COMET: COLLABORATING WITH MATHEMATICIANS TO ENHANCE TEACHING

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In this paper, we describe how mathematics education researchers and mathematicians collaborated to introduce active learning pedagogy into a proof-based linear algebra course. This description highlighted the goals, values, and obligations that mathematicians had for their pedagogical practice, and how challenging it can be to introduce active learning pedagogy into mathematics classrooms that is compatible with these. We also illustrate how mathematics educators' understanding of mathematicians' perspectives allowed mathematics educators to help create instructional techniques that mathematicians are willing to use in their practice.

Keywords: Professional Development, Curriculum, Instructional Activities and Practices, Reasoning and Proof

Introduction

The purpose of this paper is to discuss a recent collaboration project between mathematics education researchers and mathematicians to improve instruction in undergraduate mathematics. In this collaboration, mathematics education researchers and mathematicians worked together to introduce active learning into a proof-based linear algebra course in a manner consistent with the goals and values of the mathematicians who were teaching this course. We illustrate how this collaboration proceeded by describing the challenges and resolution of designing short questions that can be used during lectures that encourage student activity and elicit student thinking. As we describe what transpired, we will discuss mathematicians' values and goals and the importance of attending to them.

Literature review

Most university mathematics courses are taught by lecture (Artameva & Fox, 2011; Melhuish et al., 2022). There is a general consensus amongst researchers in undergraduate mathematics education that this situation is not ideal. Lecturing is largely viewed as an ineffective pedagogy; students who emerge from lecture-based classes in advanced mathematics typically have a poor understanding of central concepts and an inability to write proofs (e.g., Ko & Knuth, 2009; Rasmussen & Wawro, 2017). There is also evidence that students' understanding, performance, and affect improve when active-learning strategies are used (e.g., Freeman et al., 2014; Laursen et al., 2014; Rasmussen & Wawro, 2017). It follows that a key way to improve instruction in undergraduate mathematics is to introduce active learning pedagogy. However, lecture remains the dominate form of instruction across undergraduate mathematics (Johnson, 2019).

This leads to a natural question: Why aren't mathematicians using more student-centered forms of instruction when there is evidence that this pedagogy leads to better learning outcomes than lecturing? We believe that a major reason is because alternative instructional methods frequently do not address mathematicians' most pressing concerns. A primary goal of advanced mathematics courses is to enable students to successfully prove theorems (e.g., Alcock, 2010) and, as Stylianides and Stylianides (2017) observed, there are few research-based interventions that have been shown to improve students' abilities to write proofs that meet mathematicians' standards in undergraduate mathematics education research. Another often cited, but largely unaddressed concern is content coverage.

Braun et al. (2018) argued that the instructional innovations designed and promoted by mathematics educators have designed is not the only way to make university classrooms more active and improve teaching and instead emphasized that there are smaller steps that mathematicians can use to increase student activity in a lecture. For instance, think-pair-share questions and whole class discussions can increase student activity while still enabling lecturing. However, active-learning pedagogy that is compatible with lecturing has not been the subject of much mathematics education research. In this paper, we describe how we worked collaboratively with mathematicians to introduce exactly these types of active learning strategies with their lectures. In doing so, we respond to Artigue's call for projects that are "collaborative projects, building and negotiating, jointly with mathematicians and other university teachers, problématiques that make sense for all those involved, and meet their respective interests and needs" (2016, p. 12).

Theoretical perspective

We use three constructs to categorize teaching. A teacher's *values* correspond to the broad goals they want to achieve in their classroom. Values might include things such as enabling students to prove interesting theorems, preparing students to enter graduate school, of having students regularly engage in authentic mathematical debate. *Strategies* are common adaptable pedagogical techniques that teachers use to achieve their goals. These include things like modeling mathematical reasoning during lectures, asking open-ended questions with adequate wait time, or having students solve problems collaboratively and present their work. *Implementations* are the specific embodiment of a strategy. For instance, the implementation of the strategy of "use a think-pair-share questions" would involve the specific question that was chosen as well as how the question was introduced to the class. We argue that active-learning strategies are most typically strategies while the specific mathematical questions are implementations or tactics.

Broadly speaking, we believe that the lack of uptake of undergraduate mathematics education research is that these are based on strategies that do not align with many mathematicians' values and strategies that are difficult to implement. However, we believe that there are strategies that encourage active learning that are compatible with mathematicians' values. For instance, using think-pair-share questions can be done in a lecture format, or in more inquiry-based classrooms. We drew on Brownlee et al.'s (2017) conceptual description of teacher beliefs and practices as reflexively coevolving. They claimed that as teachers engage in new practices that their beliefs will change, and, as their beliefs change, their valuation of- and engage-in practices will also change.

Methods, Data, and Analysis

During the Fall 2023 semester we formed a collaborative group of mathematics educators and four mathematicians who were teaching different sections of the same proof-based linear algebra course. The research team's progressive goal was to support mathematicians in using more student-centered instructional techniques over time. To do so, we conducted weekly meetings (starting in October) with four mathematicians teaching the course and the five members of the research team. Our study followed a design research framework (Cobb et al., 2003) in which we are simultaneously trying to develop an effective collaborative framework for improving mathematicians' teaching as well as a theory for why our framework is effective and how mathematicians' beliefs and practices evolved.

