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Perspective Article

Perspectives on quantum friction, self-propulsion, and self-torque

Kimball A. Milton [a,*], Nima Pourtolami [b], Gerard Kennedy [c]

[a] Homer L. Dodge Department of Physics and Astronomy, The University of Oklahoma, Norman, OK 73019, United States of America
[b] National Bank of Canada, Montreal, Quebec H3B 4S9, Canada
[c] School of Mathematical Sciences, University of Southampton, Southampton, SO17 1BJ, UK

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ABSTRACT

This paper provides an overview of the nonequilibrium fluctuational forces and torques acting on a body either in motion or at rest relative to another body or to the thermal vacuum blackbody radiation. We consider forces and torques beyond the usual static Casimir-Polder and Casimir forces and torques. For a moving body, a retarding force emerges, called quantum or Casimir friction, which in vacuum was first predicted by Einstein and Hopf in 1910. Nonreciprocity may allow a stationary body, out of thermal equilibrium with its environment, to experience a torque. Moreover, if a stationary reciprocal body is not in thermal equilibrium with the blackbody vacuum, a self-propulsive force or torque can appear, resulting in a potentially observable linear or angular terminal velocity, even after thermalization.

1. Introduction

For nearly 50 years, it has been appreciated that when two bodies move parallel to each other, they experience a quantum frictional force that tends to retard the relative motion [1]. But more than a century ago, it was recognized [2] that a moving body experiences friction even in vacuum! Even more remarkably, in the last decade it has been shown that a stationary body out of thermal equilibrium with the background blackbody radiation will experience a force and a torque, if it is suitably asymmetric. Here we will discuss such phenomena, which are on the verge of observability, in a perturbative context, expanding in $\varepsilon - 1$, where ε is the permittivity of the object. A brief perspective of this field of research is provided in context.

We use natural units, $\hbar = c = \epsilon_0 = \mu_0 = k_B = 1$, and Heaviside-Lorentz units for polarizability, which is related to the Gaussian polarizability by $\alpha^{\rm HL} = 4\pi\alpha^{\rm G}$.

2. Friction of particle with a dielectric or metallic surface

Casimir friction refers to the dissipative retarding force experienced by a body (an atom, a nanoparticle, or a dielectric plate) moving parallel to another body (typically a metallic or dielectric plate). The subject has a long history, as noted, but the literature can be confusing because there is considerable controversy surrounding both the velocity and distance dependence, as well as the effect of temperature. There is no real experimental evidence for Casimir friction, although hundreds of theoretical papers on the subject have been published. For a review of the literature a decade ago see Ref. [3]. For a more recent detailed perspective on atom-surface friction, with many references, see Ref. [4]

Here, let us consider a particle, described by a static electric polarizability α_0, moving with velocity v parallel to a metallic plate with conductivity σ, a distance a above the plate, as illustrated in Fig. 1.

The origin of the theoretical controversy is that different physical mechanisms are considered as responsible for the dissipative effect of the polarizable particle. For an atom, the most significant mechanism arises from the resistance in the metal plate, because the image in the metal of the charge distribution of the moving polarizable particle lags behind that of the particle itself. This gives rise, on use of the fluctuation-dissipation theorem [see Eq. (4.3) below], to a frictional force on the particle that is [5,6], nonrelativistically, at zero temperature,

$$F = -\frac{135\alpha_0^2 v^3}{2\pi^3\sigma^2(2a)^{10}}. \tag{2.1}$$

If, in contrast, one were to assume that radiation reaction [see Eq. (3.2) below] were the dominant mechanism by which the polarizability of the particle acquires a dissipative part, the force would be considerably different [6],

$$F = -\frac{105}{128\pi^3}\frac{\alpha_0^2 v^5}{\sigma a^9}. \tag{2.2}$$

* Corresponding author.
E-mail addresses: kmilton@ou.edu (K.A. Milton), nima.pourtolami@gmail.com (N. Pourtolami), g.kennedy@soton.ac.uk (G. Kennedy).

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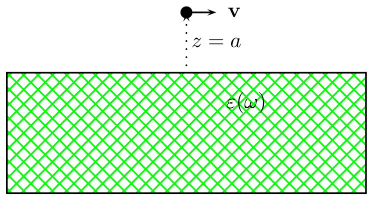

Fig. 1. A particle moving with velocity v in vacuum a distance a above a dielectric plate. Polarizability is defined by $\mathbf{d} = \boldsymbol{\alpha} \cdot \mathbf{E}$.

