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Analytic Insights into an Adapted Algorithm for the Score-Based Secretary Problem

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Abstract. In this paper, we study some basic analytic properties of a sequence of functions $\{S_n^{\mu,\sigma}\}$ that is directly derived in an adaptive algorithm originating from the classical score-based secretary problem. More specifically, we show that: 1. the uniqueness of maximum points of the function sequence $\{S_n^{\mu,\sigma}\}$; 2. the maximum point sequence of $\{S_n^{\mu,\sigma}\}$ monotone increases to infinity as n tends to infinity. All of the proofs are elementary but nontrivial.

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Key words: Secretary problem, adaptive algorithm, expected score, uniqueness of maximum points.

1 Introduction

In the classical score-based secretary problem where a decision maker is tasked with interviewing a series of candidates for a position, the goal for the decision maker is to identify and select the most qualified candidate among all the applicants. The selection process is in a sequential manner, where each candidate is interviewed one after the other. During these interviews, the decision maker assesses each candidate and assigns them a numerical score. This score represents the candidate's "value" or suitability for the position, based on factors such as qualifications, experience, and overall impression.

Upon completing an interview, the decision maker faces a critical decision for each candidate: to either accept or reject them. This decision is pivotal because of two key constraints: (1) Irreversibility of rejection: once a candidate is rejected, the decision is

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final. The candidate cannot be recalled or reconsidered at a later stage, regardless of the quality of subsequent candidates. This feature adds a significant level of risk and complexity to the decision-making process. (2) Termination upon acceptance: conversely, if the decision maker chooses to accept a candidate, the interview process is immediately terminated. The selected candidate is deemed the best choice, and no further candidates are considered. These rules create a challenging dilemma for the decision maker. They must strategically balance the risk of rejecting potentially suitable candidates early in the process against the possibility of encountering even better candidates later on. The decision maker must thus employ a judicious combination of evaluation, forecasting, and risk assessment skills to optimize the chances of selecting the best candidate out of the entire pool.

The classical score-based secretary problem is a well-explored area in decision theory with numerous significant results. In [1], the authors discuss many cases and variations. Notably, they highlight that as the number of candidates n approach infinity, the optimal strategy is to skip the first $\frac{n}{e}$ candidates. This approach yields a probability of $\frac{1}{e}$ for selecting the top candidate, focusing solely on the candidates' ranks to maximize the probability of hiring the best one. In [2], the authors provide a comprehensive overview of the secretary problem's origins, tracing its conceptual evolution. Meanwhile, Freij and Wastlund [3] make a significant advancement by demonstrating the existence of a universal algorithm applicable to any poset, guaranteeing success with a probability of at least $\frac{1}{e}$. Preater introduces in [4] an intriguing generalization of the problem, proposing an algorithm effective across all poset sets of a given size with a positive success probability. This expansion of the problem scope adds depth to its applicability in decision-making scenarios. In an interesting twist, Bearden [5] examines the scenario where the candidate data set follows a uniform distribution. In such cases, if the decision maker prioritizes the expected value of the selected candidate, an optimal policy emerges. This strategy involves skipping the first $\sqrt{n}-1$ candidates, then selecting the next candidate who ranks highest. Most recently, Sarkar [13] considers a variant of the secretary problem, in which the employer also learns the scores of the already interviewed candidates, when making the decision after the n-th interview is over.

The field has seen other notable contributions as well. Kozik [9] introduces a dynamic threshold strategy, establishing its success probability at a minimum of n/4. Kleinberg [8] examines a unique variation where the algorithm permits selecting multiple candidates, aiming to maximize expected profit. Additionally, Korula and Pál [10] explore the problem in the context of selecting elements from specially designed graphs or hypergraphs, expanding the problem's application to more complex structures.

