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# TEACHERS' COLLABORATIVE REFLECTIONS AND PLANNING ON USING DIGITAL AND NON-DIGITAL RESOURCES

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*This study investigates how two middle school teachers of mathematics reflected on and planned for the use of a digital collaborative platform embedded with a problem-based curriculum. As digital resources can help teachers enact mathematics problems that are responsive to the needs of their students, more empirical work is needed to understand and inform relevant teaching practices that leverage evidence of student thinking (Pepin et al., 2017). Drawing on a documentational approach to didactics (Gueudet & Trouche, 2009), we examine the influences of collaborative reflections on teachers' decisions about the use of digital resources. Our preliminary findings show that based on their reflective conversation, teachers considered the affordances and constraints of both digital and non-digital resources in their planning. Our findings suggest collaborative reflections help teachers critically examine digital resources.*

Keywords: Technology, Instructional Activities and Practices, Curriculum, Problem-Based Learning

## Rationale and Purpose

A critical teaching practice in student-centered, inquiry-oriented classrooms is for teachers to elicit evidence of student thinking and connect with prior understanding (Jacobs et al., 2010; Kazemi & Franke, 2004; NCTM, 2014). Given the power and potential of digital technologies, a growing number of studies are examining the important relationship between teachers and their use of digital resources (Gueudet et al., 2012; Remillard et al., 2009; Geiger et al., 2023). Yet, more empirical research is needed to understand the factors influencing teachers' decisions to integrate technology into mathematics classrooms (McCulloch et al., 2018). Further, the potentialities of using digital resources are amplified when students and teachers use digital curriculum materials or digital platform systems that are designed and developed around student thinking in problem-based classrooms (Edson & Phillips, 2021). This study examines how teachers reflect on their individual use of digital or non-digital resources on the same problems.

## Theoretical Perspectives

The notion of the teacher has shifted from teachers as curriculum implementers to teachers as enactors, and more recently, to teachers as instructional designers who make decisions on how to use curriculum materials (e.g., Jones & Pepin, 2016; Remillard et al., 2009). This study takes the perspective of teachers as instructional designers who interact with resources to achieve their instructional goals (Brown, 2009). As the scope of curriculum resources expands to include digital materials such as digital textbooks or interactive online platforms (Pepin et al., 2017; Remillard, 2016), how teachers appropriate and transform resources plays a critical role in their teaching work (Adler, 2000).

In our study, we draw on the documentational approach to didactics (Gueudet & Trouche, 2009), which centers the selection, planning, and enactment of resources (i.e., digital, non-digital) “at the core of teachers’ professional activity and professional development” (p. 199).

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The documentational approach to didactics emphasizes the dual nature of teachers' interactions with resources. When teachers shape a given artifact to achieve goals (*instrumentalization*), the affordances and constraints of the artifact influence teachers' usage of the artifact (*instrumentation*). As teachers develop schemes of how to use resources in their classroom context, their decisions are influenced by both explicit learning goals and implicit beliefs and knowledge from their teaching experiences (*operational invariants*). Over time, teachers develop documents and resource systems that entail both a set of resources and the utilization schemes of how to enact the resources (Gueudet, 2019; Ruthven, 2019).

We also draw on the perspective that discussions with other teachers are part of resources, highlighting the importance of collective documentation work among teaching colleagues (e.g., Gueudet, 2019; Gueudet & Trouche, 2009, 2012; Gueudet et al., 2016). Drawing from the perspective of understanding individual teacher learning within the context of collective practices (e.g., Lave & Wenger, 1991; Wenger, 1998), the documentational approach to didactics provides insights into the influence of collaborative reflections and planning on individual teachers' development of resource system. Given the limited evidence of how teachers navigate the affordances and constraints of resources and how teachers orchestrate them (Pepin et al., 2017; Rezat et al., 2021), this study seeks to understand how teachers extend their perceptions of digital curriculum resources and incorporate them into their teaching practices.