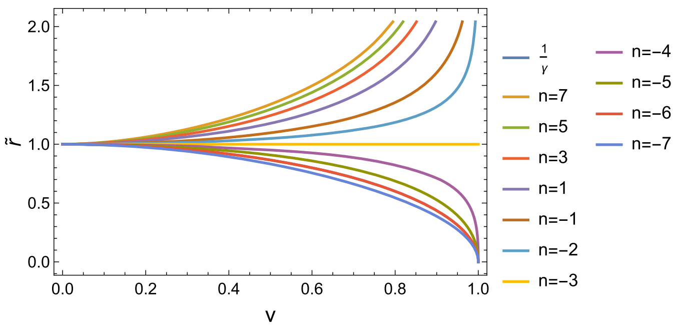

Fig. 2. The ratio $\tilde{r} = \frac{\tilde{T}}{T}$, versus velocity v, where T is the temperature of the blackbody radiation background, and $\tilde{T}$ is the temperature of the particle in NESS, where the particle neither gains nor loses energy. (The dynamic version of thermal equilibrium.) If the temperature ratio could be measured, that would be a signal of quantum friction. Note that $n = 3$ is the pure radiation reaction model (3.2), with α_0 constant. The temperature ratio is unity for $n = -3$, and is exactly $1/\gamma = \sqrt{1 - v^2}$ for $n = -6$.

In addition, the nanoparticle itself (not an atom) might well possess intrinsic dissipation of its own, say if it were composed of a realistic metal. If this were the only effect, and the conductivity of the nanoparticle were σ', while the plate had conductivity σ, the force would be first order in the static polarizability of the particle [7],

$$F = -\frac{135}{64\pi^2} \frac{\alpha_0}{\sigma\sigma'} \frac{v^3}{a^7}. \tag{2.3}$$

(These results have been recalculated using the general reformulation of quantum friction in Refs. [8,9], so there are some discrepancies in the numerical coefficients found in the original literature.) All these effects, in principle, are present, and it can be difficult to disentangle their importance, especially in the absence of any experimental guidance. We note that the force (2.1) dominates the radiation-reaction force (2.2) at low velocities and short distances; the two expressions become comparable at a separation of 10 nm if v is of order 10^6 m/s. If present, the first-order effect for a metallic nanoparticle (2.3) overwhelmingly dominates. The reader will note that these results are all given for zero temperature. This is because then the integrations are restricted to small values of frequency, where only the TM reflection coefficient need be considered, and the integrations become trivial. The general case of nonzero temperature is considerably more subtle, but less relevant in practice.

Let us give a brief survey of some of the novel features of Casimir friction with surfaces explored during the last few years. Friction may occur on a nanosphere moving parallel to a dielectric surface, if it is moving faster than the speed of light in that medium (Čerenkov friction) [10,11]. There is extensive literature concerning the friction between parallel moving plates, for which we cite Refs. [12,13], for example. A rotating particle near an isotropic surface may experience a frictional torque [14]. Exotic metamaterials may induce forces on rotating nanoparticles [15]. Nonequilibrium thermodynamics comes into play in quantum friction [16,17], as we shall see. A covariant approach to radiative friction of a particle above a surface and in vacuum is given in Ref. [18]. A very nice summary of the theory of quantum friction near surfaces is given in the thesis [19].

However, the most important issue in the field is the lack of experimental input. The quantum forces tend to be exceedingly small, so novel methods of finding signatures of friction must be sought. One exciting possibility is looking for accumulated geometric phase [20]; another scheme, reflecting the nonequilibrium nature of quantum friction, is explored in Sec. 3. There is even a report [21] of experimental observation of quantum friction, although in an analog mechanical system.

3. Quantum vacuum friction—Einstein-Hopf formula

If a particle has dissipation (which it must, if only through interaction with the electromagnetic field), it will experience quantum vacuum friction, which, nonrelativistically, is given by the Einstein-Hopf formula [2],

$$F = -\frac{v\beta}{12\pi^2} \int_0^\infty d\omega\, \omega^5 \Im\alpha(\omega) \frac{1}{\sinh^2 \beta\omega/2}, \tag{3.1}$$

which is linear in the velocity, where $\beta = 1/T$ refers to the temperature. Here, for example, if the dissipation occurs entirely from radiation reaction, as appropriate for an atom,

$$\Im\alpha(\omega) = \frac{\omega^3}{6\pi} \alpha_0^2(\omega), \tag{3.2}$$

where $\alpha_0(\omega)$ is the real, nondissipative, intrinsic polarizability of the atom.