As a well-known best-choice problem in decision theory, the secretary problem has seen a wide range of applications to real-world situations. These applications extend to various domains, such as the house-selling problem [6, 11], dynamic and stochastic knapsack problems [7], online auction strategies [8], and online matching problems, particularly in the context of internet advertising reservation systems [10].

A recent study by Zhou et al. (2021) [12] explores a novel variation of the secretary problem, where the aim is to maximize the expected value of the chosen candidate. This variant introduces a significant change in the candidate evaluation process. Unlike the traditional approach, which relies on ranking candidates, this method involves assigning a numerical score during the candidate's evaluation or "interview". This scoring system provides a more quantitative assessment of each candidate's suitability. One of the key contributions of Zhou et al.'s work is the development of an "adaptive algorithm". To the best of our knowledge, this is the first algorithm that allows a decision maker to set an expected score for the selected candidate based on the total number of candidates. This feature of the algorithm makes it uniquely adaptable and responsive to varying candidate pools.

The practical implications of the adaptive algorithm are significant, particularly for organizations. With this tool, companies can better strategize their recruitment processes, especially when they have a reasonable estimate of the number of applicants they expect for a job position over time. The algorithm's ability to adjust expectations and strategies based on applicant volume offers a more dynamic and effective approach to candidate selection, aligning with the evolving needs of modern hiring practices.

1.1 Statement of score-based secretary problem

We assume a decision maker interviews n candidates whose values are independent and identically distributed (I.I.D.) random variables $X_1,X_2,\cdots,X_n$, obeying normal distribution with known mean μ and variance σ. The decision maker has two choices for each candidate: accept or reject. Once the decision maker accepts one candidate, the interview terminates. The score-based secretary problem aims to help the decision maker obtain the expected value of the candidate selected as high as possible. Assume x is a benchmark score, then we denote by $S_n^{\mu,\sigma}(x)$ the expected score given the decision-making rule. It is important to note that our score-based secretary problem differs from the original secretary problem as presented in [1]. In the original problem, there is no predefined benchmark score x. Instead, their theory treats the expected score from interviewing $n-1$ candidates as the benchmark score for deciding the expected score from interviewing n candidates. As a result, [1] established a recursive relation between the expected scores for interviewing $n-1$ and n candidates. In contrast, our model presents a simpler form for the expected score, as detailed in Lemma 1.1 below.

Let $p(s)$ be the normal density function with mean $\mu \geq 0$ and variance $\sigma \geq 0$:

$$p(s)=\frac{1}{\sqrt{2\pi}\sigma}e^{\frac{(s-\mu)^2}{2\sigma^2}}. \tag{1.1}$$

We further denote

$$F(x)=\int_{-\infty}^{x} p(s)\mathrm{d}s, \quad E(x)=\int_{-\infty}^{x} sp(s)\mathrm{d}s. \tag{1.2}$$

We stress that the assumption of each candidate's score being satisfied by normal distribution is reasonable, then the expected score $S_n^{\mu,\sigma}(x)$ is given in the following lemma.

Lemma 1.1. *Given the decision-making rule above for the n candidates, the expected score* $S_n^{\mu,\sigma}(x)$ *is given as follows [12],*

$$S_n^{\mu,\sigma}(x)=\left[1-F(x)^{n-1}\right]\frac{\mu-E(x)}{1-F(x)}+F(x)^{n-1}\mu. \tag{1.3}$$

Proof. By applying the decision-making rule for n candidates, we consider two scenarios. In the first scenario, the scores of the first $n-1$ candidates are all lower than x. Given that the probability of a candidate's score being less than the benchmark score x is $F(x)$, and assuming that all candidates are I.I.D., the probability that the scores of the first $n-1$ candidates are all less than x is $F(x)^{n-1}$. In this scenario, the first $n-1$ candidates will be rejected, and the n-th candidate will be selected. Consequently, the expected score is the mean μ of the normal random variable X_n, conditioned on the scores of the first $n-1$ candidates being less than x. In the second scenario, at least one of the first $n-1$ candidates has a score greater than x. The probability of this event is $1-F(x)^{n-1}$. The expected score in this case should be the average value of a normal random variable X_i, for some $i=1,\cdots,n-1$, conditioned on at least one of the first $n-1$ candidates having a score greater than x. More specifically, the expected score in the second scenario is the average score conditioned on the i-th candidate's score being greater than x, which is given as

$$\frac{\int_x^\infty sp(s)\mathrm{d}s}{\int_x^\infty p(s)\mathrm{d}s}=\frac{\mu-E(x)}{1-F(x)}.$$