After the semester, we engaged in retroactive analysis. In instances where mathematicians judged an active learning strategy to be infeasible (either dismissing it before using it, or deciding it did not work for them after using it), they were required to give a rationale for their decision. Using a thematic analysis (Braun & Clark, 2017), we analyzed the mathematicians' rationales to identify what obligations (in the sense of Chazan et al., 2016) that mathematicians had to their institution, discipline, and students that made the active learning strategy infeasible. We then developed a narrative of how our active learnings strategies were ultimately adapted by the mathematicians as feasible given their perceived obligations.

Results and Discussion

In our first meeting with the mathematicians, we initially invited them to use Exit Tickets. The aim here was showing the limited understanding that students had of lectures, as well as helping the mathematicians see the value of attending to student thinking. Our suggested was rejected on the grounds that the mathematicians already knew that the most students did not understand their lectures all that well. As Mathematician D put matters, "I think most of them would say they don't understand yet". To the mathematicians, understanding only came after students had the opportunity to reflect upon the lectures. Our next suggestion was for the mathematicians to give the Exit Tickets for homework problems. Our rationale was that this would give mathematicians the opportunity to see student thinking, but since they were homework, they would not require cutting into any lecture time. This was tried, but mathematicians felt that even these questions took too much time. From our field notes on the implementation of the Exit Tickets, the mathematicians would offer complete answers to the Exit Ticket questions that they asked. Apparently, mathematicians felt an obligation to give a complete and rigorous answer to every question that they proposed.

We proposed a think-pair-share structure for questions. The mathematicians rejected the structure for multiple reasons, including that they did not want to try to force students to talk to each other and it was too late in the semester to introduce the new practice. One noted that the students had established, spatially distanced, seating patterns and asking them to move would be too much.

In general, mathematicians valued the conceptual questions that our team initially generated, as well as the student engagement that they elicited. However open-ended questions, with the nuance and detail that the mathematicians requested, required more time in class than the mathematicians were regularly able to devote to their implementation. This was largely because these questions took students a long time to process and the mathematicians felt obligated to lecture the correct solutions. A structural solution that we found that worked for the mathematicians was to ask clicker-type or voting questions immediately after they introduced a new topic or concept on a topic that they intended to lecture about anyway where the questions had to be 'understandable' within about a minute of reading them. This required careful balance between precision in the formulation of the question and the importance of the ideas.

For example, one question that Mathematician B used was, "Do we get equality or inequality in the Cauchy-Schwartz inequality if one of the vectors is a multiple of the other?" This format alleviated the concerns about time in two respects, while maintaining the conceptual focus of the question. First, because the questions were introduced within the context of the lecture, students were already familiar with many of the core ideas of the question so it would take less time to process. Second, because the mathematicians had planned to cover the topic of the questions anyway, no time was "lost" going over the solution to the questions that were asked. We also agreed that students did not necessarily formulate complete answers to the questions that were posed. They could simply make predictions, and discuss with their peers, whether certain statements were likely to be true or not. This could engage the students with thinking about the key theorems that would be covered in the class before seeing their proofs. An innovation that the mathematicians suggested was having all students commit to a yes or no vote on whether statements were likely to be true, so all students had to engage in the task and the mathematicians were able to see the thinking of all students (one even created voting cards that students would hold up).

Overall, mathematicians were satisfied with the format that we created. They liked the conceptual questions that our research team generated—our research team had the time and expertise to generate questions that elicited students thinking about the key linear algebra topics in the course—and they liked the format by which they could use the questions. Mathematician C reflected, "So that's actually, that's just like what [Mathematician D] said. It does not take much time and it is actually effective because I can ask everyone someone who would never want to raise their hand. I sort of forced her to raise her hand so that it's actually useful." Mathematician D was also enthusiastic about the questions that were used and the collaborative meetings in general:

> Yeah, I thought the suggestions for the questions [generated by the research team] were great. And that's really helpful. Also, just to see the suggestions from other instructors.

Conclusion

In this paper, we illustrated how mathematics educators and mathematicians can collaborate to introduce active learning strategies into the mathematicians' classrooms. We use our description to suggest three broader points. First, implementing active learning pedagogical techniques that are commonplace in K12 classrooms may not be immediately applicable to advanced mathematics classes. Second, mathematicians had goals and values that of which we were not initially aware that caused problems with implementing our strategies. For instance, mathematicians' desire to have questions about abstract vector spaces beyond R^n made it challenging to generate conceptual questions that could be answered (or even understood) quickly by the students. Finally, we illustrate how mathematicians and mathematics educators working together is a promising model for generating active learning strategies that mathematicians are willing to use. This supports Artigue's (2016) contention that changing university instruction may work best if we collaborate with mathematicians to solve problems that are meaningful to them with solutions that they believe are viable.

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