

TRB Annual Meeting

Analyzing Car-Following Behavior Using an Empirical Prior Statistical Learning Framework

--Manuscript Draft--

Full Title:	Analyzing Car-Following Behavior Using an Empirical Prior Statistical Learning Framework
Abstract:	<p>This study investigates the integration of traditional car-following models with machine learning techniques to analyze car-following behavior in the presence of lane-changing interactions on multi-lane road segments. Traditional car-following models, such as the Intelligent Driver Model (IDM), are effective with limited data and robust to noise but often fail to capture complex driving behaviors. In contrast, data-driven models like Gaussian Process Regression (GPR) can model intricate behaviors but require extensive, high-quality datasets. To leverage the strengths of both approaches, we propose a hybrid framework that combines IDM with GPR, using IDM as a prior to enhance GPR predictions. The analysis demonstrates that the hybrid model significantly improves the accuracy of acceleration predictions compared to IDM alone, effectively capturing the nuances of real-world driving scenarios. This approach mitigates the limitations of traditional models and reduces the dependency on large datasets. The findings underscore the potential of combining traditional and data-driven methods to improve traffic behavior simulations, offering promising applications in traffic management and autonomous driving. Future research will focus on extending this framework to other driving behaviors and integrating additional data sources for further enhancement.</p>
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1 **ANALYZING CAR-FOLLOWING BEHAVIOR USING AN EMPIRICAL PRIOR**
2 **STATISTICAL LEARNING FRAMEWORK**

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1 **ABSTRACT**

2 This study investigates the integration of traditional car-following models with machine learn-
3 ing techniques to analyze car-following behavior in the presence of lane-changing interactions on
4 multi-lane road segments. Traditional car-following models, such as the Intelligent Driver Model
5 (IDM), are effective with limited data and robust to noise but often fail to capture complex driving
6 behaviors. In contrast, data-driven models like Gaussian Process Regression (GPR) can model
7 intricate behaviors but require extensive, high-quality datasets. To leverage the strengths of both
8 approaches, we propose a hybrid framework that combines IDM with GPR, using IDM as a prior to
9 enhance GPR predictions. The analysis demonstrates that the hybrid model significantly improves
10 the accuracy of acceleration predictions compared to IDM alone, effectively capturing the nuances
11 of real-world driving scenarios. This approach mitigates the limitations of traditional models and
12 reduces the dependency on large datasets. The findings underscore the potential of combining
13 traditional and data-driven methods to improve traffic behavior simulations, offering promising
14 applications in traffic management and autonomous driving. Future research will focus on extend-
15 ing this framework to other driving behaviors and integrating additional data sources for further
16 enhancement.

17

18 *Keywords:* Car-following model, Microscopic simulation

1 INTRODUCTION

2 Car-following behavior refers to the dynamics of a driver maintaining a safe and efficient distance
3 behind a leading vehicle while traveling on a roadway. This behavior is influenced by various
4 factors such as the relative speed and distance between the vehicles, the driver's reaction time, and
5 their desire to maintain a comfortable and safe driving experience. Understanding car-following
6 behavior is crucial for traffic flow analysis, safety assessments, and the development of advanced
7 driver-assistance systems (ADAS). Researchers have developed numerous models over the years to
8 simulate this behavior, incorporating different assumptions and approaches to capture the complex-
9 ities of real-world driving. These models are essential for designing traffic management systems,
10 improving road safety, and developing autonomous driving technologies.

11 The first car-following concepts were proposed in the early 1950s (1). Since then, different
12 classic car-following models have been developed based on various assumptions. For example,
13 the Gazis-Herman-Rothery (GHR) model employs a linear form that considers reaction time, a
14 sensitivity parameter, and the speed difference to simulate the following vehicle's acceleration
15 ((2), (3)). The primary advantage of this model is its simplicity. However, it has several dis-
16 advantages, such as assuming uniform driver behavior during both acceleration and deceleration
17 phases, which overlooks potential variations in driver responses. Subsequently, (4) introduced a
18 memory function into the linear GHR model to address the limitation of using a constant to re-
19 act to speed differences. Moreover, (5) proposed an improved version of the model to simulate
20 drivers' reactions to multiple vehicles ahead, rather than solely the leading vehicle. Another cat-
21 egory of car-following models primarily focuses on the following distance rather than the speed
22 difference as the input, whereas the GHR model and its extensions mainly consider the speed dif-
23 ference. The rationale behind this approach is the drivers' intent to maintain a safe distance from
24 the leading vehicle. The most popular model based on this concept is Gipps' car-following model
25 (6), which has been used by simulation models (7). According to this model, drivers will always
26 opt for the lower speed between the free flow mode and the car-following mode, underpinning
27 the assumption that drivers aim to ensure their vehicle can be safely stopped, even if the leading
28 vehicle suddenly brakes. Bando's Optimal Velocity Model (OVM) was proposed with the notion
29 that each driver has an optimal velocity based on their space headway and will constantly adapt
30 their speed to reach this optimal velocity (8). However, a significant disadvantage of the OVM is
31 its potential to generate unrealistic acceleration rates under certain circumstances (9). To address
32 this issue, the Full Velocity Difference Model (FVDM) was introduced by incorporating a term to
33 represent the driver's reaction to the speed difference with the leading vehicle (10). Recognizing
34 that physical inputs alone do not fully capture driver behavior, another category of car-following
35 models, named psycho-physical models, incorporates perceptions into their framework. A promi-
36 nent example of this approach is Wiedemann's car-following model, widely adopted by simulation
37 software (11), segregates driving scenarios into different regimes (free driving, following, closing
38 in, and emergency braking) and develops the acceleration equation individually for each regime
39 (12). While Wiedemann's model offers a nuanced understanding of driver behavior by considering
40 psychological factors, it is not without drawbacks. One significant limitation is its reliance on pre-
41 defined thresholds to transition between driving regimes, which can oversimplify or misrepresent
42 the fluidity of human driving behavior. Also, the equations adopted for the different thresholds are
43 undisclosed (13). The Intelligent Driver Model (IDM), proposed by (14), stands out among car-
44 following models for its unique approach to simulating driver behavior. Unlike previous models,
45 IDM incorporates both the driver's desired speed and desired following distance to determine the