## Methods

This study is guided by the following question: How do individual teachers select, adapt, enact, and reflect on the use of resources in classrooms where there is readily access to both digital and non-digital resources? We situate the study within a larger design-based research project that focuses on developing a digital collaborative platform embedded with a problem-based middle school mathematics curriculum (Edson & Phillips, 2021). This platform contains various tools for digital inscriptions, such as texts, graphs, tables, drawings, and images. Also, students and teachers can access and co-opt others' digital work in real-time.

Our study focuses on two teachers, Ms. Evans and Ms. Foster, who teach seventh-grade mathematics at the same school in the Midwestern suburban area. Both teachers have more than ten years of teaching experience with a problem-based curriculum, *Connected Mathematics* (CMP; The Connected Mathematics Project, Phillips et al., in press). They have been using the developed digital platform for several years. During their daily planning meetings, they used to select problems together to utilize the digital platform. In this study, they were asked to enact the curriculum in two different learning environments—digital or paper-and-pencil. This paper focuses on two problems from one geometry unit. For Problem 1.3, Ms. Evans made use of the digital platform, and Ms. Foster did the print curriculum. For Problem 3.1, the type of resources is reversed. Ms. Foster used the digital resources, and Ms. Evans did the non-digital resources.

Teachers' collective planning meetings and their individual reflection interviews after each problem were video recorded and transcribed. Secondary data sources included classroom video recordings, researchers' field notes during classroom observations, teachers' weekly reflection survey responses, and classroom documents generated by the teachers and their students (digital and non-digital). The data analysis was guided by the reflective investigation to study teachers' documentation work (Gueudet et al., 2012). First, we identified how teachers perceived the affordances and constraints of digital and non-digital resources and how they would incorporate those resources into their teaching (*documentation work*). Then, we coded what influenced the

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teachers to develop their utilization schemes of resources (*operational invariants*). After applying descriptive codes to each teacher's data, we looked for patterns, similarities, and differences in the codes between the two teachers' cases.

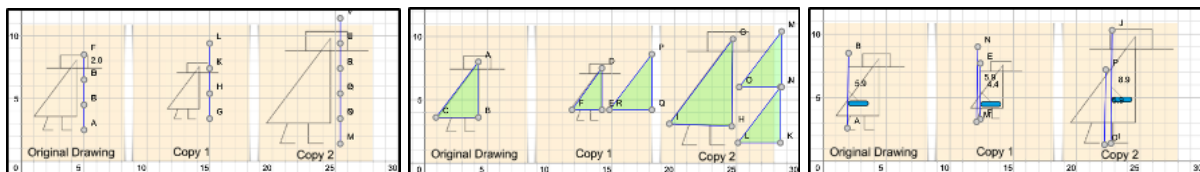
### Preliminary Findings

Our analysis revealed that after their conversations about teaching the same problem, (a) teachers considered the benefits and limitations of using both digital and non-digital resources in association with students' development of mathematical ideas, and (b) teachers changed their planning on future problems in order to integrate digital affordances.

### Seeing Both Sides: Drawing From Each Other's Reflections

When asked to select between digital and non-digital resources, Ms. Foster indicated it would make no difference in student learning experience. It was because she believed that "seventh graders are visual learners" and hands-on activities help visualization. For Problem 1.3, she had her students use patty papers to trace shapes and compare them with other shapes to explore the similarities. After having a reflective conversation with Ms. Evans, however, she recognized the differences in student engagement and understanding in different learning environments. While she appreciated how the hands-on manipulatives' color format helped her students see the corresponding sides, Ms. Foster was "rethinking" the limitations of patty papers compared to the affordances of digital manipulatives. In other words, while maintaining her belief in visualization (*operational invariant*), she could develop a scheme of using digital resources for the same problem. Her rethinking was influenced by Ms. Evan's reflection. Ms. Evans shared how her students explored various approaches using digital tools (Figure 1), whereas most of Ms. Foster's students took the same approach by which they measured the lengths of hypotenuses. Ms. Foster claimed that using patty papers was more challenging to connect with mathematical ideas during the whole-class discussions compared to using digital manipulatives:

The connections that I heard, Ms. Evans talked about today, was when they could actually go into the platform, make copies of the original, and play with the copies in the new image to where they could actually formulate a conclusion about the corresponding sides, corresponding angles, and even the area. [It is] because my kids didn't, we didn't even talk about the area, whereas Ms. Evans said some of her kids actually picked up on the area. [...] Now that I'm talking about this and reflecting, I think, timewise, the digital platform allows us a shorter amount of time but a deeper sense of questioning and understanding.



