Actively Assessing and Engaging With the Definition(s) of Active Learning

Sean P. Yee
University of South Carolina

Jungeun Park
University of Delaware

Joshua Brummer
University of Nebraska-Lincoln

Wendy M. Smith
University of Nebraska-Lincoln

Jennifer Crooks-Monastra
University of South Carolina

Michale Tepper
Pennsylvania State University, Abington

Kimberly Rogers
Bowling Green State University

Graduate student instructors (GSIs) in mathematics play a pivotal role in shaping undergraduate education and are the future of collegiate mathematics faculty. As part of their development, GSIs are expected to engage in teaching-focused professional development (TPD), particularly in evidence-based strategies like Active Learning (AL) methods. However, higher education is only beginning to explore how to effectively measure GSIs' growth in teaching skills through such TPD. This study examines the learning process of 47 novice GSIs from three universities, specifically focusing on their evolving understanding of AL before and after participating in TPD. By analyzing the GSIs' own definitions of AL, the research highlights changes in their knowledge and alignment with the intended TPD outcomes. The findings provide insight into the effectiveness of TPD on AL, while also offering recommendations for structuring future evaluations of TPD impact on GSI teaching knowledge and skills.

Keywords: Active Learning, Graduate Student Instructors, Teaching Assistants, Professional Development, Assessment

Mathematics graduate students are a substantial proportion of the undergraduate teaching workforce in the United States (Belnap & Allred, 2009; Speer et al. 2010) and their pedagogical knowledge and methods are greatly influenced during this time as novice college mathematics instructors (Deshler et al., 2015; Speer and Murphy, 2009). During their experiences as graduate student instructors (GSIs, graduate students who are instructors of record for a mathematics or statistics course), research has indicated that teaching-focused professional development (TPD) improves GSIs' knowledge and skills for effective teaching (Deshler et al., 2015; Yee & Rogers 2017; Hauk & Speer, 2023). Decades of work have also demonstrated that active learning (AL) approaches—using teaching strategies that actively engage all students in the classroom—are among the most effective (Freemen et al., 2014; Gobstein et al., 2016). AL has received significant discussion on its value within the classroom, with overwhelming recognition of its success *with appropriate TPD* (Reinholz, 2023). For example, Reinholz et al. (2022) reported on how and why supervision of collaborative learning is critical for AL strategies to be effective. Indeed, without addressing a classroom culture, student collaborations may include microaggressions that drastically limit the effectiveness of AL. Thus to prepare GSIs for implementing AL well, they need equitable and effective TPD (National Academy of Science, Engineering, and Medicine, 2023).

How can we know if TPD is effective? Specifically, how can we gather data and assess the impact among GSIs of AL-focused TPD opportunities? It requires data upon which to evaluate effectiveness, which, in turn, requires data collected around the objective or aim of the TPD (Yee et al., 2024; Deshler et al., 2015). In recent years, research in undergraduate mathematics education has begun exploring ways to evaluate the effectiveness of TPD (Yee et al, 2023; Yee et al, 2024).

Purpose and Rationale

The purpose of this study was to research how collecting and analyzing data on GSI learning in an AL-focused TPD activity might be part of evaluation of TPD effectiveness. Prior research has indicated that to improve practical understanding of AL, instructors need to (1) recognize exemplars through readings about AL methods (McConnell et al., 2017), (2) visualize and imagine themselves using varying AL methods (Yee, 2021), and (3) become aware of the nuances inherent in the enactment of AL (Laursen & Rasmussen, 2019). Our approach was to focus on how GSIs' *definitions of AL* changed during their TPD experience.

Unpacking GSIs' meaning making for educational definitions is important particularly for GSIs in mathematics where deductive and axiomatic structures put significant weight on definitions. For example, a series cannot both converge and diverge because the definition of convergence and divergence is binary, founded in the logic that a mathematical statement is either true or false, hence *dependent upon the definition* (Vinner, 2002). Thus a mathematical definition's utility is uniform. However, definitions in educational research rely on human participants and evidence, as seen in the diverse use of AL across different disciplines. How a chemistry lab uses AL is distinct from how a mathematics classroom uses AL because chemistry labs inherently engage students through lab experiments, resulting in facilitating labs being the focus for AL. Our rationale for this study emphasizes GSIs' definitions of AL as dynamic and not static. To this end, our research question for this study was: *How do novice GSIs' definitions of AL change after participating in an AL-focused professional learning activity?* For this study, novice refers to GSIs who are within their first two years of teaching.