1 acceleration rate, factoring in the current speed, speed difference, minimum safe following dis-
2 tance, and maximum acceleration. This model has proven its versatility across various following
3 regimes, offering smooth transitions between them (15). Moreover, each parameter in the IDM can
4 be related to a separate factor of the driver and has a clear physical meaning.

5 With advancements in machine learning and the increasing availability of detailed vehicle
6 trajectory data ((16), (17), (18), (19), (20)), attention has shifted towards data-driven car-following
7 models, garnering interest from researchers worldwide ((21), (22), (23), (24), (25), (26)). A signif-
8 icant branch in this domain employs Artificial Neural Networks (ANN) to analyze car-following
9 behavior, providing a novel approach to understanding and predicting driver actions. Like clas-
10 sical car-following models, these models use various inputs to predict the driver's acceleration,
11 speed, or following distance at the next time stamp (25). By categorizing drivers into different
12 driving modes, (21) utilized the following distance and the speed of the leading vehicle as inputs
13 for the ANN to predict the speed of the following vehicle. Their simulations demonstrated that this
14 model outperforms the Gipps-based model. However, it's noteworthy that their study was based
15 on a dataset limited to only 300 seconds of real-world data for training, which might not suffice for
16 comprehensive model training. (27) explored the use of speed, speed difference, and following dis-
17 tance to predict the following vehicle's acceleration rate, indicating that predictive accuracy highly
18 depends on the training data utilized. Furthermore, (28) introduced the driver's reaction delay as
19 an additional input and evaluated the model's predictive performance using the Next Generation
20 Simulation (NGSIM) dataset (16), achieving satisfactory results. (25) proposed a recurrent neural
21 network-based car-following model to examine its applicability in predicting oscillations, compar-
22 ing its performance with the IDM and demonstrating improved efficacy. Compared to classical
23 car-following models, given a sufficient supply of training data, data-driven car-following models
24 possess merits such as higher prediction accuracy, the capability to learn complex behaviors, and
25 the ability to continuously improve through ongoing data input. However, data-driven models are
26 often criticized for their notable sensitivity to data quality, their lower ability to generalize, and
27 their lack of transparency and interpretability, stemming from their nature as "black boxes" (29).

28 By recognizing the strengths and weaknesses of traditional car-following models and data-
29 driven car-following models, researchers have sought to combine both approaches to leverage their
30 respective advantages (30). (31) developed a physics-informed deep learning car-following model
31 to predict the acceleration of the following vehicle. The primary concept involves first using tra-
32 ditional car-following models to generate collocation data, which is then combined with observed
33 data. By incorporating the prediction errors from both the traditional car-following models and
34 ANNs, the results demonstrate improved accuracy in performance. Later (32) proposed a physics-
35 informed Transformer model for predicting vehicle's longitudinal trajectories.

36 The main idea of this type of physics-informed deep learning method involves changing
37 the simulation problem into a prediction problem. The collocation data is first generated by the
38 traditional car-following models, and then the collocation data and the observed data are split into
39 training and testing sets. The model is trained by modifying the loss function of the deep learning
40 model. However, the goal of analyzing car-following behavior should be focused on simulation
41 rather than prediction. In simulation, only the initial state of the following vehicle should be taken
42 as known, rather than taking the information of the following vehicle at any time stamp. Addition-
43 ally, incorporating the car-following model into the loss function of the deep learning method does
44 not always guarantee better performance, as the collocation data is purely generated without any
45 observation. Moreover, the traditional car-following model often performs unsatisfactorily when

1 dealing with lane-changing scenarios. Acknowledging this, this study proposes a new method
 2 to combine the car-following model with Gaussian Process Regression to analyze car-following
 3 behavior interacting with lane-changing behaviors. Instead of integrating the car-following model
 4 into the loss function, it is treated as the prior belief of the estimated acceleration. The performance
 5 of the proposed model under different car-following scenarios is compared with the traditional car-
 6 following model and pure GPR, showing that the proposed method outperforms both.

7 The remainder of the paper is organized as follows: Section 2 introduces the proposed
 8 methodology; Section 3 conducts the case study of the proposed model and demonstrates the
 9 performance using real-world data ; and finally, Section 4 concludes with a discussion on the
 10 proposed model and suggestions for future work.