These two expected scores for the two cases must be weighted by their probabilities $F(x)^{n-1}$ and $1-F(x)^{n-1}$, respectively, to give the expected score $S_n^{\mu,\sigma}(x)$ [12]:

$$S_n^{\mu,\sigma}(x)=\left[1-F(x)^{n-1}\right]\frac{\mu-E(x)}{1-F(x)}+F(x)^{n-1}\mu, \tag{1.4}$$

which leads to the desired result. □

Therefore, the object of finding an optimal strategy for the score-based secretary problem is to maximize the expected score:

$$\max_x S_n^{\mu,\sigma}(x). \tag{1.5}$$

In [12], the authors show that $S_n^{\mu,\sigma}$ has the following simple properties:

- $S_n^{\mu,\sigma}$ has an alternative form which is equivalent to (1.4):

$$S_n^{\mu,\sigma}(x)=\mu+\sigma p(x)+\sum_{j=0}^{n-2}F(x)^j. \tag{1.6}$$

- $S_n^{\mu,\sigma}$ is nonnegative and has asymptotic limits

$$\lim_{x\to\pm\infty}S_n^{\mu,\sigma}(x)=\mu. \tag{1.7}$$

- There is a one-to-one correspondence between the maximizers of $S_n^{\mu,\sigma}$ and those of $S_n^{0,1}$:
$$(x_n^{\mu,\sigma})^* = \mu + \sigma (x_n^{0,1})^*, \tag{1.8}$$
where $(x_n^{\mu,\sigma})^*$ is a maximizer of $S_n^{\mu,\sigma}$, and $(x_n^{0,1})^*$ is the maximizer of $S_n^{0,1}$ that corresponds to $(x_n^{\mu,\sigma})^*$.

1.2 Main result

In [12], the authors propose an adaptive algorithm for finding the optimal stopping rule to maximize the expected score of the chosen candidate. However, the implementation of this algorithm depends on a critical assumption: for each $n \in \mathbb{N}$, the function $S_n^{\mu,\sigma}$ possesses a unique maximizer. While this assumption appears valid based on numerical verification, it notably lacks a formal mathematical proof.

The goal of this paper is to validate the aforementioned assumption. Specifically, we aim to provide a formal proof of the uniqueness of the minimizers for the function $S_n^{\mu,\sigma}$ for each $n \in \mathbb{N}$. Our approach to this proof is elementary, yet highly nontrival. We summarize it in the following theorem:

Theorem 1.1. *For $x \in \mathbb{R}$, let*
$$p(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, \qquad f(x) = \int_{-\infty}^{x} p(t)\,\mathrm{d}t. \tag{1.9}$$
Then for any $n \in \mathbb{N}$, the function
$$F_n(x) = p(x)\left[1 + f(x) + f^2(x) + \cdots + f^n(x)\right] \tag{1.10}$$
has a unique maximum point in $\mathbb{R}$.

Remark 1.1. It is noted that in this theorem we simply consider the case where $(\mu,\sigma) = (0,1)$. The extension to general case is trivial due to (1.8).

