Report of the SIAM Quantum Intersections Convening

Integrating Mathematical Scientists into Quantum Research

October 7-9, 2024 | Tysons, Virginia

Society for Industrial and Applied Mathematics

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SIAM Quantum Intersections Convening

Quantum Information Science has been a priority of the federal government of the United States of America for several years and the field has advanced dramatically since the start of the National Quantum Initiative in 2018.

As quantum information science (QIS) advances, mathematical challenges and opportunities have become clearer and more urgent around issues such as quantum algorithms, cryptography, networking, simulation, dynamics, metrology, information theory, communications, and other areas. However, despite these research needs, the mathematical sciences community has largely been absent from quantum research save a few pockets of activity. Many in the mathematics community express interest in quantum and there is a clear need to provide education and training, workforce pathways, funding, and networking opportunities to bring more mathematicians into the QIS research community and address critical research needs.

The goals of the convening were to:

(i) make more mathematical scientists aware of the demand for their expertise in quantum research and articulate areas and problems where they can contribute

(ii) increase the participation of researchers in mathematical sciences in the quantum information science revolution to accelerate its research and development

(iii) provide a seeding ground for partnerships and collaborations of mathematical scientists with physicists, computer scientists, and engineers from industry and academia

(iv) recommend activities to develop a quantum science and technology workforce pipeline in the mathematical and computational sciences

With funding from the United States National Science Foundation under award DMS-2425995, the Society for Industrial and Applied Mathematics (SIAM) hosted a 3-day interactive workshop on October 7-9, 2024 in Tysons, Virginia that brought quantum-curious mathematical scientists together with leading experts in quantum science. The *SIAM Quantum Intersections Convening (QIC)* was designed to foster and increase the involvement and visibility of mathematicians and statisticians in quantum science research and education. Recognizing the critical role of mathematical scientists, this convening aimed to promote multidisciplinary collaborations that bridge the gap between mathematics and quantum sciences. The convening also developed recommendations for NSF and other federal research and development agencies to better foster mathematical sciences engagement in QIS research.

At the convening, many areas of QIS research were discussed where mathematics can help research and discovery including quantum computing, quantum algorithms, quantum optimization, quantum error corrections, quantum information theory, quantum cryptography, quantum sensing and metrology, and quantum networks. Additionally, recommendations surfaced in areas including education and workforce, funding and networking, publications, and infrastructure.

There were many ideas brought forth by participants at the workshop. After discussion and a process to identify the most promising, **the convening ultimately resulted in four broad recommendations, including:**

01 Support critical research and development in key areas at the intersections of QIS and mathematics:

- Mathematical foundations of quantum algorithms and applications
- Noisy intermediate scale quantum devices and noise
- Data science, machine learning and optimization to advance quantum computing
- Mathematics to advance quantum networks
- Mathematics to advance quantum sensing and learning theory

02 Increase funding and create additional networking mechanisms to build mathematics QIS capacity and enhance collaboration and involvement of the mathematics community in QIS research and development:

- Develop core research funding, programs focused on building capacity in the mathematics community, and programs focused on expanding the number and types of institutions in QIS
- Support for workshops and other networking activities to engage mathematics researchers in QIS communities
- Create collaborative funding mechanisms to encourage QIS convergent research including support for small and mid-sized interdisciplinary teams and encouraging cross-sector partnerships between academia, industry, and federal laboratories
- Expand accessibility of QIS infrastructure through better sharing of information and investing in new resources and assets

03 Enhance QIS education and workforce development efforts to better prepare students and create workforce pathways:

- Support for curricular advancements to support QIS relevant topics and math preparation such as related to linear algebra in the K-12 and undergraduate domains
- Create a new quantum education and workforce hub to coordinate education, workforce, and outreach efforts across the QIS community
- Expand internships and other workforce opportunities for students to engage in and prepare for QIS related careers
- Create marketing and public engagement materials to expand awareness of QIS topics to the general public

04 Work with SIAM and other mathematics societies to deepen the math-centered quantum science community

- Include more QIS-related content in SIAM and other mathematical publications
- Deepen the QIS math community by creating a new SIAM activity group in QIS or expanding QIS-focused sessions at mathematics community conferences
- Create mechanisms to share relevant quantum challenges among the math community

In the next sections of this report, these recommendations are described in more detail. The greater SIAM community looks forward to working with the National Science Foundation and other federal agencies and partners to help implement these recommendations.