11 METHODOLOGY

12 Deterministic model

13 Traditional car-following models exhibit significant drawbacks when interacting with lane-changing
 14 maneuvers. As shown in [Figure 1](#), when vehicle $j - 1$ cuts in, the gap between vehicle i and
 15 $i - 1$ changes abruptly, leading to a sudden drop in the predicted acceleration for vehicle i . This
 16 deficiency stems from the inherent accident-free assumption of traditional car-following models.
 17 These models assume that the following vehicle will anticipate the worst-case scenario, where the
 18 leading vehicle suddenly brakes to a complete standstill. Similarly, for vehicle j when the leading
 19 vehicle abruptly leaves the current lane, the sudden increase in the gap can result in an unrealisti-
 20 cally high predicted acceleration. For the proposed framework, the empirical deterministic model
 21 plays a crucial role in accurately predicting the longitudinal trajectory. In this study, the IDM is
 22 chosen to serve as the deterministic model for the following reasons:

23 1). Comprehensive Consideration of Driver Behavior: IDM incorporates both the driver's
 24 desired speed and desired following distance to determine the acceleration rate, taking into account
 25 factors such as current speed, speed difference, minimum safe following distance, and maximum
 26 acceleration. This model has demonstrated its versatility across various following regimes, offering
 27 smooth transitions between them.

28 2). Parameter Interpretability and Flexibility: Each parameter in the IDM can be associated
 29 with a distinct aspect of driver behavior and has a clear physical meaning. One of the advantages of
 30 the IDM is its ability to represent a wide range of driving styles by adjusting its parameter values.

31 3). Realistic Accelerating Behaviors: When simulating accelerating behavior, the IDM
 32 generates realistic acceleration rates than other car-following models, avoiding the unrealistic val-
 33 ues that can sometimes be produced by alternative models.

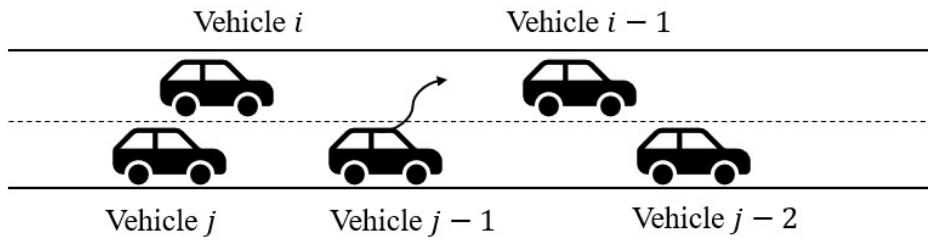


FIGURE 1: Illustration of the lane changing behaviors

1 The original IDM can be expressed as follows:

2
$$\frac{dv_n}{dt} = a_n \left[1 - \left(\frac{v_n}{v_{n,0}} \right)^\delta - \left(\frac{s^*(v_n, \Delta v_n)}{s_n} \right)^2 \right] \quad (1)$$

3

4
$$s^*(v_n, \Delta v_n) = s_0 + \max(v_n T + \frac{v_n \Delta v_n}{2\sqrt{a_n b_n}}, 0) \quad (2)$$

5

6
$$\Delta v_n = v_n - v_{n-1} \quad (3)$$

7

8
$$S_n = x_{n-1} - l_{n-1} - x_n \quad (4)$$

9 The free accelerating term $a[1 - (\frac{v_n}{v_0})^\delta]$, governs the acceleration of the vehicle. Here, a represents
 10 the maximum acceleration, and v_0 denotes the vehicle's desired speed. Given an unobstructed path
 11 for a stationary vehicle, the vehicle would first accelerate at the rate of a , and the acceleration
 12 gradually decreases as the speed increases. Such reduction is controlled by the exponent term
 13 δ , and the vehicle would not exceed its desired speed. In accordance with the IDM author's
 14 recommendation ((15)), this paper assigns the value of 4 to δ . On the other hand, the vehicle's
 15 decelerating process is regulated by the braking term $(\frac{s^*(v_n, \Delta v_n)}{s_n})^2$, where $s^*(v_n, \Delta v_n)$ represents
 16 vehicle's desired gap and s_n is the actual gap as shown by Equations 1 - 4. The term $s_0 + v_n T$
 17 denotes the vehicle's desired following distance at the steady state. Here, the minimum gap, s_0 ,
 18 represents the space gap between the standstill vehicles, T is the time gap that the driver aims to
 19 maintain while in motion. The dynamic term $\frac{v_n \Delta v_n}{2\sqrt{a_n b_n}}$, symbolizes the driver's response to the speed
 20 difference Δv_n based on its own comfortable deceleration b .

21 Stochastic modeling

22 In this study, we employ Gaussian Process Regression (GPR) to model the underlying dynamics
 23 of car-following behavior. This section would introduce the GPR from function-space view (to
 24 understand the GPR from weight-space view, readers could refer to (33), A Gaussian Process (GP)
 25 is a collection of random variables, any finite number of which have a joint Gaussian distribution.
 26 Specifically, a distribution over functions $f(\mathbf{x})$ are described by a Gaussian process. It is fully
 27 specified by its mean function $m(\mathbf{x})$ and covariance function $k(\mathbf{x}, \mathbf{x}')$ as shown below.