Remark 1.2. While the scores of candidates can be influenced by various factors such as diverse skills, knowledge areas, and question types, etc, we assume that the interview questions are designed and structured to differentiate among candidates of varying ability levels, leading to a spread of scores that can resemble a normal distribution. Therefore, in this paper, we fix the assumption of normal distribution for each candidate's score. On the other hand, the expected score formulation (1.4) and existence and uniqueness of the maximizer of the expected score still hold for some other distribution such as uniform distribution. The proof of the main result for case of uniform distribution is rather trivial since $p(f), f(x), F_n(x)$ al have simple expressions. It is the normal distribution case that make the main result highly challenging.

In the rest of the paper, we will prove Theorem 1.1 in Section 2. Besides, as a byproduct of Theorem 1.1, we show that the sequence of unique maximizers, denoted by $\{x_n^*\}_{n>0}$ in the rest of the paper for simplicity, monotone increases to ∞.

2 Proof of the main result

We first introduce two technical lemmas, which are the essence of the proof of the main result.

Lemma 2.1. *For any* $x \geq 0$ *and* $n \in \mathbb{N}$*, it holds*

$$2\left[f(x)+f^2(x)+\cdots+f^n(x)\right]-nf^{n+1}(x)<n. \tag{2.1}$$

Proof. Let us define

$$H_n(x)\stackrel{\text{def}}{=}2\left[f(x)+f^2(x)+\cdots+f^n(x)\right]-nf^{n+1}(x)-n, \qquad x\geq 0.$$

Then the following facts hold

$$\begin{aligned}
&H_n(0)=-\frac{1}{4}<0, \qquad \text{for } n=1,\\
&H_n(0)=2\left(\frac{1}{2}+\frac{1}{2^2}+\cdots+\frac{1}{2^n}\right)-\frac{n}{2^{n+1}}-n<2-\frac{n}{2^{n+1}}-n<0, \qquad \text{for } n\geq 2,\\
&\lim_{x\to+\infty} H_n(x)=2n-n-n=0.
\end{aligned}$$

Meanwhile, note that for any $x>0$,

$$\begin{aligned}
H_n'(x)&=2p(x)\left[1+2f(x)+\cdots+nf^{n-1}(x)\right]-n(n+1)p(x)f^n(x)\\
&=p(x)\left[2(1+2f+\cdots+nf^{n-1})-n(n+1)f^n\right]\\
&>p(x)\left[2(f^n+2f^n+\cdots+nf^n)-n(n+1)f^n\right]\\
&=p(x)f^n(x)\left[2(1+2+\cdots+n)-n(n+1)\right]\\
&=0,
\end{aligned}$$

which implies that $H_n(x)$ is strictly monotone increasing on $(0,+\infty)$. Therefore, inequality (2.1) is verified. □

Lemma 2.2. *For any* $n \in \mathbb{N}$*, there exists a unique* $y_n>0$*, such that*

$$p(y_n)=\frac{2y_nf(y_n)}{n}. \tag{2.2}$$

Further, it holds

$$p(x)>\frac{2xf(x)}{n}, \qquad \forall 0<x<y_n, \tag{2.3}$$

$$p(x)<\frac{2xf(x)}{n}, \qquad \forall x>y_n. \tag{2.4}$$

Proof. Let

$$K_n(x) \overset{\text{def}}{=} p(x) - \frac{2xf(x)}{n}, \qquad \text{for } x \geq 0.$$

Then it is easy to see that

$$K_n(0) = \frac{1}{\sqrt{2\pi}} > 0, \lim_{x\to+\infty} K_n(x) = -\infty,$$
$$K_n'(x) = -xp(x) - \frac{2f(x)}{n} - \frac{2xp(x)}{n} < 0, \qquad \forall x > 0.$$

Hence $K_n(x)$ is strictly monotone decreasing on $(0,+\infty)$. In all, for each $n \in \mathbb{N}$, there exists a unique $y_n > 0$ satisfying (2.2), (2.3), (2.4). □

Now we are ready to prove Theorem 1.1.