Relevant Literature for Framing the Professional Development

Dimensions and Pillars of Active Learning

Four pillars for AL support instructors in developing a definition of AL: (1) Students engaging deeply with coherent and meaningful mathematical tasks, (2). Students collaboratively processing mathematical ideas, (3) Instructors inquiring into student thinking, (4) Instructors fostering equity in their design and facilitation choices (Gobstein et al., 2016; Laursen & Rasmussen, 2019; Smith et al., 2021; Yee et al., 2024). Recent work in developing TPD for operationalizing a definition of AL has come out of the SEMINAL project (and coding of over 100 mathematics instructor definitions of AL; Williams et al., 2020). In leveraging this previous work, we sought to attend to novice GSIs with their beginning understanding of AL. This led to another dimension of interest: interconnectedness. Together with the four pillars, the dimensions reflected on and considered in our AL-focused TPD and research were: (1) Role of students, (2) Role of mathematics, (3) Role of the teacher, (4) Role of equity, (5) Interconnectedness of the other roles. The fifth dimension was added because our research team agreed that how the four roles are connected is just as important as any single role. For example, the interconnectedness dimension focused on if novice GSIs could recognize that the role of equity affects the role of the teacher and students.

Critical Reflection by Assessing Definitions

The larger scope of this work strives to understand how GSI conceptions of AL for productive use in the classroom change over time. This study focuses on definitions for multiple reasons. We understand how educational definitions can be perceived differently from mathematicians' use of definitions (Tall & Vinner, 1981). Ultimately, learning to integrate definitions for teaching is a process. This study is not trying to argue or search for a "right" or "correct" definition of AL. Indeed, Williams et al.'s (2022) work shows the diverse ways in which experienced mathematicians view AL. The goal of this study was for GSIs to reflect on how their AL definition might change as they built experience as instructors. Consequently, the AL-focused TPD activity was designed to have GSIs critically reflect (Brookfield, 2017) on AL definitions by actively engaging in assessing their own and others' definitions. To that end, similar to other classroom practices used to help novice GSIs use AL (e.g. Observation Protocols, Yee et al., 2022), this study did not use assessment *of* AL understanding, but rather assessment *for* AL understanding.

Description of Module, Lessons, and Activities

The AL-focused TPD module included two 50-minute sessions with novice GSIs as (Lesson 1 and Lesson 2) as well as pre- and post-lessons assignment. Figure 1 describes the module.

<table>
<tr><td colspan="2">Overarching Goal of Module*: The goal of this unit is to have novice GSIs grow their definition, understanding, and implementation of AL.
Full description of activity is at (Yee et al. 2024)</td></tr>
<tr><td colspan="2">Pre-Module Assignment:
1. Read Active learning in the college classroom by Faust and Paulson, 1998.
2. In the context of Mathematics teaching and learning, please type your own definition of active learning in as much detail as possible.</td></tr>
<tr><td>Lesson 1 Objectives:
● GSIs will Identify what students and instructors can do with a selected set of active-learning strategies.
● Justify their decisions on what students and instructors can do with a selected set of active-learning strategies in their classroom.</td><td>Lesson 1 Activities (50 Minutes)*
● GSIs are given a list of 10 common GSI strategies
● GSIs are given a table with rows indicating ways students are engaging during the activity and columns about how the instructor is engaged during the activity. GSIs then work in small groups to list each AL strategy in what cells they find most relevant.
● GSIs negotiate with one another how they would enact the activity relative to what the instructor is doing and what the students are doing.
● GSIs recognize that direct instruction is just one of many teaching methods and quite limited within the scope of what students can and can't do.</td></tr>
<tr><td>Lesson 2 Objectives:
● Notice and articulate distinctions and dimensions of definitions of Active Learning
● Revise their initial definition of Active Learning.</td><td>Lesson 2 Activities (50 Minutes)
● GSIs review the rubric (see Figure 2) and practice evaluating three pre-generated definitions according to the rubric.
● GSIs then review their own definition
● Classroom discussion is had around any issues of confusion with the rubric and what they notice and wonder about their own pre-module AL definition</td></tr>
</table>

Figure 1. Active learning module structure and components

GSIs began by reading a collegiate AL article (Faust & Paulson, 1998) and creating their own initial definition of AL. Lesson 1 had them engage with 10 AL strategies by asking the GSI to envision (1) what the student is doing and (2) what the instructor is doing for each. Lesson 2 had the GSIs see a rubric for evaluating definitions of AL along the five dimensions using a scale from 1 to 5 (see Figure 2). GSIs then practiced and made sense of the rubric with three AL definition examples that were pre-generated to identify how the dimensions of AL are used. Finally, GSIs reviewed their own definition and discussed what GSIs noticed and how they might want to change their definition.