**Figure 1: Students' Different Strategies Used on the Digital Platform**

Although teachers used one format for each problem, either digital or non-digital resources, they drew on each other's different experiences and considered both resource types to better help students "actually see" embedded mathematical ideas and their conversations "go deeper." After teaching Problem 3.1 on the digital platform, Ms. Foster imagined the possible limitations of

using patty paper. She continued to say, “It’s an aha moment for me because I used to be the queen of paper-and-pencil” who always used patty paper for these two problems.

### **Looking Ahead: Adapting Plans for Future Problems**

Teachers adapted their plans for the same problem to integrate digital affordances. This means that teachers revised their documents as they perceived that digital manipulatives could help students focus on exploring mathematical ideas (*operational invariant*). For example, after teaching Problem 3.1 using patty paper, Ms. Evans claimed it took longer for students to work on the problem because her class did not have sufficient time to facilitate as deep conversations about scale factors as Ms. Foster’s class. Ms. Evans further pointed out students’ tracing was not accurate enough to examine scale factors, because she learned that from the digital representations, Ms. Foster’s students saw underlying mathematics “right away.” After this conversation, Ms. Evans planned to discard patty papers during her next class, and instead, she provided each group with a set of plastic shapes. She hoped students would struggle less with drawing and focus more on the relationships with scale factors. At the end of the day, however, Ms. Evans said that using plastic shapes was no better than patty paper for seeing the patterns. She noticed that most students stopped drawing after making a shape twice as big. With digital tools, she imagined students would easily have duplicated shapes to stack them into larger shapes more than twice. Thus, Ms. Evans wanted to revisit the same problem using the digital platform:

I’ll just make some different shapes [...] and then have them try and rep-tile it [because] some of them are still not sure about the scale factor. [In the platform,] they’re still looking at how many shapes versus side lengths, so I think that will help with connecting that a little bit better for them. [...] Getting to do it again in a slightly different format will help solidify it.

For both problems, teachers compared the benefits and limitations of digital and non-digital resources and concluded that the digital platform would be more beneficial for student learning. Interestingly, they mentioned that if they had to teach these problems in a paper-and-pencil environment, they would prepare multiple pre-cut shapes so students could skip the drawing step and focus on manipulating shapes to explore mathematical ideas. That is, teachers changed the scheme of using hands-on manipulatives to remove unnecessary distractions and help focus on deepening mathematical understanding, inspired by the digital affordances they observed.

### **Discussion and Conclusions**

This study provides empirical evidence of how teachers shifted from using non-digital, hands-on manipulatives toward using digital resources. This shift was based on collaborative reflections about student understandings of the key mathematical ideas. The teachers developed operational invariants that digital resources help students focus on visualizing their thinking more efficiently than hands-on manipulatives. Such belief will influence the ways teachers consider digital affordances and represent their resource systems (Ruthven, 2019).

Findings from the study pose important implications as teachers select, adapt, and use digital and non-digital resources in their mathematics classrooms. First, we provide evidence that teachers can utilize co-planning meetings to share the different affordances of resources for the same problems that inform how to use technology in supporting student learning (McCulloch et al., 2018). As our preliminary findings focused on the two teachers’ cases, further research should investigate the impact of collaborative reflections compared to individual reflections. Second, we provide evidence that digital features in the platform prompted teachers to re-

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examine the benefits and challenges of using digital resources (Geiger et al., 2023). As our preliminary findings were specific to geometry problems involving patty paper, further research is needed, e.g., How do teachers develop resource systems regarding their use of digital/non-digital resources in problem-based classrooms?

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