Proof of Theorem 1.1. Firstly, since $p(x)$ and $1+f(x)+\cdots+f^n(x)$ are positive and strictly monotone increasing on $(-\infty,0)$, there will be no critical point on this interval. Next we calculate

$$\begin{aligned}
F_n'(x) &= -xp(x)\left[1+f(x)+\cdots f^n(x)\right] + p^2(x)\left[1+2f(x)+\cdots nf^{n-1}(x)\right] \\
&= -xp(x)\left[\frac{1-f^{n+1}(x)}{1-f(x)}\right] + p^2(x)\left[(1+f+\cdots+f^{n-1})+\cdots+(f^{n-2}+f^{n-1})+f^{n-1}\right] \\
&= -xp(x)\left[\frac{1-f^{n+1}(x)}{1-f(x)}\right] + \frac{p^2(x)}{1-f}\left[(1-f^n)+f(1-f^{n-1})+\cdots+f^{n-1}(1-f)\right] \\
&= -xp(x)\left[\frac{1-f^{n+1}(x)}{1-f(x)}\right] + \frac{p^2(x)}{1-f}\left(1+f+\cdots+f^{n-1}-nf^n\right) \\
&= -xp(x)\left[\frac{1-f^{n+1}(x)}{1-f(x)}\right] + \frac{p^2(x)}{(1-f)^2}\left[1-f^n-n(1-f)f^n\right] \\
&= \frac{p(x)}{(1-f)^2}\Big\{-x(1-f)(1-f^{n+1}) + p(x)\left[1-f^n-n(1-f)f^n\right]\Big\}.
\end{aligned}$$

Our goal is to show that $F_n'(x)$ has a unique zero over $(0,+\infty)$ such that $F_n(x)$ has a unique maximum point over $(0,\infty)$. To this end, we define

$$g_n(x) \overset{\text{def}}{=} -x\left[1-f(x)\right]\left[1-f^{n+1}(x)\right] + p(x)\Big\{1-f^n(x)-n\left[1-f(x)\right]f^n(x)\Big\}, \qquad x \geq 0.$$

Then,

$$g_n(0) = \frac{1}{\sqrt{2\pi}}\left(1-\frac{2+n}{2^{n+1}}\right) > 0.$$

Meanwhile, given y_n defined in Lemma 2.2 and $\forall x \geq y_n$, we have

$$\begin{aligned} g_n(x) &= (1-f)\Big[-x(1-f^{n+1}) + p(x)\big(\underbrace{1+f+\cdots+f^{n-1}-nf^n}_{>0}\big)\Big] \\ &\leq (1-f)\Big[-x(1-f^{n+1}) + \frac{2xf}{n}(1+f+\cdots+f^{n-1}-nf^n)\Big] \qquad \text{by inequality (2.4)} \\ &= (1-f)x\Big[-1+\frac{2}{n}(f+f^2+\cdots+f^n)-f^{n+1}\Big]<0, \qquad \text{by inequality (2.1).} \end{aligned}$$

It remains to prove $g_n(x)$ is strictly monotone decreasing on $(0,y_n)$. To this end we check

$$\begin{aligned} g_n'(x) &= -(1-f)(1-f^{n+1}) + xp(x)(1-f^{n+1}) + (n+1)xp(x)f^n(1-f) \\ &\quad - xp(x)\left[1-f^n-n(1-f)f^n\right] - p^2(x)\left[n(n+1)f^{n-1}-n(n+1)f^n\right] \\ &= (1-f)\Big\{-(1-f^{n+1}) + xp(x)(1+f+\cdots+f^n) + (n+1)xp(x)f^n \\ &\qquad - xp(x)\left[(1+f+\cdots+f^{n-1})-nf^n\right] - n(n+1)p^2(x)f^{n-1}\Big\} \\ &= (1-f)\big[\underbrace{-(1-f^{n+1})}_{<0} + 2(n+1)xp(x)f^n - n(n+1)p^2(x)f^{n-1}\big] \\ &< (1-f)n(n+1)p(x)f^{n-1}\left[\frac{2xf(x)}{n}-p(x)\right]<0, \qquad \forall x\in(0,y_n), \end{aligned}$$