This report focuses on the Lesson 2 objectives. The activity was designed after multiple iterations of a smaller activity (Lesson Plan 1) so that results could be used to interpret and justify conclusions about GSI's change in definitions. By looking for changes in definitions (especially relating to equity), we collected data to determine if the TPD objectives had been met. Furthermore, through assessment and reflection, we could determine how to improve the TPD itself. Ultimately, the rubric is not just to rate definitions, but a means to understand the components of the definition (i.e. assessment *for* AL understanding).

Methods

To answer our research questions, this study used a sequential mixed methods approach where qualitative coding was followed by quantitative analysis. GSIs' original definitions (pre-module assignment) and the GSIs' revised definition were qualitatively coded using the rubric by the research team after the entire module TPD was completed (post-module assignment). Figure 2 offers an abridged rubric, while the full rubric can be found at https://bit.ly/RUME2025.

	(1) Missing Element (α)	(2) Basic Definition (α)	(3) Specific Example(s) (β)	(4) Sufficient Definition (γ)	(5) Rich and Nuanced Definition (γ)
Role of Students	No mention of students or what students are doing	Includes broad mentions that students are active or actively engaged	Can articulate what the student is doing during a specific example of active learning	Describes what students are doing inside and/or outside the classroom; definition may include active engagement with mathematical learning, or peer-to-peer interaction	Describes what students are doing inside and outside the classroom, including active engagement with mathematical learning, peer-to-peer interaction to communicate mathematical reasoning
Role of Mathematics	No mention of the mathematics	Mention of math class or doing math (in general)	Gives examples of specific mathematics content (e.g., chain rule)	Reflect on the mathematical content that was difficult to understand, that has become clearer conceptually than it was.	Defines how specialized mathematics knowledge for teaching is directly involved with the active learning strategy, including how mathematical reasoning is embedded to build student understanding
Role of Teachers	No mention of the teacher or what teachers are doing	Mention of the teacher in general ("the teacher plans active learning")	Gives specific examples of active learning strategies such as think-pair-share	Describes formative assessment(s) and how student feedback will be incorporated to inform teaching	Describes what the teacher is doing to orchestrate active learning, including planning considerations, instructor use of student thinking, and how the instructor engages students
Role of Equity	No mention of equity	Mention equity within the classroom around content, student, or teacher access; reference to "all students" such as active learning	Gives specific inclusive teaching strategies related to implementing active learning	Illustrate how the specific strategies will provide equitable access to the learning activity. May provide examples linking ideas to practice such as equity versus equality	Describes equity considerations and intentionality for using inclusive teaching practices, such as norm setting for engaging students in discussions; includes discussion of how active learning is not automatically equitable; may talk about other equitable or inclusive teaching strategies.
Interconnected-ness	No connections among the four different roles	Defines active learning by what it is not ("not lecture"), and does not connect across dimensions	At least one connection is stated between two of the roles (teacher, students, math, equity)	Definition touches on more than one connection across more than two of the roles (teacher, student, math, and equity)	Definition touches on roles of students, teachers, and mathematics, with attention to equity, interactions, and mathematical sense making and communication; acknowledgement that the roles are interconnected

Figure 2. Abridged rubric used to assess active learning definitions by graduate student instructors

The data was then quantitatively analyzed to determine what changes, and frequency of those changes, occurred among participants' definition components and qualities. α, β, and γ are categorical codes described further in the findings.

Participants and Data Collection

47 novice GSIs from three universities in the midwestern and southeastern United States participated in the GSI TPD. All novice GSIs had just begun as full instructor-of-record in introductory mathematics and statistics courses. All three providers of the TPD were researchers in this study. Participants submitted their initial and final definitions of AL via an online form, either during a TPD activity or as homework. Participants' prior understanding of AL from the providers was limited. Students had perhaps heard the language around students should be "active," but AL was never formalized beyond personal experiences with teaching.

Data Analysis

Three members of the research team completed the qualitative coding of all 47 GSIs' pre- and post-definitions using the rubric (Figure 2). They all coded 20 GSIs' pre- and post-definitions along all five rubric dimensions, then met to compare, discuss, and agree on initial codes. They discussed how to align the rubric, then discussed and revised their individual codes until the whole research team agreed on coding for the initial set of 20 GSIs. The remaining 27 GSIs were split evenly among this same group so that each researcher coded for 9 GSIs' pre- and post-definitions along all five rubric dimensions on their own. Inconsistencies were discussed until interrater reliability was 95%.

