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Learning to Teach Teachers: Community College Faculty Explore Fraction Tasks for Teaching

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Teaching mathematics for future elementary teachers is fundamentally different from other forms of mathematics and thus requires different knowledge. As community colleges become increasingly involved in the process of training future teachers, it is essential to explore how instructors at these institutions develop as mathematics teacher educators. This paper reports on a preliminary exploration of how community college faculty grappled with teaching-oriented mathematical tasks involving fractions. Choices of mathematical representation, selection of answer before and after discussion, and overall themes are discussed, with a focus on development of mathematical content knowledge for teaching.

Keywords: Professional Development, Math Teacher Educators, Preservice Elementary Teachers

Mathematics teacher educators (MTEs) carefully consider the preparation and development of preservice elementary teachers (PTs). Though four-year institutions have predominantly prepared students for teaching and provide certification/licensure for elementary teachers through credential programs (Masingila et al., 2012), community colleges have begun to focus attention on widening the teacher preparation pipeline "as more students [turn] to them to take required mathematics and education courses" (Blair et al., 2018, p. 185). Masingila et al. (2012) found in a survey of 207 two-year college math departments that over 80% offered math content courses for PTs, implying that "two-year schools play a key role in the mathematical preparation of teachers" (p. 352).

Our study is framed by the perspective that teaching math for future teachers entails a fundamentally different approach than teaching other math courses, as learning to teach math requires different and complex ways of understanding (Ball et al., 2008). Just as teachers of math require a knowledge of math different than those *not* engaged in teaching, MTEs require knowledge of teaching mathematics that is developed and held in a way different than how teachers know it (Beswick & Goos, 2018). While the content being taught in elementary math content courses may appear simple, conceptual meaning underlying topics, addressed at both the level of the PT and the future elementary school student, is deceptively complex. Masingila et al. (2012) argue that "instructors teaching mathematics content courses designed for [PTs] may not be prepared to teach those courses in ways that will provide the type of mathematical support needed by [PTs]" (p. 355). While faculty may hold strong mathematical knowledge, many have not had extensive pedagogical training nor training for how to be a MTE. This paper focuses on the following question: How do community college math faculty reason through teaching-oriented mathematical tasks involving fractions?

Methods

This paper focuses on the mathematical work collected from a one-week professional development (PD) of 15 math faculty who are developing MTEs. None of the faculty had specific training as MTEs prior to the PD. Ten of the faculty were full-time math instructors from three community colleges, while six were part-time math instructors at a university and/or community college. Teaching experience ranged from two years to over 20 years, with over half of participants having some kind of K-12 teaching experience. Five instructors had taught a math course for PTs at least once before, while ten instructors had never taught a math course for PTs and therefore had never engaged with the ways that PTs think mathematically. All participants expressed a desire to develop their understanding of how to teach mathematics content in a first mathematics course for PTs.

Each morning, faculty engaged in a selected task from the Learning for Mathematics Teaching (LMT) Project from the University of Michigan (Hill et al., 2004). These LMT tasks were designed to be used in many different contexts. For purposes of the PD, we used tasks as "open-ended prompts which allow for the exploration of teachers' reasoning about mathematics and student thinking." (Hill et al., 2004, p. 2). We utilized a total of five tasks, each focusing on a mathematical topic related to the PD activities for that day. This paper highlights participants' responses for two of these tasks, shown in Figure 1. Task 1 targets the knowledge needed by a teacher to develop children's reasoning about comparing and ordering fractions. Task 2 demonstrates a task related to fractions, providing sequences of questions that may help a child determine how many 4s are in 3.

Task 1: Comparing and Ordering Fractions

Mr. Foster's class is learning to compare and order fractions. While his students know how to compare fractions using common denominators, Mr. Foster also wants them to develop a variety of other intuitive methods.

Which of the following lists of fractions would be best for helping students learn to develop several different strategies for comparing fractions?

a) $\frac{1}{4}$ $\frac{1}{20}$ $\frac{1}{19}$ $\frac{1}{2}$ $\frac{1}{10}$

b) $\frac{4}{13}$ $\frac{3}{11}$ $\frac{6}{20}$ $\frac{1}{3}$ $\frac{2}{5}$

c) $\frac{5}{6}$ $\frac{3}{8}$ $\frac{2}{3}$ $\frac{3}{7}$ $\frac{1}{12}$

d) Any of these would work equally well for this purpose

Task 2: How many 4s in 3?

Mrs. Brockton assigned the following problem to her students: How many 4s are there in 3?

When her students struggled to find a solution, she decided to use a sequence of examples to help them understand how to solve this problem. Which of the following sequences of examples would be best to use to help her students understand how to solve the original problem?

a) How many:
4s in 6?
4s in 5?
4s in 4?
4s in 3?

b) How many:
4s in 8?
4s in 6?
4s in 1?
4s in 3?

c) How many:
4s in 1?
4s in 2?
4s in 4?
4s in 3?

d) How many:
4s in 12?
4s in 8?
4s in 4?
4s in 3?