Steering Committee

The workshop was planned and managed by a steering committee:

Di Fang
Assistant Professor, Mathematics
Duke University

Lior Horesh
Senior Manager, Mathematics & Theoretical Computer Science
IBM Research

David Hyde
Assistant Professor, Computer Science
Vanderbilt University

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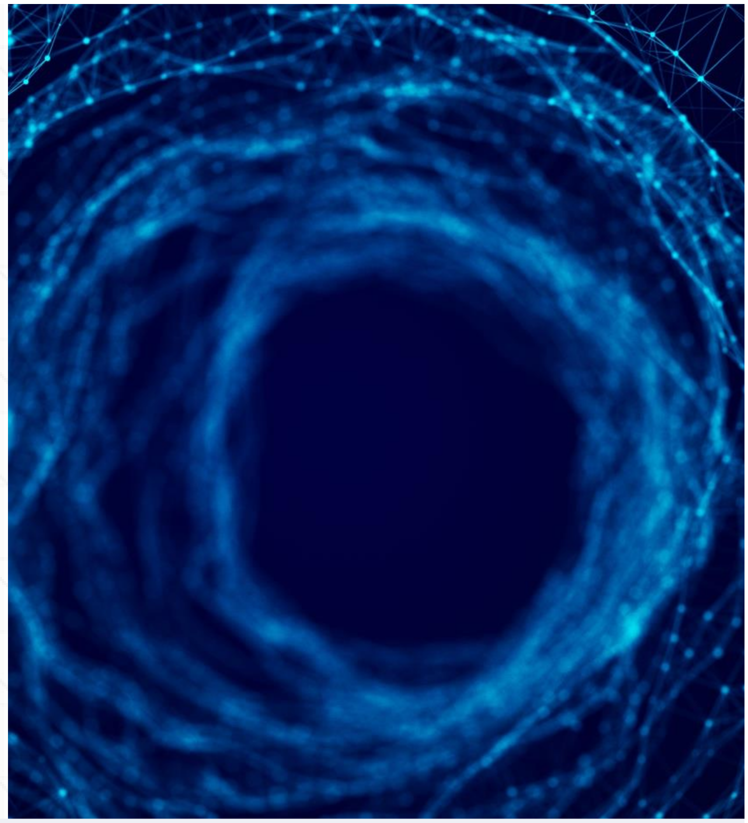
Assistant Professor, Industrial & Systems Engineering
University of Southern California

Alex Pothen
Professor, Computer Science
Purdue University

Suzanne Weekes
Chief Executive Officer
SIAM
(Principal Investigator DMS 2425995)

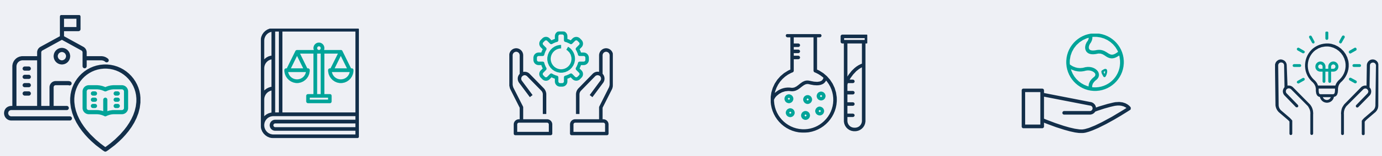
A liaison from Knowinnovation (KI) supported the design and planning process and, for the workshop itself, KI provided a team of professionals who specialize in guiding and accelerating academic, scientific, and interdisciplinary innovation to facilitate the discussions and the generation of recommendation ideas. Additional facilitation support was provided by SIAM's government relations partners Lewis-Burke Associates LLC.

This report was written by the steering committee along with Kirk Jordan, IBM Distinguished Engineer Emeritus; Jillian Kunze, Associate Editor, *SIAM News*, SIAM; and Miriam Quintal, Managing Principal, Lewis-Burke Associates LLC.