For example, one student wrote in their pre-definition, "My own description of it would include things such as collaborative problem solving, hands-on activities, flipped classroom, peer teaching, project based or discovery learning," which the research team agreed was a score of 1 in all categories, except Role of Students (score 5). The post definition for the same student was:

> Multiple participants are involved in AL inside a classroom: the instructor creates learning opportunities with equitable access points and then facilitates tasks so that everyone in the class is involved with the task, while simultaneously collecting feedback from the students to navigate and decide on the next learning actions and goals. All students have an opportunity to be actively involved in the learning process, while following the established classroom norm of promoting the construction of their own knowledge and ideas simultaneously respecting others' experiences and perspectives. Students engage in tasks to develop understanding and become an active participant/ a primary stakeholder in their learning and further investigate their understanding in and outside of the classroom by taking ownership of their own learning outcomes.

Researchers scored this student's post definition as still a 1 in Role of Mathematics and a 4 for Interconnectedness, but a 5 for the Role of Student, Instructor, and Equity.

Findings

Pre- and Post-Definition Descriptive Discussion

Upon completion of the initial coding, the research team decided to group some rubric score values into broader categories. The α category comprises scores of 1 (Missing Element) and 2 (Basic Definition), representing, at best, the inclusion of a rubric dimension in a GSI's definition with little specificity. The β category represents a score of 3 (Specific Examples(s)). And the γ category comprises scores of 4 (Sufficient Definition) and 5 (Rich and Nuanced Definition), representing the inclusion of a rubric dimension in a GSI's definition at a level of complexity beyond a basic statement or concrete example. These three broader categories were introduced to help identify trends via quantitative analysis, both in describing pre- and post-definitions individually and in understanding changes from pre- to post-definitions overall.

Table 1 provides a summary of how GSIs' pre- and post-definitions were coded and organized by rubric item category (ranging from less rich in the α category to more nuanced in the γ category). This provides evidence for the initial and end states of GSIs' definitions through the activity.

Table 1. Number of GSIs' pre- and post-definitions by category

	Pre-Definition (N=47)			Post-Definition (N=47)		
	α	β	γ	α	β	γ
Role of Students	25	9	13	6	11	30
Role of Mathematics	44	0	3	36	2	9
Role of Teachers	28	17	2	9	32	6
Role of Equity	46	1	0	38	5	4
Interconnectedness	20	20	7	0	12	35

α = rubric score of 1 or 2; β = rubric score of 3; γ = rubric score of 4 or 5

As seen in Table 1, the category with the largest number of GSIs' pre-definitions was the α category for every rubric dimension. Pre-definitions were especially highly concentrated in the α

category for both Role of Mathematics and Role of Equity, meaning that the Roles of Mathematics and Equity were either missing or included in a basic way for nearly all GSIs' pre-definitions. The other rubric dimensions (Role of Students, Role of Teachers, And Interconnectedness) were distributed more widely across all categories.

Regarding post-definitions, Table 1 shows that Role of Mathematics and Role of Equity remain relatively highly concentrated in the α category, but these are the only rubric dimensions for which the α category had the largest number of post-definitions. Researchers expected that GSIs' definitions would change after having worked with the rubric in Figure 2, but it is noteworthy that no post-definitions fell into the α category for Interconnectedness. This is likely because many of GSIs' pre-definitions completely lacked mention of certain rubric elements (such as the "Role of Mathematics" or the "Role of Equity") and therefore had fewer opportunities to make connections between roles. The rubric prompted GSIs to think about and include all roles (and naturally make more connections among the roles) in their definitions.

Changes in Definition

We observed a deepening in richness of GSIs' definitions of AL by observing their change in category from their pre-definition to their post-definition. For simplicity, a GSI's definition became *richer by category* if it was assigned rubric scores for pre- and post-definitions that represented a shift in category from α to β, from β to γ, or from α to γ. Among the 47 GSI participants, it was most common (17 of 47 participants, 36%) for a definition to become richer by category in three different rubric dimensions. Two GSIs saw their definitions become richer by category across all five rubric dimensions (15%), while seven GSIs saw no increased richness by category in any rubric dimension.