Figure 1: Task 1 and Task 2 from the LMT sample tasks.

Participants were first given five minutes individually to review the task, select a response, and explain their reasoning. In groups of four, participants were then given eight minutes to discuss the question with their peers before we discussed as a whole group. Finally, we asked the participants to reflect on their thinking after group discussion. Fifteen people responded to Task 1 and 13 responded to Task 2. This paper discusses the written reflections from participants. Data were analyzed through constant comparison analysis (Corbin & Strauss, 2008). All authors first read through each individual response, noting which choices were selected initially and after group discussion and notable themes from their reasoning. This guided our second read which focused on all participants' initial choice reasoning. Our third read focused on all participants' final choice reasoning. During the second and third read, we specifically focused on participants' use of language, inclusion of visual representations, and overarching themes in their reasoning.

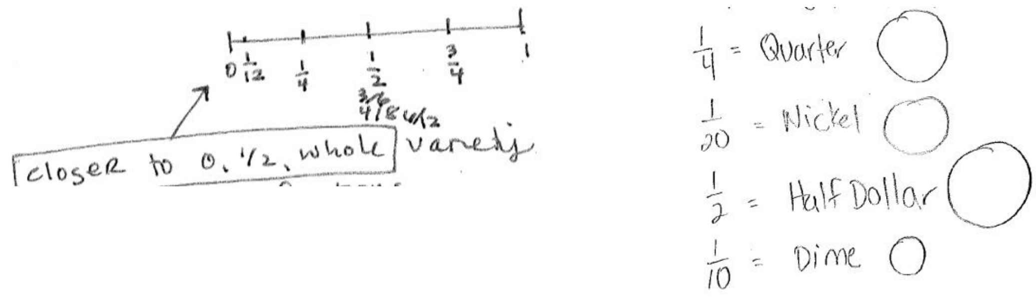
Findings

Findings are discussed for each task, with a focus on language/vocabulary participants used to explain their reasoning, types of visual mathematical representations provided in the response, and participants' initial and final answer selection.

We first discuss Task 1. Some language trends used by the participants were the words: *unit fraction*, *size*, *easiest*, and *variety*. Initially, option A was the most selected answer. Eight of the 15 participants identified (in some way) the list of fractions as unit fractions. Nine participants referred to the size of the fractions in option A, indicating that option A could help students focus on the value in the denominator and its meaning. One participant, Heidi, noted that "it is important for students to understand the 'size' of fractions first and also determine how the denominator affects the size of a fraction before comparing them." Across participants, there was significant overlap between thinking in terms of *unit fraction* and *size*, implying that the only difference in the fractions in option A was the denominator, which may help children determine the size of the fraction. Four participants labeled option A as *easy,* mentioning that the values were "easiest to compare." For example, Stephanie wrote, "Beginning by understanding fractions with 1 in the numerator makes comparing them easier to access," further sharing that most of the fractions "can be re-written with a common denominator of 20 fairly easily." The idea of *variety* was also something participants discussed, although it was not clear what they meant by the word. Four participants selected option C and one participant option B because of variety in the numerators and denominators. One participant initially liked option C because of the variety of prime and composite values in the denominators. The participant who initially chose option B changed to option C, this time mentioning variety to describe difficulty. Variety was also used to describe the many strategies that could be used to help students compare fractions.

Four participants drew diagrams in their response, while three provided real-life contexts related to the fractions in the options. The types of diagrams drawn included a tape diagram, fraction circles, a number line, and a coin model. For example, one participant drew a number line from 0 to 1, showing tick marks for 1/12, 1/4, 1/2, and 3/4 (see Figure 2a). Another bridged the idea of student knowledge and fractions in option C, stating that the denominator in 3/7 could represent days in a week or the denominator in 3/8 could represent the number of slices in a medium pizza. Courtney drew a diagram of coins to connect the denominators in option A (Figure 2b), which included different American coins in relation to a one dollar whole, drawing them to their relative size, noting that 1/19 caused a challenge for this model, but that students could compare it to a nickel.

There were substantial changes to the final selection in Task 1, with 12 participants changing their selection after group discussion. Initially, eight participants selected option A, three selected option B, two selected option C, three selected a combination of options, and one was unclear on their selection. After discussion, many participants selected a different option, and many struggled to select one option. Overall, 11 participants chose option C in some way, and *no* participants selected option A. Most participants' choices included some type of conditional statement, which indicated that they liked an option, but with some adjustments. One participant stated that she would pick options A, B, and C, sharing she would start with option A, move to option C, and then depending on class time, also use option B. It appeared that participants were highly open to hearing the perspective of others and multiple ways of thinking of the same problem. Participants noted that option C included many interesting values, could highlight the concept of benchmark fractions, and how the values could be organized in relation to closeness to 0, 1/2, and 1, as shown in Figure 2a.