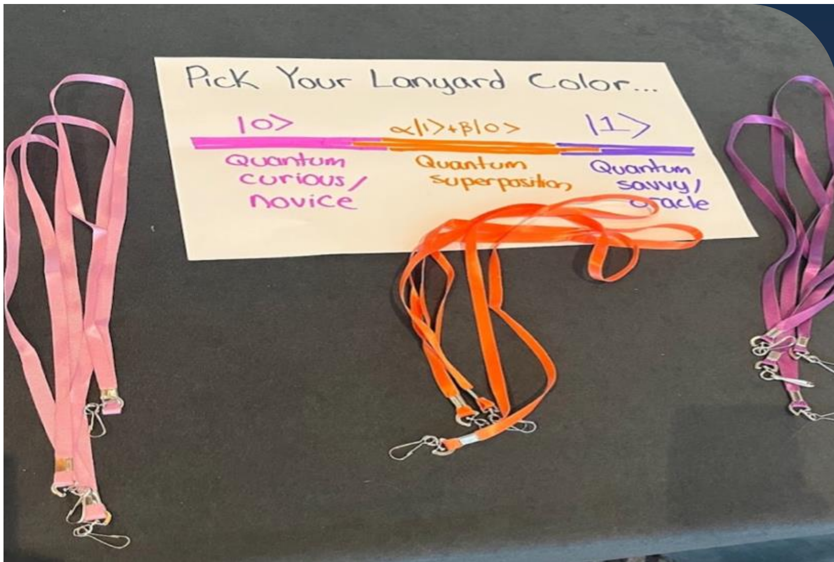
Workshop Participants and Structure

There were over 80 attendees at the three-day SIAM Quantum Intersections Convening. The participants ranged from quantum-curious mathematicians to quantum experts with backgrounds in chemistry, computer science, engineering physics, and mathematics. Half of the participants work in academia, 13% in government, 14% in the private sector, 11% in national laboratories, 2% in the non-profit sector, and 10% were students. Just over one-third of the participants were early career professionals, 22% were mid-career, 31% were senior, and 10% were students. Convening participants came from 27 states and Washington, DC with one third of the states represented being EPSCOR states.



Quantum science experts from various subdomains were *invited* to the Quantum Intersections Convening. These invitees took center stage to give broad overviews of their field including remarks and slides on:

- The future of research in the field
- Thoughts on how mathematics is involved and needed
- Key challenges and gaps encountered, and what is needed to advance the field
- How to best foster partnerships and collaborations to drive progress



Attendees of the SIAM Quantum Intersections Convening ranged from quantum science experts to quantum-curious novices. Photo Courtesy of SIAM.

The list of topics and the featured speakers were as follows:

Quantum Science Needs Mathematicians

Bert de Jong, Senior Scientist, Department Head, Computational Science Department, Applied Computing for Scientific Discovery, Lawrence Berkeley National Laboratory

The National Quantum Initiative

Jacob Taylor, Senior Advisor for Critical and Emerging Technologies, National Institute of Standards and Technology

Federal Outlook and Opportunities for Quantum Research and Workforce Development

Miriam Quintal, Managing Principal, Lewis-Burke Associates LLC

Quantum Algorithms Teaser

Jeffrey Larson, Computational Mathematician, Argonne National Laboratory

Quantum Error Corrections

Michael Perlin, Vice President - Quantum Error Correction, JPMorgan Chase

Quantum Cryptography

Qipeng Liu, Assistant Professor, University of California, San Diego

Post Quantum Cryptography

Angela Robinson, Mathematician, National Institute of Standards and Technology

Quantum Science – Hope, Hype, Hamiltonians, and Homology Panel Discussion

Lior Horesh (Moderator), Senior Manager, Mathematics and Theoretical Computer Science, IBM Research
Di Fang, Assistant Professor, Mathematics, Duke University
Marcos Crichigno, Senior Quantum Algorithms Scientist, Phasecraft
Alex Dalzell, Research Scientist, AWS Center for Quantum Computing

Quantum Machine Learning

Alex Dalzell, Research Scientist, AWS Center for Quantum Computing

Quantum Chemistry

James Whitfield, Associate Professor, Dartmouth College & Amazon Visiting Academic, AWS

Quantum Optimization

Ruslan Shaydulin, Head of Quantum Engineering Research - Executive Director, JPMorgan Chase

Industry Panel Discussion

Kirk Jordan (Moderator)**,** IBM Distinguished Engineer Emeritus, IBM Research
Marcos Crichigno, Senior Quantum Algorithms Scientist, Phasecraft
Mariam Kiran, Group Lead for Quantum Communications and Networking, Oak Ridge National Laboratory
Ruslan Shaydulin, Head of Quantum Engineering Research - Executive Director, JPMorgan Chase