For low temperature ($< 10^6$ K), low velocity ($v \ll c$), and if dispersion is neglected, this frictional force reduces to

$$F(v) = -\frac{32\pi^5 \alpha_0^2}{135\beta^8} v = m\frac{dv}{dt}, \tag{3.3}$$

which means the time required for a atom to slow from an initial velocity v_i to a final velocity v_f is

$$\Delta t = -t_0 \ln \frac{v_f}{v_i}, \quad t_0 = \frac{135 m\beta^8}{32\pi^5 \alpha_0^2} = 1.7 \times 10^{25}\,\text{s} \tag{3.4}$$

for gold at $T = 300$ K. For a 10% decrease in velocity, Δt would drop to the lifetime of a US physics graduate student, 5.9 years, if the temperature could be raised to 30,000 K!

For higher velocities, we need to distinguish between the temperature, T, of the blackbody background, and the temperature, T', of the body. There is a special ratio of T'/T, which is the generalization of thermal equilibrium, in which the body neither gains nor loses energy in its rest frame. This is called the "nonequilibrium steady state" (NESS) [16]. This ratio is illustrated in Fig. 2 for a monomial dissipation law, that is, where $\Im\alpha(\omega) \propto \omega^n$. Measuring this temperature ratio might be a way of finding a signal of quantum friction, but would require velocities comparable to the speed of light. For more detail on relativistic quantum vacuum friction at arbitrary temperatures see [8,9]. This, of course, was discussed earlier, for example in Refs. [22–24], which also discuss the NESS temperature.

4. Quantum vacuum torque: nonreciprocal media

Classically, the torque on a stationary dielectric body with polarization vector $\mathbf{P}$ is given by [25]

$$\boldsymbol{\tau} = \int (d\mathbf{r}) \frac{d\omega}{2\pi} \frac{d\nu}{2\pi} e^{-i(\omega+\nu)t} \left[\mathbf{P}(\mathbf{r};\omega) \times \mathbf{E}(\mathbf{r};\nu) + P_i(\mathbf{r};\omega)(\mathbf{r} \times \boldsymbol{\nabla}) E_i(\mathbf{r};\nu)\right]. \tag{4.1}$$

The first term here is called the *internal* torque and the second the *external* torque, because the latter is reflective of the force on the body. Now we use the first-order expansions

$$\mathbf{E}^{(1)}(\mathbf{r};\omega)=\int(d\mathbf{r}')\boldsymbol{\Gamma}(\mathbf{r}-\mathbf{r}';\omega)\cdot\mathbf{P}(\mathbf{r}';\omega),\quad \mathbf{P}^{(1)}(\mathbf{r};\omega)=\chi(\mathbf{r};\omega)\cdot\mathbf{E}(\mathbf{r};\omega). \tag{4.2}$$

We evaluate the EE and PP contributions to the torque by use of the fluctuation-dissipation theorem (FDT):

$$\langle\mathbf{P}_i(\mathbf{r};\omega)\mathbf{P}_j(\mathbf{r}';\nu)\rangle=2\pi\delta(\omega+\nu)\delta(\mathbf{r}-\mathbf{r}')\chi^A_{ij}(\mathbf{r};\omega)\coth\beta'\omega/2, \tag{4.3a}$$

$$\langle\mathbf{E}_i(\mathbf{r};\omega)\mathbf{E}_j(\mathbf{r}';\nu)\rangle=2\pi\delta(\omega+\nu)\Im\boldsymbol{\Gamma}_{ij}(\mathbf{r}-\mathbf{r}';\omega)\coth\beta\omega/2, \tag{4.3b}$$

where symmetrization is understood for the field products, $T=1/\beta$ is the blackbody vacuum temperature, and $T'=1/\beta'$ is the body temperature. Here, $\boldsymbol{\Gamma}$ is taken to be the usual vacuum retarded Green's dyadic, which at coincident points is (rotationally averaged)