28
$$m(\mathbf{x}) = \mathbb{E}[f(\mathbf{x})] \quad (5)$$

29

30
$$k(\mathbf{x}, \mathbf{x}') = \mathbb{E}[(f(\mathbf{x}) - m(\mathbf{x}))(f(\mathbf{x}') - m(\mathbf{x}'))] \quad (6)$$

31 Thus, the Gaussian process can be expressed as:

32
$$f(\mathbf{x}) \sim \mathcal{GP}(m(\mathbf{x}), k(\mathbf{x}, \mathbf{x}')) \quad (7)$$

33 Generally, for a pure Gaussian Process Regression (GPR) problem, the mean function is set to
 34 zero, represents the prior belief of the value of $f(\mathbf{x})$. On the other hand, the covariance function
 35 determines the relationship between variables. Consider the observed data $(\mathbf{x}_i, f_i | i = 1, 2, 3, \dots, n)$
 36 and the estimated mean function \mathbf{f} , the goal is to predicted new data \mathbf{x}_* as \mathbf{f}_* . The joint distribution
 37 of the \mathbf{f} and \mathbf{f}_* is multivariate Gaussian distribution, which can be expressed as:

38
$$\begin{bmatrix} \mathbf{f} \\ \mathbf{f}_* \end{bmatrix} \sim \mathcal{N} \left(\begin{bmatrix} m(\mathbf{x}) \\ m(\mathbf{x}_*) \end{bmatrix}, \begin{bmatrix} \mathbf{K}_{nn} & \mathbf{K}_{n*} \\ \mathbf{K}_{*n} & \mathbf{K}_{**} \end{bmatrix} \right) \quad (8)$$

39 Knowing the joint probability of \mathbf{f} and \mathbf{f}_* , the conditional distribution of \mathbf{f}_* could be derived:

40
$$\mathbf{f}_* | \mathbf{f}, \mathbf{x}, \mathbf{x}_* \sim \mathcal{N}(\mathbf{K}_{*n} \mathbf{K}_{nn}^{-1} \mathbf{f}, \mathbf{K}_{**} - \mathbf{K}_{*n} \mathbf{K}_{nn}^{-1} \mathbf{K}_{*n}) \quad (9)$$

41 It should be noticed the $f(\mathbf{x})$ is a noise-free function. For a more realistic scenario, the noisy
 42 versions $y = f(\mathbf{x}) + \varepsilon$, where ε is a independent identically distributed Gaussian noise with variance

1 σ_n^2 . The prior on the noisy observations now is:

2 $\text{cov}(\mathbf{y}) = \mathbf{K}_{nn} + \sigma_\epsilon^2 \mathbf{I}$ (10)

3 and Equation 10 now becomes:

4
$$\begin{bmatrix} \mathbf{y} \\ \mathbf{f}_* \end{bmatrix} \sim \mathcal{N} \left(\begin{bmatrix} m(\mathbf{x}) \\ m(\mathbf{x}_*) \end{bmatrix}, \begin{bmatrix} \mathbf{K}_{nn} + \sigma_\epsilon^2 \mathbf{I} & \mathbf{K}_{n*} \\ \mathbf{K}_{*n} & \mathbf{K}_{**} \end{bmatrix} \right)$$
 (11)

5 And the conditional probability of \mathbf{y} over \mathbf{f}_* becomes:

6 $\mathbf{y}_* | \mathbf{y}, \mathbf{x}, \mathbf{x}_* \sim \mathcal{N}(\mathbf{y}_* | \mathbf{K}_{*n}[\mathbf{K}_{nn} + \sigma_\epsilon^2 \mathbf{I}]^{-1} \mathbf{y}, \mathbf{K}_{**} - \mathbf{K}_{*n}[\mathbf{K}_{nn} + \sigma_\epsilon^2 \mathbf{I}]^{-1} \mathbf{K}_{n*})$ (12)

7 Although GPR is a promising and widely-used non-parametric method that performs well across
8 various tasks, it faces criticism when dealing with large datasets due to its computational complexity.
9 The time complexity of GPR is $\mathcal{O}(n^3)$ and the storage requirement is $\mathcal{O}(n^2)$, where n is the
10 number of samples. These limitations make it challenging to scale GPR to larger datasets.

11 Integration of deterministic and stochastic model

12 Notice that the input data used for calibrating the IDM only requires the variables: the following
13 distance S_n , the speed difference Δv_n , and the speed of the following vehicle v_n . Consistent with
14 the IDM, this study also adopts these three variables as inputs. Define the observed data set as $\mathbf{x} =$
15 $\{x^i = (S_n^i, v_n^i, \Delta v_n^i) | i = 1, 2, \dots, N\}$ and a set of scalars $\mathbf{y} = \{a^i | i = 1, 2, \dots, N\}$, representing the
16 observed acceleration rates of the following vehicle. The data \mathbf{x} is first calibrated using $f_{IDM}(\mathbf{x} | \lambda)$
17 to obtain the optimal set of parameters λ . The mean function of the GPR is then set as $m(\mathbf{x}) =$
18 $f_{IDM}(\mathbf{x} | \lambda)$, indicating that the IDM-predicted acceleration of the following vehicle represents the
19 prior belief of the acceleration value in the GPR. After training the GPR, the performance of the
20 proposed model is evaluated. Unlike previous work (31), this study does not use the observed
21 information of the following vehicle as input for prediction. This approach mitigates the risk of
22 overfitting and ensures a fair comparison with traditional car-following behavior models. The
23 initial state $x^0 = (S_n^0, v_n^0, \Delta v_n^0)$ is set to the known condition, and the predicted acceleration \hat{a}^0 is
24 used to update the new state $x^1 = (\hat{S}_n^1, \hat{v}_n^1, \hat{\Delta v}_n^1)$ for the next timestamp as shown by Equation 13 and
25 14 where Δt is set to 0.1 seconds in this study. In other words, for the prediction at time step $k+1$,
26 the input is the simulated $\hat{x}^k = (\hat{S}_n^k, \hat{v}_n^k, \hat{\Delta v}_n^k)$.