Quantum Education Panel Discussion

Bashir Mohammed (Moderator), Senior Staff AI Architect, Intel Corporation
Emily Edwards, Associate Research Professor, Duke University
Saif Rayyan, Manager, Quantum Computational Science Research, IBM

Quantum Software & Compilers

Kate Smith, Assistant Professor of Computer Science, Northwestern University

Quantum Communications & Networking

Mariam Kiran, Group Lead for Quantum Communications and Networking, Oak Ridge National Laboratory

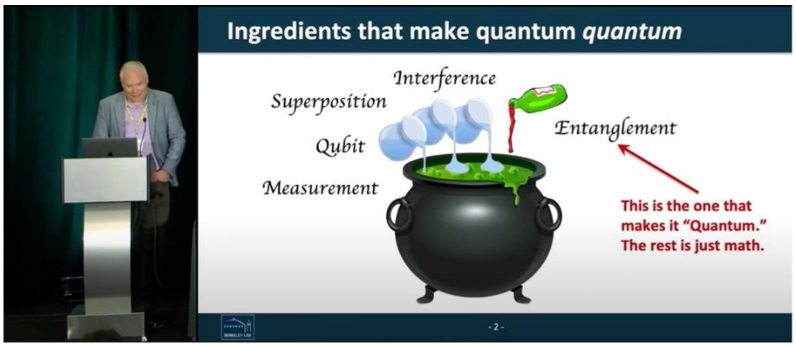
Qiskit Demo

David Hyde, Assistant Professor, Computer Science, Vanderbilt University

Unlike a traditional conference, where active question-and-answer sessions follow presentations and panels, this event featured roundtable discussions. Participants shared ideas, questions, and suggestions by posting them on sticky notes.

The schedule for the convening is in Appendix I of this report and the presentation slides, recorded presentations, summaries of recommendations, and this document are all available at the SIAM Quantum Intersections Convening website.

The SIAM Quantum Intersections Convening. Photo courtesy of SIAM.



Bert de Jong from Lawrence Berkeley National Laboratory gave the first expert talk entitled "Quantum Science Needs Mathematicians". Photo courtesy of SIAM.

Based on insights and ideas from expert speakers, participants of the SIAM Quantum Intersections Convening worked in small groups to craft recommendations that will help to bridge the gap between mathematics and quantum science. Photo courtesy of SIAM.

Speakers, facilitators, and participants of the SIAM Quantum Intersections Convening gathered in the plaza of the Tysons Corner Center in Tysons, Va., for a group photo during the workshop, which took place in early October. Photo courtesy of Knowinnovation.

Research and Development

Emerging research in QIS and quantum technology intersects with mathematics across a variety of interesting and challenging areas. A goal of the SIAM Quantum Intersections Convening was to gather a list of critical research topics where mathematical sciences play a key role in advancing QIS and quantum technology—and where quantum advances may inform new mathematical insights. Workshop participants recognized similarities with classical scientific computing, where applied mathematicians play different roles, including, notably: determining which problems are suitable for numerical solutions with current technology; writing proper mathematical models to characterize the problem of interest; and characterizing and improving the efficiency of computational methods. With this in mind for QIS, several research challenge themes became clear, including:

Mathematical foundations of quantum algorithms and applications

Noisy intermediate scale quantum devices and noise

Data science, machine learning and optimization to advance quantum computing

Mathematics to advance quantum networks

Mathematics to advance quantum sensing and learning theory

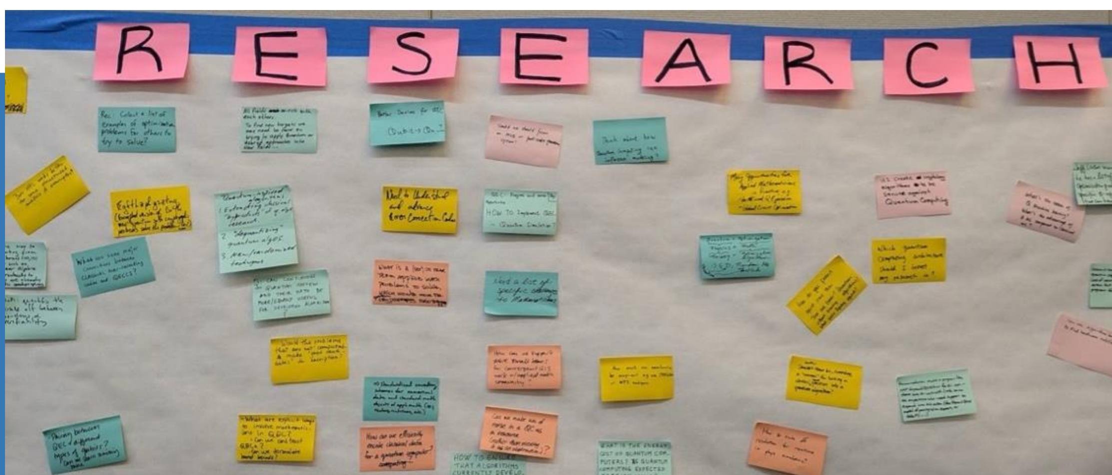


Photo Courtesy of SIAM.

Each of these research areas is poised for substantial advances by leveraging applied mathematics and its practitioners; and contemporaneously, advances in QIS can spur the development of novel mathematical techniques. This section offers concrete research and development recommendations on how mathematicians can contribute to quantum computing across a variety of fronts (and vice versa).

Mathematical Foundations of Quantum Algorithms and Applications

At a theoretical level, for any given computational problem, one can ask whether quantum computers can provide a measurable speedup compared to all best-known classical approaches. We emphasize that quantum speedup is not merely about accelerating the implementation of existing classical algorithms on quantum devices. Instead, it offers fundamentally new perspectives on many problems, enabling approaches that are infeasible classically and reshaping the mathematical framework for understanding these problems. In several cases, quantum algorithms can promise theoretical exponential (superpolynomial) advantage, a term typically used in the quantum computing community for superpolynomial cost separation between quantum and classical algorithms.

Some prominent examples of applications with expected significant quantum speedups include:

State Preparation Tasks: Ground state preparation and thermal state preparation for quantum systems that have broad applications in physics, chemistry, and beyond.

Integer Factoring: Pioneered by Shor's algorithm [1] - [3], this remains a landmark example of exponential quantum advantage with profound implications for cryptography.

Hamiltonian Simulation: Proposed by Feynman in the 1980s [4] as the original motivation for building quantum computers, Hamiltonian simulation, i.e. quantum dynamics simulation via quantum computers, remains one of the most fundamental and active areas of quantum computing research.

Quantum Linear System Problems: These involve preparing a quantum state that encodes the solution to large linear systems much faster than classical algorithms. Pioneered by the Harrow–Hassidim–Lloyd (HHL) algorithm [5], such methods are typically referred to as quantum linear system algorithms.

While substantial theoretical quantum speedups have been established for the tasks mentioned above, these results mark the beginning of the exploration of the mathematical foundations of quantum algorithms. Further research is needed to fully realize the potential of quantum algorithms and meet the following goals:

Theoretical Quantum Advantage

Establish mathematical understandings of quantum algorithm performance for tasks that achieve substantial (e.g., superpolynomial) improvements over all classical methods.