$$\boldsymbol{\Gamma}(\mathbf{r}-\mathbf{r}';\omega)\to\mathbf{1}\left(\frac{\omega^2}{6\pi R}+i\frac{\omega^3}{6\pi}+O(R)\right),\quad R=|\mathbf{r}-\mathbf{r}'|\to 0, \tag{4.4}$$

while χ^A is the anti-Hermitian part of the electric susceptibility:

$$\chi^A=\frac{1}{2i}(\chi-\chi^\dagger),\qquad \Im\chi^A_{ij}=-\frac{1}{2}\Re[\chi_{ij}(\omega)-\chi_{ji}(\omega)], \tag{4.5}$$

where we recognize that only the antisymmetric part of χ^A, which is even in ω, contributes to the torque. The real part of χ^A is symmetric in indices and odd in ω. ($\Im\chi^A=0$ for a reciprocal body).

Using this, we readily calculate [26,27] the torque on a nonreciprocal body in vacuum, due to PP and EE fluctuations, which arises only from the internal torque,

$$\tau_i=-\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{\omega^3}{6\pi}\left[\coth\frac{\beta'\omega}{2}-\coth\frac{\beta\omega}{2}\right]\epsilon_{ijk}\Re\alpha_{jk}(\omega),$$
$$\alpha_{jk}(\omega)=\int(d\mathbf{r})\chi_{jk}(r;\omega). \tag{4.6}$$

The system must be out of thermal equilibrium for a torque to occur. This result exactly agrees with that of Refs. [28,29]. Closely related is the observation that an external magnetic field may result in such a torque on a stationary spherical particle [30], which makes explicit the nonreciprocity arising from the applied field [26,27]. However, there is no quantum vacuum force in this static situation. A nonreciprocal medium typically requires an external magnetic field. If the nonreciprocity is generated by $B=1$ T, the corresponding torque for a gold ball of radius 100 nm is $\sim 10^{-24}$ N m, for $T'/T=2$.

A topological insulating film in a magnetic field can also experience a nonequilibrium torque [31]. The angular momentum flux radiated by a magneto-optical nanoparticle, an example of a nonreciprocal body, was discussed in Ref. [32]. Forces and torques can also be induced on a particle or body in the near field of a surface made nonreciprocal by the presence of a magnetic field [33,34].

5. Self-propulsive force

The Lorentz force on a dielectric body is [25]

$$\mathbf{F}=\int(d\mathbf{r})\int\frac{d\omega}{2\pi}\frac{d\nu}{2\pi}e^{-i(\omega+\nu)t}\left\{-\left(1+\frac{\omega}{\nu}\right)[\boldsymbol{\nabla}\cdot\mathbf{P}(\mathbf{r};\omega)]\,\mathbf{E}(\mathbf{r};\nu)-\frac{\omega}{\nu}\mathbf{P}(\mathbf{r};\omega)\cdot(\boldsymbol{\nabla})\cdot\mathbf{E}(\mathbf{r};\nu)\right\}. \tag{5.1}$$

Now we expand the fields out to second order (4th order in generalized susceptibilities, χ and $\boldsymbol{\Gamma}$, upon use of FDT), by iterating the expansion in Eq. (4.2). Then using the symmetries of the integrand we find the general expression for the force on a body composed of isotropic material. It is evident that, in this order, there is no force on a homogeneous body. So, we consider the simplest example of an inhomogeneous body, one composed of two homogeneous parts, A and B, as shown in Fig. 3(a). Then, the force reduces to [35]

$$F_z=8\int_0^\infty\frac{d\omega}{2\pi}X_{AB}(\omega)I_{AB}(\omega)\left[\frac{1}{e^{\beta\omega}-1}-\frac{1}{e^{\beta'\omega}-1}\right], \tag{5.2}$$

where the geometric integral is ($\mathbf{R}=\mathbf{r}-\mathbf{r}'$, $\tilde{R}=\omega R$)

$$I_{AB}(\omega)=\int_A(d\mathbf{r})\int_B(d\mathbf{r}')\frac{1}{2}\nabla_z\left[\Im\Gamma_{ji}(\mathbf{r}'-\mathbf{r};\omega)\Im\Gamma_{ij}(\mathbf{r}-\mathbf{r}';\omega)\right]$$
$$=\int_A(d\mathbf{r})\int_B(d\mathbf{r}')\frac{1}{(4\pi)^2}\frac{R_z}{R^8}\phi(\tilde{R}), \tag{5.3}$$

in terms of the function ϕ,

$$\phi(\tilde{R})=-9-2\tilde{R}^2-\tilde{R}^4+(9-16\tilde{R}^2+3\tilde{R}^4)\cos 2\tilde{R}+\tilde{R}(18-8\tilde{R}^2+\tilde{R}^4)\sin 2\tilde{R};\qquad \phi(\tilde{R})\sim-\frac{4}{9}\tilde{R}^8+\frac{28}{225}\tilde{R}^{10}+\cdots,\quad \tilde{R}\ll 1. \tag{5.4}$$