27 $v_n(t + \Delta t) = v_n + a_n(t) \cdot \Delta t$ (13)

28

29 $x_n(t + \Delta t) = x_n(t) + v_n(t + \Delta t) \cdot \Delta t$ (14)

30 CASE STUDY

31 Experiment setup

32 To validate the performance of the proposed framework, real-world vehicle trajectory data col-
33 lected by the NGSIM was utilized. The detailed vehicle trajectory data was gathered on the east-
34 bound I-80 in Emeryville, CA, on April 13, 2005. The study segment featured five freeway lanes
35 and a high-occupancy vehicle (HOV) lane, spanning approximately 1,640 feet. Given the pres-
36 ence of errors in the original dataset, this study adopted the reconstructed version proposed by
37 (34). This study aims to investigate car-following behavior in the context of lane-changing in-
38 teractions. To this end, five vehicle trajectories were selected, encompassing various scenarios:
39 car-following without lane-changing (vehicle 1593 and vehicle 1700), car-following with cutting-
40 in (vehicle 797), car-following with the leading vehicle leaving the current lane (vehicle 1649),
41 and car-following with both cutting in and leading vehicle leaving the current lane (vehicle 3222).
42 The following gap of each target vehicle is shown in Figure 2.

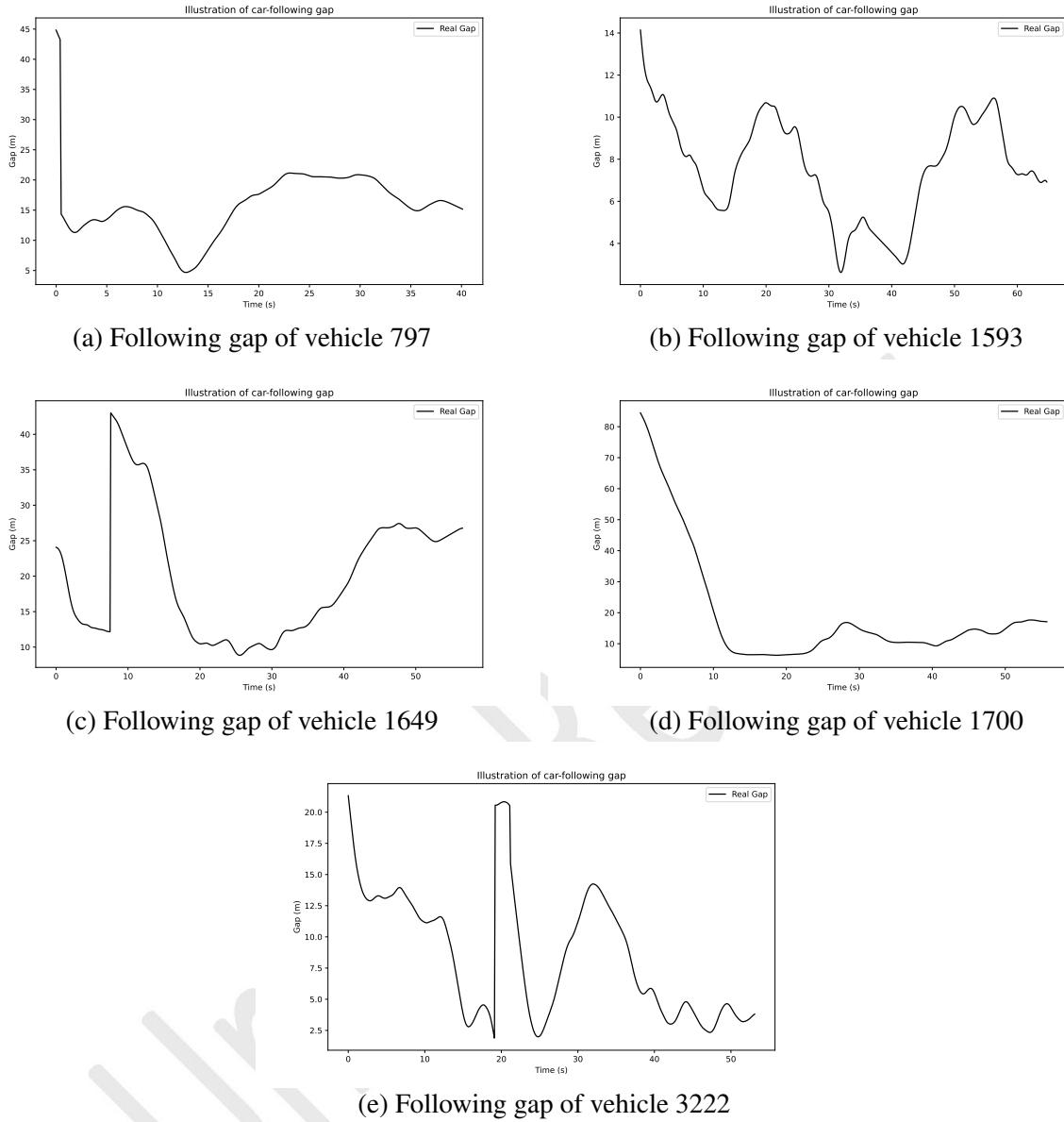


FIGURE 2: Illustration of the following gap of each target vehicle

1 Before testing the performance of the proposed model, each individual trajectory was first
 2 calibrated using the IDM. Various methods can be used to calibrate the IDM, such as maximum
 3 likelihood estimation (35) and least square estimation (36). Given the complexity of the IDM, this
 4 study adopts the Genetic Algorithm (GA) for calibration due to its heuristic nature and gradient-
 5 free approach (37). The parameters for executing the GA in this study are as follows: the GA
 6 runs for a maximum of 200 generations, with each generation consisting of a population of 100
 7 individuals. The mutation rate is set to 0.05. On the other hand, it is essential to ensure that the
 8 calibrated parameters of the IDM remain within realistic bounds to maintain interpretability and
 9 prevent overfitting. The boundaries for these parameters are defined as follows: the time gap T s set
 10 between 0.1 and 3 seconds, and the minimum spacing s_0 is limited to 1 to 5 meters. The maximum
 11 acceleration a restricted to 0.1 to 4 m/s^2 , which corresponds to a maximum acceleration rate of