Design Quantum Algorithms with Optimal Asymptotic Cost

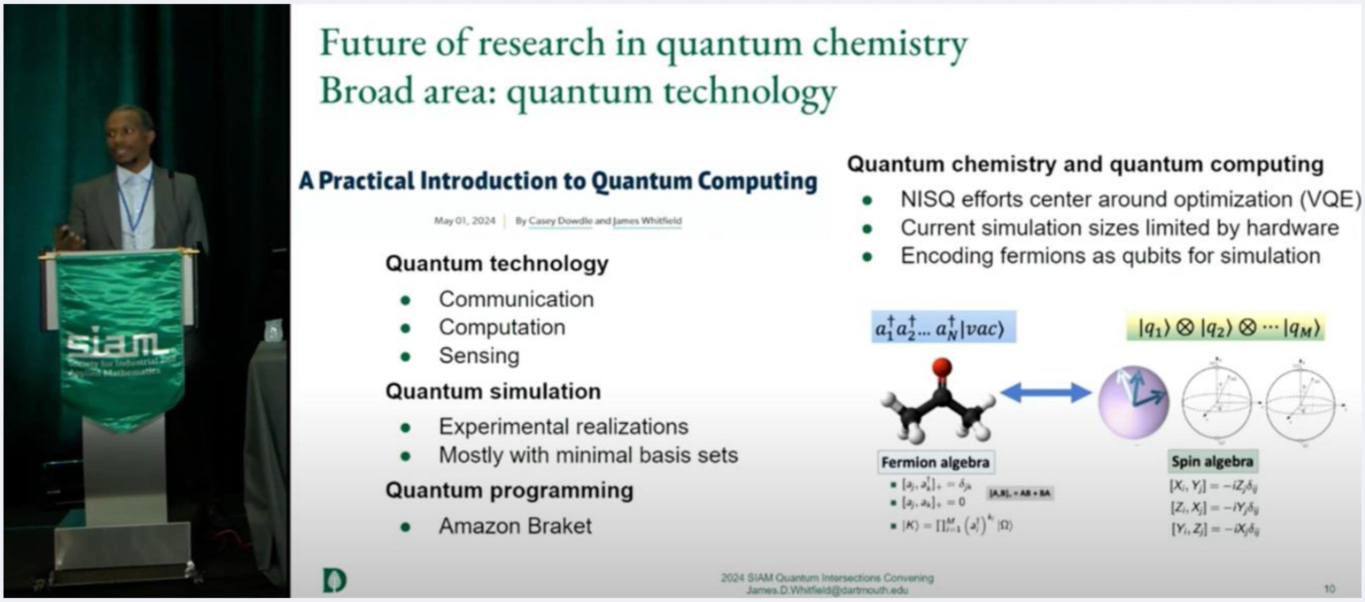
For tasks with significant potential advantage, design quantum algorithms with the best possible asymptotic scaling, achieving better quantum costs across all parameters such as system size, precision, and runtime.

Expanding Practical Applications and Resource Estimation

Identify new applications for quantum algorithms and provide concrete quantum resource estimate to evaluate feasibility in real-world scenarios, especially those motivated by industrial and societal challenges.

Improving Hardware Compatibility

Create algorithms that are implementation-friendly for near-term quantum hardware by minimizing circuit depth, reducing the need for ancilla qubits, enhancing error robustness, and so on.



James Whitfield of Dartmouth College and AWS talked to the convening participants about quantum chemistry and the role of mathematics in the field. Photo Courtesy of SIAM.

In addition to purely quantum innovations in algorithms, many promising candidates for near-term quantum benefits come from hybridizing classical and quantum resources. For instance, in processing a large-scale dataset, one may classically decompose the dataset into smaller subsets, each of which may be sent to a quantum computer for analysis. Classical algorithms in computational chemistry [6] can also be combined with quantum algorithms, such as preparing the initial state for quantum algorithms in quantum chemistry applications, leveraging the strengths of both approaches to address complex chemical problems more effectively. Hybrid classical-quantum approaches involve their own mathematical challenges (e.g., jointly analyzing the different error profiles arising from classical and quantum devices) and innovations (e.g., quantum hardware-aware domain decomposition strategies) that are critical for advancing algorithms and applications in these fields.

Noisy Intermediate Scale Quantum (NISQ) Devices and Noise

Quantum computing poses a distinct computational model from classical computing. For instance, quantum computers promise—in the best case—to perform computations exponentially faster or with exponentially less storage, than what is possible with a classical computer. On the other hand, due to the inherent randomness of quantum mechanics, programs on quantum computers generally need to be run a large number of times to assess the result of a program, in stark contrast with a classical, deterministic code. On top of inherent randomness, quantum computers are extremely environmentally sensitive devices, and their delicate physical state quickly develops errors despite experimentalists' best efforts to isolate any sources of error and noise. Thus, hefty error correction is generally a requirement of today's noisy intermediate-scale quantum (NISQ) devices.