The susceptibility product is

$$X_{AB}(\omega)=\Im\chi_A(\omega)\,\Re\chi_B(\omega)-\Re\chi_A(\omega)\,\Im\chi_B(\omega). \tag{5.5}$$

In order to apply our weak-susceptibility approximation to a metal, described by the Drude model,

$$\chi(\omega)=-\frac{\omega_p^2}{\omega(\omega+i\nu)}, \tag{5.6}$$

where ω_p is the plasma frequency, and ν ia the damping frequency, we will assume that the object is no thicker than the skin depth, which is approximately given by [25]

$$\delta=(\omega\sigma/2)^{-1/2}=(\omega^2\Im\chi/2)^{-1/2}=\sqrt{\frac{2(\omega^2+\nu^2)}{\omega\omega_p^2\nu}}\sim 50\,\mathrm{nm}, \tag{5.7}$$

putting in parameter values for gold.

As an example, consider a Janus ball, of radius a, with one half being a dispersionless dielectric, and the other half a Drude metal. In that case, the geometric integral I_{AB} is shown in Fig. 4. The spontaneous force on a small Janus ball (A being a dispersionless dielectric and B gold) is

$$F^{\mathrm{JB}}=\frac{1}{27\pi}\chi_A\omega_p^2(\nu a)^7\hat{F}\approx 4\times 10^{-25}\chi_A\hat{F}\,\mathrm{N},\quad \hat{F}=f_7(\beta\nu)-f_7(\beta'\nu), \tag{5.8}$$

for a ball of radius 100 nm. Here, the dimensionless force $\hat{F}$ is given in terms of

$$f_n(y)\equiv\int_0^\infty dx\frac{x^n}{x^2+1}\frac{1}{e^{yx}-1}. \tag{5.9}$$

$\hat{F}$ is shown as a function of the temperature of the ball in Fig. 5 for $T=300$ K. The spontaneous force on a Janus ball was first considered, perturbatively, in Ref. [37], and nonperturbatively, in Ref. [36].

A considerably larger force might arise on a planar structure, one half being absorbing material and the other being a metal. This was studied first in Ref. [38] and later in Ref. [35].

6. Thermal relaxation leads to terminal velocity

Once the body starts to move due to the nonequilibrium quantum force, it will experience quantum friction, but this is very small. More important than quantum friction is thermalization. Unless some mechanism is provided to maintain the temperature difference, the body will eventually cool or heat to the temperature of the environment. From Newton's law, the terminal velocity is, if we assume an adiabatic change in the temperature,

$$v_T=\frac{1}{m}\int_0^\infty dt\,F(T'(t),T). \tag{6.1}$$

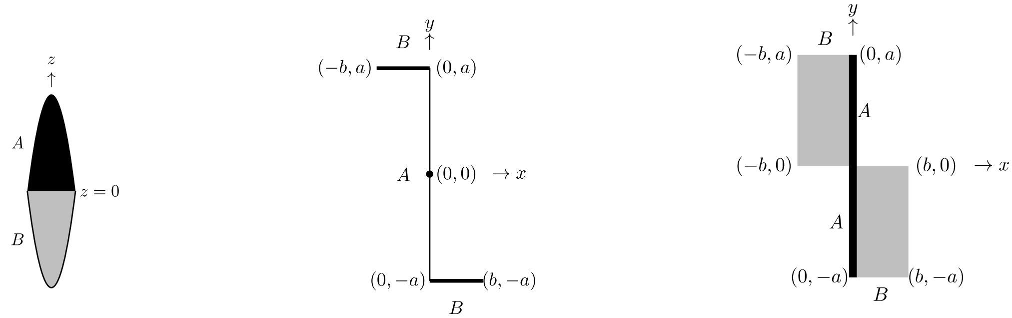

Fig. 3. (a) Generic object with two parts. Axial symmetry is shown for simplicity, so the force is in the z direction. (b) An inhomogeneous wire of small cross section bent in the shape of a dual Allen wrench. The end pieces ("tags") B are taken to be dispersionless dielectric, while the central wire A is a Drude-type metal. The Cartesian coordinates of the various junctions are shown, as is the center of mass. (c) Dual flag, consisting of a metal central wire, with dielectric flags attached antisymmetrically.