1 equivalent to 0-100km/h in 6 seconds, while the comfortable deceleration boundary is set at 0.1 to
 2 9 m/s^2 . The upper limit for the desired velocity v_0 is set to 33.6 m/s (120km/h). The lower limit
 3 must exceed the highest velocity observed in the dataset to avoid excessive deceleration due to the
 4 power of 4 applied in the term $(\frac{v_n}{v_0})^4$ This consideration is crucial because v_0 in the IDM is primarily
 5 designed for modeling acceleration rather than deceleration scenarios (15). By minimizing the
 6 error of following gaps, The calibrate parameters of each following vehicle is shown by Table
 7 1. As mentioned before, the calibrated IDM model for each single trajectory is taken as the mean
 8 function of the GPR, and for the selection of the kernel function, after evaluating multiple available
 9 kernel functions, the study adopts the radial basis function (RBF) kernel as shown by Equation 15,
 10 where σ and θ are hyper-parameters. The goodness-of-fit (GoF) function and the measure of
 11 performance (MoP) in this study is the root mean square error (RMSE) of acceleration as shown
 12 by Equation 16.

$$13 \quad k(\mathbf{x}, \mathbf{x}') = \sigma^2 e^{-\frac{\|\mathbf{x}-\mathbf{x}'\|^2}{2\theta^2}} \quad (15)$$

$$14 \quad 15 \quad RMSE(a) = \sqrt{\frac{1}{T} \sum_{t=1}^T [a_i(t) - \hat{a}_i(t)]^2} \quad (16)$$

TABLE 1: Illustration of calibration result of IDM

Vehicle	v_0	T	s_0	a	b
797	31.79 m/s	1.08 s	4.96 m	0.77 m/s 2	2.09 m/s 2
1593	32.71 m/s	0.79 s	3.40 m	3.97 m/s 2	1.28 m/s 2
1649	29.51 m/s	1.10 s	5.00 m	0.40 m/s 2	1.25 m/s 2
1700	21.3 m/s	0.96 s	5.00 m	0.97 m/s 2	1.47 m/s 2
3222	31.14 m/s	0.99 s	1.25 m	4.00 m/s 2	8.99 m/s 2

16

17 Results discussions

18 To compare the performance of the IDM, pure GPR with no empirical prior, and GPR with IDM
 19 prior, three groups of experiments were conducted. The corresponding RMSE values and the
 20 estimated accelerations and the hyperparameters of the proposed model are presented in Figure
 21 3 and Table 2. In the plots, the red lines represent the real acceleration rate of each following
 22 vehicle, the green dashed lines depict the simulated acceleration by the IDM model, and the blue
 23 lines show the predicted acceleration simulated by the GPR with the IDM prior, while the light blue
 24 areas represent the 95% confidence intervals of the simulated accelerations. Across all trajectories,
 25 the observed acceleration fluctuates over time, capturing the dynamic nature of real-world vehicle
 26 behavior.