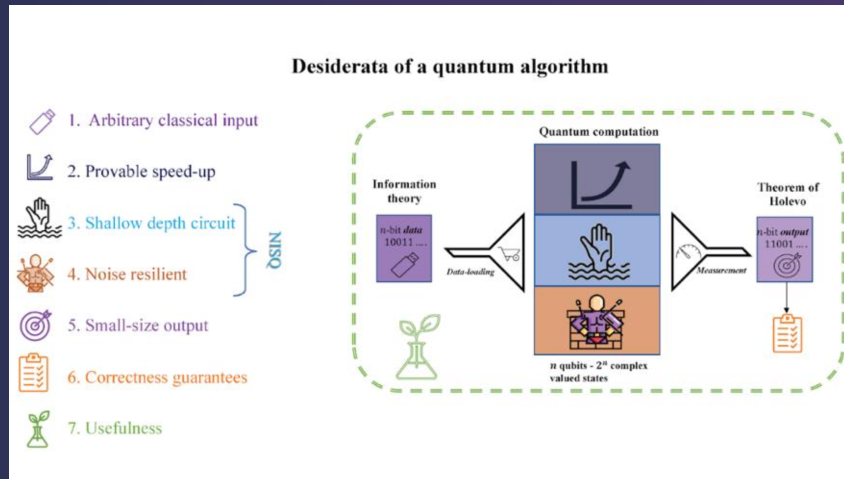
Even without error correction overhead, these intrinsic restrictions leave many quantum algorithms irrelevant, or with merely small asymptotic advantages over their classical counterparts. Moreover, with NISQ devices, noisy computation introduces many further challenges. Dealing with the physical realities of quantum devices is a substantial mathematical challenge.

Algorithms involve different types of operations, each characterized by different forms of noise, order, and noise resiliency characteristics. Additionally, the number of qubits to be employed and the circuit depth often depend upon the scale of the problem addressed. The figure below offers an illustrative desideratum for quantum algorithms in the NISQ context [7].

Equally, each hardware technology design and hardware control have individual multi-dimensional unique noise signatures. It is therefore essential to think of noise and error tolerance not as binary features (present/not present), but as complex, multi-faceted features that can account for numerous factors.

Desiderata of a Quantum Algorithm

The armed quantum computing advantage race: a set of seven criteria that an aspirational quantum algorithm should satisfy (left), and the end-to-end quantum computational model (right), where the funnels symbolize the narrow input and output bottlenecks, and the boxes represent the other desired properties. [7]



To assess the viability of quantum computing from a comprehensive perspective, it would be crucial to address the following aspects from a multi-disciplinary angle:

Noise Characterization

Employ and develop analysis tools from uncertainty quantification, mathematical modeling, information theory, experimental design, and extrapolation theory. Introducing new uncertainty quantification techniques can enhance quantum error correction and system design, improving reliability and performance. Fidelity should be seen as a multi-dimensional measure requiring complex characterization based on hardware and control characteristics. Proper analysis of noise factors is essential for accurate performance assessment. Embracing noise, rather than eradicating it, may be viable by leveraging knowledge from other disciplines.

Algorithm Design

NISQ computing focuses on designing algorithms that are lean in circuit depth and resilient to noise. Transitioning to fault-tolerant quantum computing will depend on the algorithm and input size, requiring reconsideration of existing algorithms and their economic viability compared to classical computing. Designing broadly error-resilient algorithms can benefit both quantum and classical computing, where the meta-level consideration is to identify computational primitives (if they exist), where quantum can offer proven advantage, while delegating all remaining computations to classical systems. Focusing on algorithms that provide low dimensional summaries of entities involving linear algebra primitives with k-local/sparse Hamiltonians seems like a possible avenue—leveraging quantum computing strengths while being conscious of its innate limitations.

Algorithm Analysis

Comprehensive comparative quantum and classical complexity and noise analysis study is essential (a great example is [8]) to map the landscape of noisy quantum computation. Such analysis should go beyond worst-case analysis and consider average case and smooth analysis to provide fine-grained understanding. Further clarity can be attained through explicit derivation of noise resiliency bounds. Uncertainty quantification and characterization of input/output distributions, alongside mitigation benchmark failures can be instrumental in providing a more concise understanding of algorithms' performance under various considerations. Collaboration with physicists and engineers is crucial for identifying uncertainty sources.

Error-Correction Codes

Error-correction codes are crucial in maintaining the integrity of quantum information by identification and pro-active correction of errors due to quantum noise. In the NISQ regime, a balance between error-correction codes (that must compete for limited computational resources) vs. error mitigation and resilience should be considered. Each hardware technology has its own noise characteristics, connectivity economy, gate rates, etc. Thus, development of topological error-correction codes as well as the co-design of codes with hardware will be helpful. Lastly, the delegation of error mitigation to compilers or hardware controls, and properly characterizing physical qubits, are essential for improving quantum computing performance.