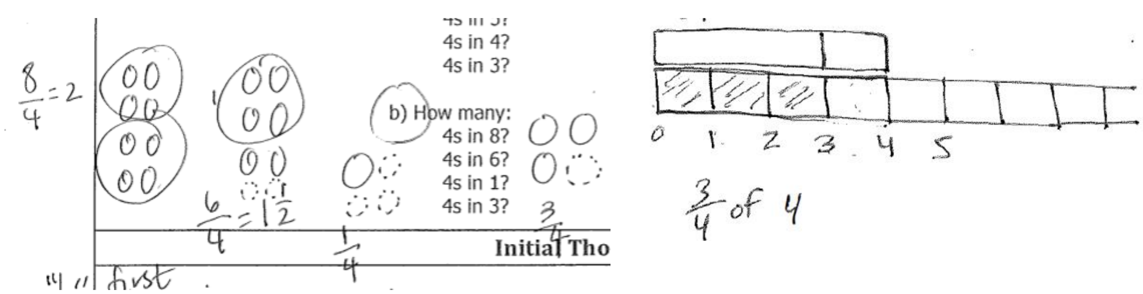


(a) (b)

Figure 2. (a) Diagram of Jane's number line. (b) Diagram of Courtney's thinking.

Next, we discuss Task 2, which asked for the best sequence to help students understand the concept of "4s in 3." Participants responding to Task 2 differed in how they conceptualized the problem, either as a division problem, fraction problem, or both. From the 13 participants, only one viewed the problem as solely a *division* problem, stating that option D led students "to recognize the use of division." Eight participants had writing indicating that they viewed this problem as a *fraction* problem, using language like whole, mixed number, or unit fraction, or by writing fractions. Four participants used language that indicated thinking of the problem as pertaining to both division and fractions.

Six participants utilized diagrams in Task 2, three of which had also drawn a diagram in the first task. One participant, Mara, had a visually distinct drawing for Task 2 involving a discrete model for her initial choice, option B, showing circles in groups of four with dotted lines to indicate fractional parts (see Figure 3a). The other four participants, in contrast, used number lines and tape diagrams. Figure 3b shows Patricia's use of a tape diagram to visually demonstrate that there should be less than one 4 in 3.



(a) (b)

Figure 3. (a) Mara's discrete drawing representing option B. (b) Patricia's tape diagram showing 4's in 3.

Task 2 also had a high number of people unable to choose one option after group discussion. Eight people had more than one answer listed, with six settling on options B and D. Reasoning included comments like "I think both explanations are valid" and "Perhaps I would use a combination of B and D," often showing slight preference for one or the other but not making a clear decision. Furthermore, one person had no answer listed but instead wrote, "context matters," with a list of the different factors that would affect a teacher's decision for which response to select, such as grade level and whether the teacher used discovery learning. Two more participants also did not choose an answer after group discussion, with one participant only including their ideal list of nine examples that would help students to understand the concept. In total, there were 11 participants who struggled to pick an answer in some way.

Due to the general indecisiveness of the group, participants were given the opportunity to create an ideal list of four example problems to lead up to the "4s in 3" question, creating a list of five examples. Twelve participants had a mixture of examples from options B and D, with six people from this group adding in the example "4s in 2" into the list. One person had a visually distinct list that was sourced from option C, using the same problems but instead listed as 4s in 4, 4s in 1, 4s in 2, then 4s in 3. Three people had lists that were longer than five examples, and after some large group discussion one participant remarked that in his classroom, he could give as many problems as he wanted. Rather than settling on a specific answer, the group felt relieved to create their own lists, agreeing that no option in Task 2 gave the "best" sequence of examples.

Discussion

The findings described above highlight both the challenges and affordances that may be leveraged in training community college faculty to become effective MTEs. Participants' work with two LMT tasks revealed differences in how they described and understood fractions and division, choices of visualization, and how they incorporated their experience into the work.

One of the most surprising features highlighted across both tasks was participants' limited use of visualizations. Very few participants provided number lines, tape diagrams, or other visuals in their work. Within their explanations and in discussions, many participants noted the importance of visualization when working with fraction ideas yet did not include a diagram themselves. It remains unclear whether this was due to a lack of need for diagrams personally or a perception that diagrams were not required when explaining to their peers. It has been shown that MTEs struggle to know when and how to incorporate visual fractional representations (Petit et al., 2016); without evidence of visualizations in the responses, it was not yet clear whether the MTEs had developed knowledge on how visualizations may support student thinking.

Across both tasks, it became clear that participants had a difficult time selecting a single answer after the whole group discussions. Many participants placed a strong emphasis on instructional context, providing qualifiers next to multiple choices. This indecisiveness and need for additional information were likely influenced by the background of the participants. As experienced teachers, they were acutely aware of the need to adapt materials to the specific class. They were also willing to engage in group discussions with colleagues and subsequently adapt or modify their choices - a necessary component of learning new ideas and building a community amongst their peers. These changes support the idea that participants were beginning to reshape their knowledge of teaching PTs, especially in the context of fractions and division, and were open to substantive change in their practice as future MTEs.

Questions for further discussion: (1) How have others transitioned from mathematicians to MTEs, specifically around elementary-school mathematics content? (2) How might lessons learned in this space transfer to teaching and learning in other mathematics courses?

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