Identifying and Categorizing Instructor Considerations for Active Learning Implementation

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In this paper, we discuss our experience collaborating with mathematicians to increase their use of active learning pedagogy in a proof-based linear algebra course. We use this experience to attend to three primary research objectives. First, we identified three primary categories of instructor considerations that would determine whether or not they would incorporate a proposed strategy. Second, we observed and made sense of which of these were most prominent for these mathematicians. Third, we determined what combination of considerations needed to be satisfied to warrant the implementation of a strategy by these mathematicians.

Keywords: Active learning; changing pedagogical practice; linear algebra; university mathematics instruction

When Michelle Artigue (2016) reflected on her career as a researcher in undergraduate mathematics education, she lamented that one of the field's biggest weaknesses was its lack of impact on how undergraduate mathematics courses were taught. Specifically, Artigue was alarmed by the "insufficient dissemination of research results towards the relevant communities or practitioners, and the very limited influence of our research on university teaching practices." (p. 12). Indeed, it is still the case that most university mathematics courses are taught predominantly by lecture (Fukawa-Connelly et al., 2017; Johnson et al., 2018; Johnson, 2019), even though effective, research-based, alternatives are available and have shown positive effects on student learning outcome (see Freeman et al., 2014).

To redress the lack of impact of undergraduate mathematics education research, Artigue (2016) recommended that mathematics educators form "collaborative projects, building and negotiating, jointly with mathematicians and other university teachers, problématiques that make sense for all those involved, and meet their respective interests and needs" (p. 12). In this study, we follow Artigue's suggestion of utilizing collaborative teams between mathematics education researchers and mathematicians to offer insight into instructional change. We describe a collaborative project between mathematics education researchers and mathematicians in which we together attempted to integrate active learning pedagogy into mathematicians' lectures in a proof-based linear algebra course.

In conducting this study, we found that some active-learning activities that seem easy to integrate into a lecture from the perspective of mathematics education researchers were rejected as infeasible from the perspective of the mathematician. Our research aim here was to better understand features of active learning strategies that contribute to their feasibility. In order to do so, we needed to identify considerations, such as obligations and goals, held by the mathematicians we were working with. We then considered the features of the instructional strategies in relation to these considerations to determine what combination of criteria needed to be met for a suggested instructional strategy to be feasible.

Literature Review

Fukawa-Connelly et al. (2016) found that among lecturers "56 percent agreed (and 26 percent more slightly agreed) with the statement I think students learn better when they do mathematical work (in addition to taking notes and attending to the lecture) in class" (p. 279). Yet when mathematics educators have observed class meetings in university mathematics courses, active learning strategies were rarely used (e.g., Fukawa-Connelly et al., 2017). It may be the case that some mathematicians feel that the questions that they ask during class time are sufficient to engage students and have them actively think about the material, or it may be the case that some mathematicians believe the limited active learning that teacher questioning provides is all these teachers can offer if they have the obligation to cover the course material (Woods & Weber, 2020). Alternatively, some mathematicians might be unaware of plausible alternatives to lecturing and still others might feel they lack the time to learn a new way of teaching due to other professional obligations (Johnson et al., 2018). Or perhaps, mathematics educators do not fully understand what it means, to mathematicians, for students to 'do mathematical work' in class. Perhaps this is partially due to "active learning" itself not being well-defined.

Braun et al. (2017) observed that, although recommendations to introduce active learning into university mathematics classrooms are commonplace, there is not a shared understanding for what active learning pedagogy is. In this paper, we follow the Conference Board of the Mathematical Sciences in defining *active learning pedagogy* broadly, where active learning pedagogy refers "to classroom practices that engage students in activities, such as reading, writing, discussion, or problem solving that promote higher-order thinking" (as cited in Braun et al., 2017, p. 124). In other words, active learning in which students are engaged in some activity involving mathematical content that does not involve passively listening to a lecture or practicing procedures. One approach is to use short active learning activities, such as think-pair-share questions or clicker questions, which can augment lectures. While these activities have been used in primary and secondary mathematics classrooms, as well as university physics classrooms (e.g., Crouch & Mazur, 2021), they have been the subject of limited research in undergraduate proof-based mathematics courses (Melhuish et al., 2022, but see Alcock, 2018, and Iannone & Miller, 2019, for exceptions).