Researchers note that since GSIs' were given the rubric prior to constructing their post-definitions, one might expect to observe GSIs' definitions becoming somewhat uniformly richer by category across all rubric dimensions (after considering rubric dimensions where GSIs' pre-definitions were concentrated in richer categories, see Table 1). This was not the case at all. The "Interconnectedness" rubric dimension saw the most substantial enrichment of definitions, with 35 of 47 participants' definitions (74%) becoming richer by category, even though this rubric dimension started with the fewest pre-definitions in the α category. Conversely, the "Role of Mathematics" and "Role of Equity" rubric dimensions began with the most pre-definitions in the α category and yet saw the least amount of enrichment by category.

Role of Mathematics and Role of Equity: All but three GSIs' pre-definitions fell into the $\boldsymbol{\alpha}$ category for the "Role of Mathematics" rubric dimension, yet only nine definitions became richer by category in this dimension through the activity. Looking at the "Role of Equity" rubric dimension is even more extreme—all but one pre-definition fell into the $\boldsymbol{\alpha}$ category, yet only eight definitions became richer by category. To better understand the changes in richness for the Role of Mathematics and the Role of Equity, a more granular analysis of the participants' post-definitions was done with participants' whose pre-definition was an $\boldsymbol{\alpha}$ category. Table 2 details the exact change in rubric score (not just category) for all pre-definitions within the $\boldsymbol{\alpha}$ category.

Table 2 illustrates 8 out of 47 post-definitions (17%) received a score of 1 to represent the Role of Mathematics as being a "Missing Element." However, 23 of 35 of the GSIs' definitions (65%) that started at a pre-definition score of 1 ("Missing Element") grew to a post-definition score of 2 ("Basic Definition"). These GSIs' definitions became richer within the α category. Ultimately, pre-definitions that began in the α category lacked concrete examples of how mathematics takes a role in the AL process (β category) or how mathematical content and reasoning shapes or is shaped by the AL process more broadly (γ category).

Table 2. Number of GSIs pre- to post-definition rubric score shifts in α category

	Pre-Definition Rubric Score (α category)	Post-Definition Rubric Score 1	2	3	4	5
Role of Mathematics	**1 (N=35)**	6	23	1	5	0
	2 (N=9)	2	4	1	2	0
Role of Equity	**1 (N=41)**	20	15	2	2	2
	2 (N=5)	1	2	2	0	0

Table 2 also states 21 out of 47 post-definitions (45%) received a score of 1 to represent the Role of Equity as being a "Missing Element." 15 of the 41 (37%) post-definitions began with a pre-definition of score of 1 shifted richness towards a "Basic Definition" (score 2) for the Role of Equity, yet 20 of the 41 (49%) had a pre- and post-definition score of 1 within the Role of Equity. Even more so than for the Role of Mathematics, GSIs' definitions infrequently incorporated specific examples or sufficiently rich/nuanced conceptualizations of the Role of Equity in AL.

Discussion & Implications

Despite limited research on GSI learning (e.g., Deshler et al. 2015; Yee et al., 2023) this study represents a transferable model for assessing the impacts of GSI TPD. Like Williams et al. (2022), we found that AL definitions rarely included any mention of equity or mathematics, much less nuanced and rich treatments of these complex topics. In answering our research question, this AL-focused TPD, overall, did lead to more nuanced and richer GSI AL definitions. Although one might expect that exposure to the rubric for AL definitions would result in uniform improvements to GSIs' definitions of AL, we did not see that. Instead, we saw that most GSIs improved in their definition with the Role of the Teacher, Role of the Student, and Interconnectedness. Although the considerations of mathematics and equity did not show overall categorical growth beyond the α category, we did see more mentions of mathematics and/or equity. The surface attention to equity was not surprising; it typically takes far more equity-focused TPD for GSIs to develop nuanced conceptualizations of equity (Reinholz, 2023; Reinholz et al., 2022). Thus, one implication for GSI TPD is that GSIs need longitudinal TPD experiences focused on developing their conceptions of equitable mathematics teaching and learning.

An additional implication for TPD providers is that the model we used for assessing the impacts of the AL TPD on GSIs' definitions is transferrable to other pedagogical topics. Thus, this study answers the call of many for more research on what makes for effective TPD (e.g., Deshler et al., 2015; Hauk & Speer, 2023; Yee & Rogers, 2017; Yee et al., 2023; Yee et al., 2024). One limitation of our study is that we just focused on GSIs' definitions of AL; future studies should look for connections between growth in GSIs' AL definitions and enactment practices of AL. We recommend some longitudinal future studies to look at more lasting impacts, and to assess if sustained TPD can further improve not only GSIs' AL definitions, but also their AL practices.

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