27 The analysis of the five trajectory plots comparing observed accelerations, IDM predicted
 28 accelerations, and GPR predicted accelerations, along with the GP confidence intervals, reveals
 29 several key insights. The trajectory of vehicle 797 depicts a cutting-in scenario where a vehicle
 30 from the adjacent lane cuts in and the leading vehicle switches lanes around 2 seconds. The target
 31 vehicle continues to follow the new vehicle until the end of the data. The plot shows that the
 32 IDM model predicts a drastic deceleration when the new vehicle cuts in. In contrast, the GPR with
 33 IDM prior provides a more accurate estimation, maintaining accuracy throughout the car-following

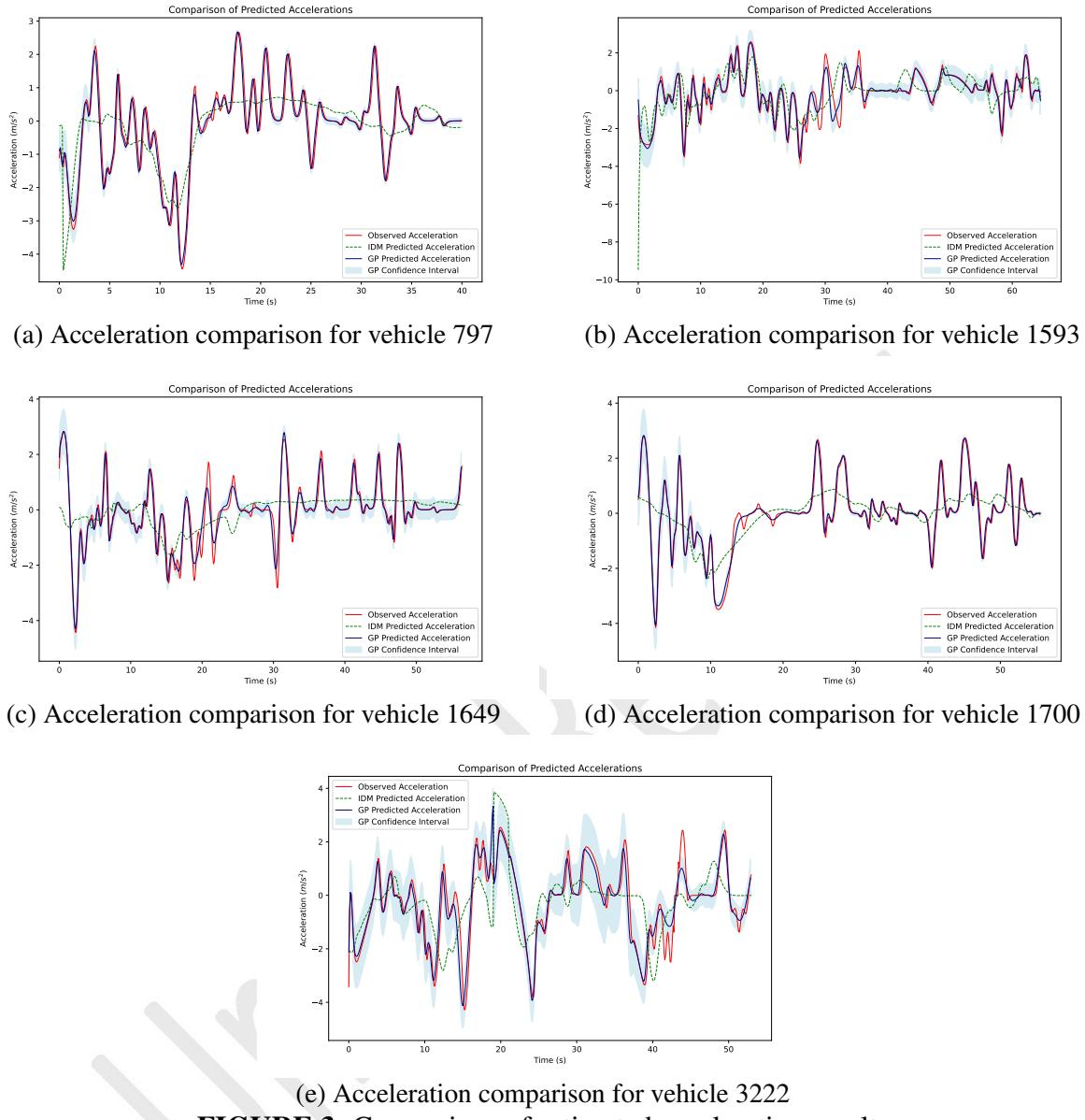


FIGURE 3: Comparison of estimated acceleration results

1 behavior simulation, whereas the IDM model fails to capture the fluctuations in acceleration. The
 2 RMSE values reflect this performance, with the IDM having an RMSE of 0.95, while the pure
 3 GPR and GPR with IDM prior have much lower RMSE values of 0.11 and 0.09, respectively. For
 4 vehicle 1593, although there are no lane-changing behaviors, the IDM model predicts a significant
 5 deceleration rate near -10 m/s^2 at the beginning, while the actual deceleration rate is around -3
 6 m/s^2 . The GPR with IDM prior performs better in this scenario, accurately reflecting the observed
 7 behavior. This is supported by the RMSE values, where the IDM has an RMSE of 1.02, and the
 8 pure GPR and GPR with IDM prior have RMSE values of 0.51 and 0.27, respectively. For vehicle
 9 1649, where the leading vehicle leaves the current lane around 8 seconds, the IDM model does
 10 not show an increase in acceleration as the following gap increases, likely due to the estimated
 11 maximum acceleration being 0.40 m/s^2 . Conversely, the GPR with IDM prior shows a better