Herbst and Chazan (2003) observed that when mathematics teachers do not adapt reform-oriented recommendations for changing instruction, some mathematics educators attribute this to the teachers suffering from a "lack of knowledge or a paucity of vision" (p. 3). Herbst and Chazan argue that this stance is counterproductive. Teachers are not free to teach in any manner that they choose. Rather, they chose to honor obligations to their discipline, to their students, and to their institutions (Chazan et al., 2016). Many instructional decisions, especially in choosing whether to use, or how to adapt, reform-oriented instruction, involve managing conflicting obligations. Here, in understanding our mathematicians' reactions to suggested instructional strategies, we are looking to understand their obligations and their goals for instruction.

Context and Methods

This study took place at a large northeastern university in the United States and involved a collaboration between mathematics educators and mathematicians, where both groups worked together in six weekly meetings to incorporate active learning strategies into a proof-based linear algebra course. The course presented an axiomatic treatment of abstract vector spaces and linear transformation on those spaces and covered topics such as eigenvectors and eigenvalues, the

Jordan canonical form and its associated theorems, and inner product spaces. The research team included the three final authors and four mathematicians agreed to participate in this study. All meetings were recorded over Zoom and transcribed. Only three of the four instructors could fully attend meetings regularly, so only three were included in this analysis.

During each meeting, the mathematics education research team proposed an active learning pedagogical strategy that mathematicians were asked to incorporate into their lessons. After a strategy was agreed upon, its implementation was discussed. The implementation involved choosing specific questions (e.g., if an exit ticket was to be used, what would the specific prompt be?) and the practical issues of how the strategy would be employed in the classroom context (e.g., if an exit ticket was to be used, how would the prompt be given to students and what student artifacts would the instructor collect?) In general, the mathematics education research team would provide mathematicians with an array of questions which mathematicians could then choose to use or adapt. In following meetings, the groups would debrief on how the active learning strategy went, what benefits and shortcomings this might have had from the mathematicians' perspectives, and what changes might be needed for this to be viable in the future.

In analyzing transcripts from the six collaborative planning meetings, we used the method of constant comparison through the grounded theory approach (Strauss and Corbin, 1994) to form a preliminary codebook. First, we began by looking at the data through the lens of Obligations, Instructor Goals, and Features for Implementation. Then, we flagged instructor comments which aligned with these larger themes, resulting in a codebook including these three categories as parent codes. Subcodes were identified by looking for commonalities and recurring themes within each parent code.

The Obligations category included *Time*, *Institutional*, and *Student Preference* as subcodes. The Instructor Goals category included *Building Math Understanding* and *Attending to Social/Emotional Needs of the Student* as subcodes. The final category, Features for Implementation, included *Ease of Use*, *Features of Question Posed*, *Mathematical Features of Question*, and *Student Cognitive Resources and Perspective* as subcodes. Then, we checked for co-coding to observe potential relationships between (1) obligations and features and (2) features and goals for these three linear algebra instructors. Below are three tables defining the subcodes from each category with examples and occurrence counts between all three instructors.

Table 1. A table of Obligations subcode definitions with examples and counts of their emergence.

Obligations			
Subcode	Definition	Example	Count
Time	Available time for class meetings and to cover topics.	"My biggest adversary in the classroom when I'm teaching is the clock on the wall."	74
Institutional	University-set expectations for the course	"The material should really emphasize vector spaces which are not $\mathbb{R}^n$	50

Table 1. A table of Obligations subcode definitions with examples and counts of their emergence.

	and students' prior knowledge.	because otherwise it's 250 material."	
Student Preference	What students are likely to do or how they will react.	"If you ask them to watch a video, I suspect a lot of them will not watch. That is the problem."	37

Table 2. A table of Instructor Goals subcode definitions with examples and counts of their emergence.