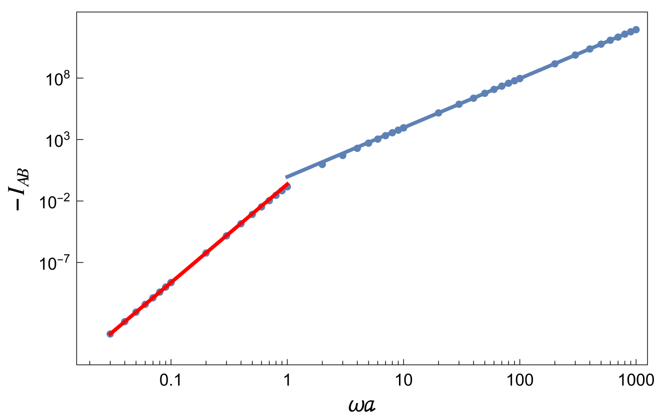

Fig. 4. Geometric integral for a Janus ball, multiplied by $8\pi a$. The lines show the large $\sim (\omega a)^4$ and small $\sim (\omega a)^8$ behaviors.

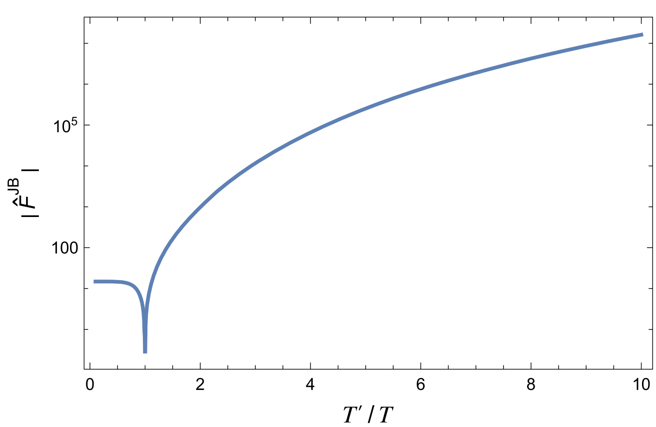

Fig. 5. Force on Janus ball. Results are comparable with those of Ref. [36], but scaling with a is different. The force is negative, that is, toward the metal side, if $T' > T$.

Here the cooling rate is given by, for a slowly moving body,

$$\frac{dQ}{dt} = C_V(T')\frac{dT'}{dt} = P(T',T),$$
$$P(T',T) = \frac{1}{3\pi^2}\int_0^\infty d\omega\,\omega^4\,\mathfrak{I}\,\mathrm{tr}\,\boldsymbol{\alpha}(\omega)\left[\frac{1}{e^{\beta\omega}-1} - \frac{1}{e^{\beta'\omega}-1}\right], \tag{6.2}$$

where $C_V(T')$ is the specific heat of the body at temperature T'. In terms of $u = T'/T$, the time taken for a homogeneous body to cool from temperature T'_0 to T'_1 is, for $T'_0 > T'_1 > T$,

$$t_1 = \int_{T'_0}^{T'_1} dT'\,\frac{C_V(T')}{P(T',T)}, \quad \text{or} \quad \frac{t_1}{t_c} = \int_{u_0}^{u_1}\frac{du}{p(u,T)}, \tag{6.3}$$
$$p(u,T) = f_3(\beta\nu) - f_3(\beta'\nu), \quad t_c = \frac{3\pi^2 nT}{\nu^3\omega_p^2} \sim 10^{-4}\,\mathrm{s},$$

where n is the number density. Here, we've used the weak susceptibility model of a metal, and the assumption that $T \gg \Theta_D$, the Debye temperature. (The latter approximation is well-satisfied for $T = 300$ K.) This means we can write the terminal velocity as

$$v_T = \frac{t_c}{m}\int_{u_0}^{1} du\,\frac{F(u,T)}{p(u,T)}. \tag{6.4}$$

The terminal velocity turns out to be only about 0.1 nm/s for a Janus ball of radius 100 nm, half made of gold and half dielectric, initially twice as hot as the background. Since this seems impossible to observe directly, in the next Section we turn to a more accessible possibility.

