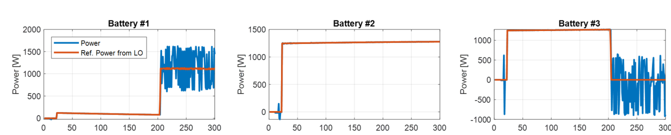

Fig. 3. Case 2 HIL results, where batteries 2 and 3 are prioritized.

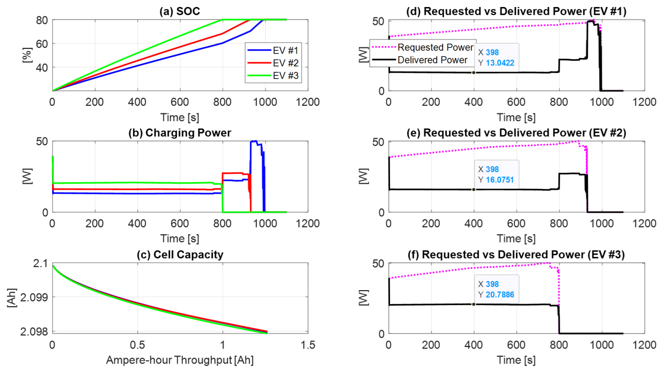

Fig. 4. Case 3 results, where batteries 2 and 3 are prioritized. Each EV uses Algorithm 2 to find its optimal charging current.

in batteries 2 and 3 are attributed to different ports of OPAL-RT, as the calculated reference lines (in red) do not fluctuate.

Fig. 3 portrays the case 2 results where $\omega_t^1 = 1.1$, $\omega_t^2 = 1$, and $\omega_t^3 = 0.9$. This means the first EV pays less than the other two EVs, while the third EV pays a higher rate than the second EV. Therefore, the first EV is allocated nearly 100 watts of charging power, and the other two are prioritized. Interestingly, EVs 2 and 3 receive roughly equal share of the available power as enough power is available to distribute between them. Hence, the higher payment rate by EV 3 is deemed unnecessary in this scenario. After just over 200 (s), EV 3 finishes its charging session, and EV 1 receives a higher share of the charging power (almost equal to EV 2's share).

Fig. 4 presents the outcome of case 3 with $\omega_t^1 = 1.2$, $\omega_t^2 = 1$, and $\omega_t^3 = 0.8$. Since EV #3 pays the highest rate, it receives higher charging power, followed by EVs #2 and #1. Subplots (d)-(e) show EVs' requested charging power and FCS's delivered power. In Figs. 4(d)-(e), EVs do not consider the FCS power limit to calculate their requested power. However, they consider the FCS power limit when calculating their charging current based on the available power. In other words, at each time step, EVs run Algorithm 2 twice, once with the imposed FCS power limit and once without it. The former is used to find their optimal charging power, while the latter is utilized to submit their charging power demand.

It takes 799 (s) for EV #3 to finish its charging session, while EVs #1 and #2 need 991 (s) and 931 (s), respectively. Hence, EV #3 is charged 3.2 minutes faster than EV #1. Additionally, after 931 (s), EV #1 receives its requested power without restriction as it is the only EV at the station. Furthermore, the imposed charging power limit has led to slight differences between cell capacity drop for each EV, as Fig. 4(c) visualizes. Interestingly, EV #2 has the least capacity drop despite being charged faster than EV #1. Nonetheless, the differences in capacity loss of the EVs are minimal as Algorithm 2 considers capacity loss.

IV. Conclusion

This paper formulates joint optimization of an FCS QOS and EV fast charging. The FCS problem considers the power cap at the station and distributes it optimally among connected EVs. On the other hand, EVs optimize their charging current according to their share of allocated power. Both FCS and EV problems are solved via LO, as it enables real-time updates on allocated power on the EV side and customer power demand on the FCS side. Three case studies are examined. The first case study optimizes power allocation in a fair manner, while the second case study prioritizes those who pay more. The third case study combines FCS QOS maximization with EV fast charging optimization. The results show that, compared to the baseline method, the proposed approach reduces the increment in charging time by over 30%. The increment occurs due to the limited power supply.

Acknowledgments

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