Instructor Goals			
<u>Subcode</u>	<u>Definition</u>	<u>Example</u>	<u>Count</u>
Building Math Understanding	Developing students' mathematical rigor and intuition.	"There are students who understand the mechanics and the concepts, but what they have difficulty with is the flow of the logic."	84
Social/Emotional Needs of the Student	Engaging with students, attending to what they want to learn, and potential social influences.	"One thing that I'm trying to get out of active learning is a chance to talk to the students one on one… they'll ask questions they're embarrassed to ask."	44

Table 3. A table of Features for Implementation subcode definitions with examples and counts of their emergence.

Features for Implementation			
<u>Subcode</u>	<u>Definition</u>	<u>Example</u>	<u>Count</u>
Ease of Use	Planning required to implement.	"This is something I should have in my pocket. I need to know what I'm giving at the end."	74

Table 3. *A table of Features for Implementation subcode definitions with examples and counts of their emergence.*

Features of Question Posed	Easy to comprehend or short questions.	"I wanted to make these so that students don't need a lot of time and can just get to what they want to decide."	38
Mathematical Features of Question	Questions emphasizing particular math concepts.	"What's wrong if you just define the distance as the inner product of a vector within itself?"	85
Student Cognitive Resources and Perspective	Instructor beliefs about students' abilities.	"If you want them to create something, that's impossible for the beginning. So I have to give them an example."	37

Results and Discussion

Using Atlas.ti software, a table of code occurrences was developed to identify the prominent Obligations, Instructor Goals, and Features for Implementation influencing the use of suggested strategies by the three linear algebra instructors. The predominant Obligation from the collective analysis was *Time* which was referenced 74 times between all three instructors. The predominant Instructor Goal was *Building Mathematical Understanding* which was discussed 84 times. The predominant Feature for Implementation was *Mathematical Features of Question* which was mentioned 85 times. Although these aspects were identified to be most significant in the collective analysis, other key factors surfaced when investigating the respective profiles of the three instructors.

Time was the most referenced Obligation and *Building Math Understanding* was the most prominent Instructor Goal for all three instructors respectively; however, variation emerged between instructors regarding the Features for Implementation category. The primary Feature for Implementation for both Instructor C and Instructor Y was *Mathematical Features of Question*. Of all three categories, Instructor C spoke most often about concerns regarding the *Mathematical Features of Question* which was followed by *Time*. Instructor Y also spoke most often about the *Mathematical Features of Question* and *Time*. However, Instructor S primarily discussed *Ease of Use* and *Building Math Understanding*. *Mathematical Features of Question* was one of the least mentioned considerations for Instructor S.

In addition to how frequently certain topics were mentioned, the instructors expressed their openness to more engaging practices such as student discussions but would conclude that there was simply not enough time allotted. This underscores the need to align with institutionally imposed obligations and its precedence over instructor interest. Similarly, instructors' need to attend to specific mathematical features stems from this dominant institutional priority to cover required content and distinguish the course from its calculation-based predecessor. Instructors at times chose to modify questions to attend to these considerations and reinforce their goal of building students' intuition and rigor. With these findings, the first two authors constructed Figure 1 to showcase the collective impact of the most commonly discussed institutional requirements and student-related considerations on the implementation of proposed strategies.