7. Spontaneous torque on an inhomogeneous chiral body

In Sec. 4, we calculated the torque in first order, which required the body be composed of nonreciprocal material, which usually necessitates an external field be applied. In second order, a torque can arise for an ordinary (reciprocal) body, but again only if the body is *inhomogeneous*. It must further be *chiral*, in that any mirror reflection cannot be turned into the original object by translations or rotations. For a body with isotropic but inhomogeneous susceptibility there is only an external torque. If the body, again, consists of two homogeneous parts, A and B, the external torque (4.1) yields [39]

$$\boldsymbol{\tau} = \frac{1}{2\pi^2}\int_0^\infty \frac{d\omega}{2\pi} X_{AB}(\omega)\left(\frac{1}{e^{\beta\omega}-1} - \frac{1}{e^{\beta'\omega}-1}\right)\mathbf{J}_{AB}(\omega), \tag{7.1}$$
$$\mathbf{J}_{AB}(\omega) = -\int_A (d\mathbf{r})\int_B (d\mathbf{r}')\frac{\mathbf{r}\times\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^8}\phi(\tilde{R}),$$

where the function ϕ is defined in Eq. (5.4).

We consider a "dual Allen wrench" as illustrated in Fig. 3(b). The object will experience a quantum vacuum torque, perpendicular to its plane, but not a net force, because it is reflection invariant in the center of mass, $\mathbf{r} \to -\mathbf{r}$. The geometric factor is

$$J_{AB}(\omega) = 2S_A S_B \int_{-a}^{a} dy \int_0^b dx\, xy\frac{\phi(\tilde{R})}{\tilde{R}^8}, \tag{7.2}$$
$$\tilde{R} = \omega R, \quad R = \sqrt{x^2 + (a+y)^2}.$$

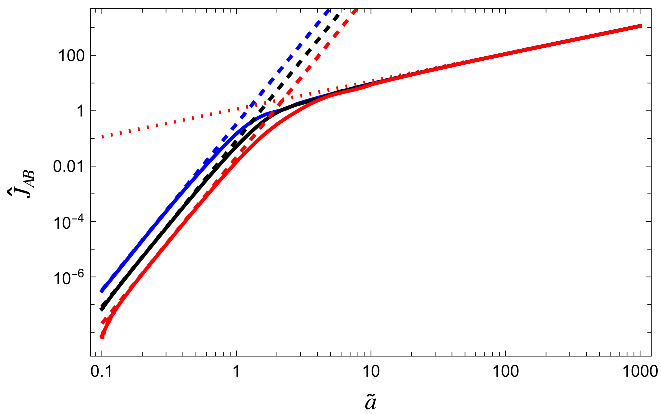

Fig. 6. Geometric factor for dual Allen wrench, $J_{AB} = 2\omega^4 S_A S_B \hat{J}_{AB}$, for different values of $b = a/2$, $b = a$, $b = 2a$ from bottom to top. Here $\tilde{a} = a\omega$.

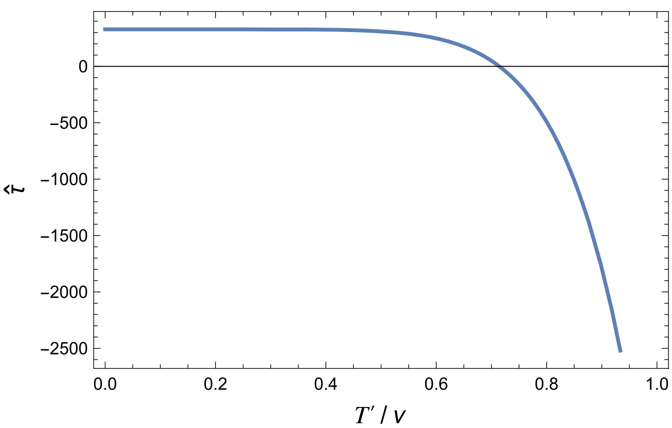

Fig. 7. Dimensionless torque on a small Allen wrench, for $T = 300$ K and dissipation appropriate for gold.