1 performance, accurately capturing the increase in acceleration. The RMSE values indicate this
 2 as well, with the IDM having an RMSE of 1.18, while the pure GPR shows a worse performance
 3 that IDM with RMSE of 2.68, while GPR with IDM prior have RMSE values 0.22, shows a better
 4 performance. For vehicle 1700, during the first 10 seconds of the simulation, both the IDM and GPR
 5 models fail to simulate the drastic fluctuations in acceleration. However, the GPR still outperforms
 6 the IDM by providing a closer match to the observed data. This is reflected in the RMSE values,
 7 where the IDM has an RMSE of 0.95, and the pure GPR and GPR with IDM prior have RMSE
 8 values of 0.51 and 0.15, respectively. Vehicle 3222 presents a more complex scenario where the
 9 leading vehicle leaves the current lane around 20 seconds, followed by another vehicle from the
 10 adjacent lane cutting in. The IDM model generates a sudden change from deceleration to maximum
 11 acceleration and then another drastic change back to deceleration. In contrast, the GPR with IDM
 12 prior avoids such abrupt transitions, offering a more realistic result. The RMSE values show this
 13 clearly, with the IDM having an RMSE of 1.29, while the pure GPR and GPR with IDM prior have
 14 RMSE values of 1.01 and 0.40, respectively.

TABLE 2: Results comparison

Vehicle	θ	σ^2	RMSE(a) (IDM)	RMSE(a) (pure GPR)	RMSE(a) (GPR with IDM prior)
797	3.06	1.03	0.95	0.11	0.09
1593	1.16	0.726	1.02	0.51	0.27
1649	2.99	1.81	1.18	2.68	0.22
1700	3.64	6.43	0.95	0.51	0.15
3222	3.41	2.64	1.29	1.01	0.40

15 The IDM predicted acceleration generally follows the observed trends but exhibits deviations
 16 at various points, indicating that while IDM captures overall behavior, it lacks the precision
 17 to accurately model complex interactions and abrupt changes in real driving scenarios. The tra-
 18 ditional car-following model tends to show drastic switches in acceleration when dealing with
 19 lane-changing behavior. On the other hand, pure GPR can sometimes produce worse estimation
 20 results than IDM. In contrast, the predicted acceleration by GPR with IDM prior closely aligns
 21 with the observed acceleration across all trajectories, demonstrating the higher accuracy of the
 22 proposed method. The confidence intervals typically encompass the observed acceleration, sug-
 23 gesting reliable predictions and realistic uncertainty estimation. This consistency is evident in all
 24 five trajectories, where the proposed model consistently outperforms both the IDM model and pure
 25 GPR in terms of accuracy and robustness. Specifically, the proposed model's flexibility and ca-
 26 pacity to incorporate both observed data and a physics-informed prior from IDM allow it to better
 27 adapt to variations in acceleration, capturing more intricate driving behaviors. This integration of
 28 data-driven approaches with traditional models proves beneficial, as it mitigates the risk of over-
 29 fitting and ensures a fair comparison with traditional car-following models. Consequently, the
 30 GPR with IDM prior's superior performance highlights its potential advantages for applications
 31 in traffic management and autonomous driving, offering more precise and reliable predictions for
 32 vehicle accelerations. Overall, the comparative analysis of these trajectories underscores the ef-
 33 fectiveness of the proposed model in modeling car-following behavior, demonstrating significant
 34 improvements over traditional IDM estimations.

1 **CONCLUSION**

2 This study investigates car-following behavior in conjunction with lane-changing interactions on a
3 multi-lane road segment by combining a traditional car-following model with a machine learning
4 method, specifically Gaussian Process Regression (GPR). Traditional car-following models, while
5 robust to varying sampling rates and noise, often struggle to accurately replicate complex empirical
6 driving behaviors. On the other hand, data-driven car-following models can capture these
7 intricate behaviors but demand high-quality, large-scale datasets. To leverage the strengths of both
8 approaches, this study proposes a new framework that integrates traditional car-following models
9 with GPR. The proposed method utilizes the IDM as a prior in the GPR framework to enhance
10 the accuracy of acceleration predictions. The analysis demonstrates that GPR, when informed by
11 IDM, provides superior performance in predicting vehicle accelerations compared to using IDM
12 and pure GPR alone. The results indicate that the GPR with IDM prior is more effective at mod-
13 eling the complex interactions between car-following and lane-changing behaviors, capturing the
14 nuances of real-world driving scenarios more accurately. This hybrid approach not only mitigates
15 the limitations of traditional models but also reduces the dependency on large-scale, high-quality
16 datasets typically required by purely data-driven methods.

17 Overall, the findings highlight the potential of combining traditional car-following mod-
18 els with advanced machine learning techniques to improve the accuracy and reliability of traffic
19 behavior simulations. This integrated approach offers significant advantages for applications in
20 traffic management and autonomous driving, providing more precise and robust predictions of ve-
21 hicle dynamics. Future research could extend this framework to other aspects of driving behavior
22 and explore the integration of additional data sources to further enhance model accuracy and appli-
23 cability. The continued development of such hybrid models holds promise for advancing the field
24 of traffic flow analysis and contributing to safer and more efficient transportation systems.

25 **CREDIT**

26 **Kaitai Yang:** Conceptualization, Methodology, Writing - original draft. **Yuan-Zheng Lei:** Con-
27 ceptualization, Methodology, Writing - original draft. **Xianfeng Terry Yang:** Conceptualization,
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