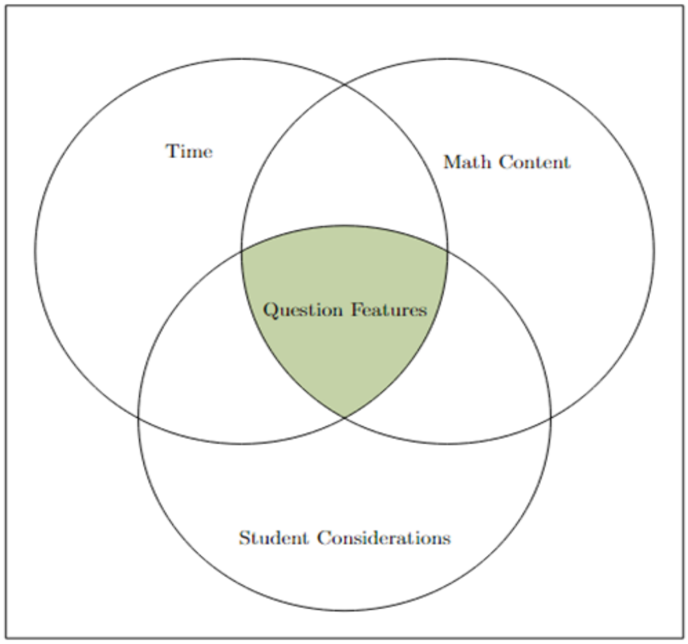

Figure 1. A three-circle Venn diagram illustrating the prominent considerations for implementing a proposed strategy.

Considering its prevalence from the collective analysis, Time reserves its own circle of the diagram. Time impacts some previously mentioned considerations for implementation, such as needing to satisfy university-set course expectations and the planning required to do so while incorporating a new strategy. Math Content was assigned its own circle given the emphasis placed on Building Math Understanding and Mathematical Features of Question. Finally, Student Considerations encompass the collective influence of Student Preference, Social/Emotional Needs of the Student, and Student Cognitive Resources and Perspective. Although these may not have been as commonly mentioned compared to the other two circles, each instructor did attend to how their students may respond or did respond to their efforts to implement a strategy or expressed beliefs concerning their students' mathematical or cognitive abilities.

This Venn diagram also provides the criteria these instructors deemed important to satisfy when considering implementing a proposed strategy. More specifically, the nature of the questions being used for a proposed strategy sits at the intersection of all three circles. Short, easy to comprehend, and mathematically precise questions seemed to be perceived as the most feasible option for instructors to adapt and incorporate into their lessons and planning. The instructors expressed that shorter and easy to comprehend clicker questions would meet their need for questions that could be answered quickly to alleviate timing concerns and reduce anticipated difficulty unrelated to the mathematical content. Mathematical precision was also emphasized, especially by Instructor Y and Instructor C, to mitigate potential issues regarding term definitions and emphasize the significance of aspects such as vector spaces.

To distinguish this course from the previous, conceptual questions seemed to be of greater interest than calculation-based questions, which may also be due to interest in quicker questions. Additionally, such questions which also pinpoint whether a common misconception has been addressed or not was observed to be an important feature. A final component which resonated with all instructors was planning certain questions to be given after students had time to reflect on what they had learned outside of class time. The instructors expressed apprehension toward providing Exit Tickets on concepts students had just learned based on their beliefs that students would not fully understand the concepts that soon. Instructor S, for instance, instead chose to incorporate Do Now activities in the following class to check students' understanding of previous concepts before introducing new material.

Conclusion

Due to our small sample, we cannot claim that our results generalize to other mathematicians or to a context beyond a proof-based linear algebra course. In particular, we do not have grounds to claim that the obligations or goals that we outlined would be shared by other mathematicians or that others would agree with the features that made instructional strategies more feasible would be valued by others. However, we are able to make some observations about what appeared to matter most to these mathematicians as they considered implementing different active learning strategies.

In essence, the interaction between time spent covering the mathematical content and addressing student understanding is important as a constraint for the group of instructors we assessed. It was important for researchers to find the most time-efficient way to determine student understanding of the mathematics involved if they are hoping for their strategies to be implemented. This suggests that research requires the input of instructors who are aware of the most common misconceptions in the material to bridge the gap between research and what happens in the classroom. These findings also suggest that instructors may have found Do Now activities more time-efficient than Exit Tickets based on their intuition regarding students' window of retention.

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