In terms of $\tilde{a} = \omega a$, J_{AB} is shown in Fig. 6, apart from a prefactor of $2\omega^4 S_A S_B$, where S_i is the cross-sectional area of the ith wire. We see saturation in $\tilde{b}$, and linear behavior in $\tilde{a}$ for large $\tilde{a}$, which is easily understood. The interactions between the parts are local, so increasing b beyond a certain point does not increase the torque. The local forces on A are also saturated as a increases, but the lever arm increases linearly. The asymptotic values of $\hat{J}_{AB}$ are

$$\hat{J}_{AB} \sim \frac{11}{30}\pi\omega a, \quad \omega a \gg 1; \qquad \hat{J}_{AB} \sim \frac{56}{675}\omega^6 a^4 b^2, \quad \omega a, \omega b \ll 1. \tag{7.3}$$

Around room temperature, the transition from large to small occurs at a, $b \sim 10$ μm.

The situation is most favorable for a small object. The corresponding torque is

$$\tau = \frac{28}{675\pi^3}\chi_B \nu^9 \omega_p^2 S_A S_B a^4 b^2 \hat{\tau} \equiv \tau_0 \hat{\tau}, \quad \hat{\tau} = f_9(\beta\nu) - f_9(\beta'\nu), \tag{7.4}$$

in terms of the function defined in Eq. (5.9), where $\hat{\tau}$ is shown in Fig. 7. The moment of inertia of the object is

$$I = \rho_A S_A \frac{2}{3} a^3 + \rho_B S_B 2b\left(a^2 + \frac{1}{3}b^2\right), \tag{7.5}$$

so the resulting terminal angular velocity due to thermal cooling is

$$\omega_T = \frac{t_c \tau_0}{I}\hat{\omega}_T, \quad \hat{\omega}_T = \int_{T_0'/T}^{1} du \frac{\hat{\tau}(u,T)}{p(u,T)}, \tag{7.6}$$

where the prefactor $t_c\tau_o/I \sim 2\times 10^{-7}$ s^{-1} for a, b of order 1 μm, with cross-sectional radius about 50 nm. Because the dimensionless torque is large, so is $\hat{\omega}_T \sim 20{,}000$, if the initial temperature of the object is twice that of the room temperature environment, which leads to a large terminal angular velocity $\omega_T \sim 4\times 10^{-3}$ s^{-1}. This should be quite observable. Further enhancement, by a factor of 10 or so, occurs if the tags in the dual Allen wrench are unfurled into flags, as shown in Fig. 3(c). Many more details are supplied in Ref. [39].

So, seeking to observe the torque on a small chiral object seems a promising experimental direction.

8. Conclusions and perspective

After briefly reviewing quantum friction with a surface, or with vacuum blackbody radiation, we turned to consideration of spontaneous torques and forces in vacuum. In first order in electric susceptibility, a vacuum torque, but no force, can arise for a body made of *nonreciprocal* material, if the body is out of equilibrium with the blackbody vacuum environment. In second order, a vacuum force can arise only if the body is *inhomogeneous*, but no exotic material properties are required. A vacuum torque can also arise for ordinary *chiral* bodies in second order, but again only if the body is also *inhomogeneous*. This is in contrast to the nonperturbative findings of Ref. [36], which discrepancy is resolved by considering third-order effects (work in progress). We consider some examples of bodies which should exhibit possibly observable vacuum forces and torques, although cooling (or heating) to equilibrium with the vacuum environment may make it somewhat challenging to observe the resulting linear and angular terminal velocities, but estimates of terminal angular velocities on chiral bodies are promising. For dielectric-metal bodies, the force is toward the metal side, due to the low emissivity and high reflectivity of the metal. The corresponding torque is in the same sense.

Many intriguing theoretical ideas have emerged in the last few years concerning forces and torques exerted on small bodies by exotic surfaces and even by the pervasive blackbody radiation. The thermal vacuum acts as a non-trivial medium, against which a body with suitable anisotropy can move or rotate. This has been a subject totally dominated by theory; however, ideas are now emerging which we hope in the next decade can reveal experimental signatures of these dynamical nonequilibrium Casimir effects.

CRediT authorship contribution statement

Kimball A. Milton: Writing – review & editing, Writing – original draft, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Conceptualization. **Nima Pourtolami:** Writing – review & editing, Methodology, Investigation, Formal analysis, Conceptualization. **Gerard Kennedy:** Writing – review & editing, Writing – original draft, Methodology, Investigation, Formal analysis, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

No data was used for the research described in the article.

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