where we use inequality (2.3) to obtain the last inequality. Therefore, for any $n\in\mathbb{N}$, $g_n(x)$ has a unique zero, denoted by $x_n^*\in(0,+\infty)$, so does $F_n'(x)$. Note that

$$\lim_{x\to-\infty} F_n(x) = \lim_{x\to+\infty} F_n(x) = 0,$$

hence x_n^* is the unique maximum point of $F_n(x)$. The proof is complete. □

For the maximizing sequence $\{x_n^*\}$ established in the proof of Theorem 1.1 above, we can further reveal that it is a sequence monotone increasing to ∞, which is summarized in the following theorem.

Theorem 2.1. *For each $n\in\mathbb{N}$, we denote x_n^* the unique maximum point of the function $F_n(x)$ defined in* (1.10). *Then*

$$x_n^* < x_{n+1}^*, \quad \lim_{n\to\infty} x_n^* = +\infty. \tag{2.5}$$

Proof. To begin with, we show that the sequence $\{x_n^*\}$ is strictly monotone increasing. Since $g_n(x_n^*)=0$, $0<x_n^*<y_n$ for each $n\in\mathbb{N}$, together with Lemma 2.2, one can further derive

$$\begin{aligned} g_{n+1}(x_n^*) &= \left[1-f(x_n^*)\right]\Big\{g_n(x_n^*) - x_n^* f^{n+1}(x_n^*)\left[1-f(x_n^*)\right] + (n+1)p(x_n^*)f^n(x_n^*)\left[1-f(x_n^*)\right]\Big\} \\ &= f^n(x_n^*)\left[1-f(x_n^*)\right]^2\left[(n+1)p(x_n^*) - x_n^* f(x_n^*)\right] \end{aligned}$$

$$\geq f^n(x_n^*)\left[1-f(x_n^*)\right]^2\left[\frac{2(n+1)}{n}x_n^*f(x_n^*)-x_n^*f(x_n^*)\right]>0.$$

Meanwhile, it follows from the proof of Theorem 1.1 that

$$\begin{cases} g_{n+1}(x)>0, & \forall 0<x<x_{n+1}^*, \\ g_{n+1}(x)<0, & \forall x>x_{n+1}^*. \end{cases}$$

Therefore, we conclude $x_n^*<x_{n+1}^*$ for each $n\in\mathbb{N}$.

Now we prove the unboundedness of $\{x_n^*\}$. Suppose the sequence $\{x_n^*\}$ is bounded above, that is, $\exists M>0$ such that

$$x_n^*\leq M, \quad \forall n\in\mathbb{N}.$$

Since x_n^* is the maximum point for $F_n(\cdot)$, we have that

$$F_n(x)\leq F_n(x_n^*)=p(x_n^*)\frac{1-f^{n+1}(x_n^*)}{1-f(x_n^*)}<\frac{p(0)}{1-f(M)}=\frac{1}{\sqrt{2\pi}\left[1-f(M)\right]}\doteq L, \quad \forall x\in\mathbb{R},\forall n\in\mathbb{N}. \tag{2.6}$$

On the other hand, note that

$$\lim_{x\to\infty}\lim_{n\to\infty}F_n(x)=\lim_{x\to\infty}\frac{p(x)}{1-f(x)}=\lim_{x\to\infty}\frac{-xp(x)}{-p(x)}=+\infty,$$

hence there exists $\tilde{x}>0$, such that

$$\lim_{n\to\infty}F_n(\tilde{x})>2L,$$

which is in contradiction with (2.6). Therefore, $\{x_n^*\}$ is monotone increasing but unbounded. The proof is complete. □

Remark 2.1. It remains to be a challenging problem to estimate the growth rate of $\{x_n^*\}$ as $n\to\infty$.

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