Hardware Control

Hardware control involves control errors, misspecified models and actuation, crosstalk effects, and other undesirable implications. To overcome these, it is essential to incorporate reinforcement learning machinery, adopt notions from partially observable Markovian systems, noisy channel signal processing, and numerical optimization

Hardware Design

Quantum device characterization is a precursor for design and co-design decisions (see noise characterization section). Such characterization should account for hardware architecture /technology choices (connectivity, substrate), as well as variability between devices of similar designs. In the process, it is important to question hardware design prejudices and substantiate hardware design upon clear metrics (e.g., importance of connectivity over qubit count, quantum volume considerations, etc). Mathematical modeling can serve a key role in experimenting with design propositions. In the context of NISQ computing, given the unique noise signature of each algorithm, it is also important to consciously trade-off between the specificity of the hardware (to excel potentially in solution of a specific problem) and a more general purpose, master-of-none design.

Secured Quantum Information Transfer

Analysis of communication channels and design of communication protocols should be pursued as a means for quantum computing, as well as independently for advancing quantum communication itself. Beyond the obvious benefits of developing novel, secured communication channels for inter- and intra-system communication, an information theoretic communication channel across noisy channels lens can be instrumental in modeling noise, analyzing multipartite systems, as well as in quantifying channel capacity.

Data Science, Machine Learning, and Optimization to Advance Quantum Computing

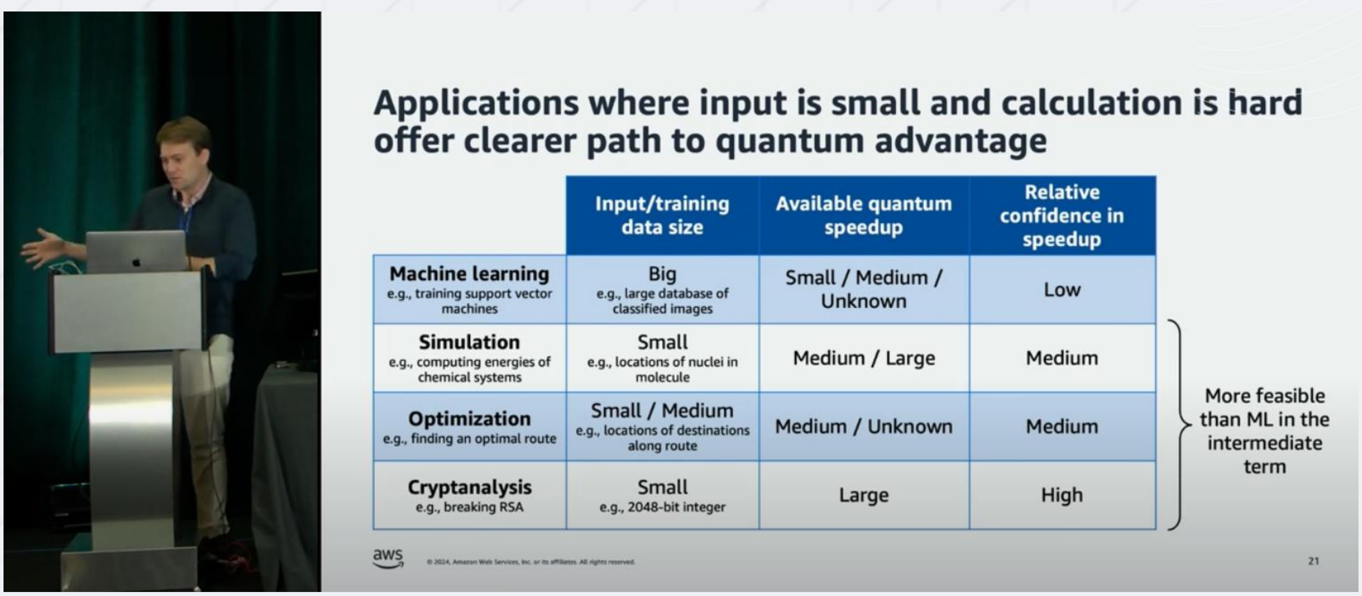
From a mathematical perspective, there is a great deal of opportunity for applied mathematicians in the areas of data, machine learning, and optimization to realize potential value from quantum computing. Applied mathematicians have already made valuable contributions to these areas in the realm of classical computing, and now their skills may be brought to bear on quantum devices. A key to accelerating the practical use of quantum computing is encouraging applied mathematicians to use their expertise and insight to impact quantum computing. Incentivizing mathematicians, quantum experts, and domain scientists to collaborate on researching some of the many challenges and current barriers will accelerate the eventual practical use of quantum computing for problems in data science, machine learning, and optimization.

Quantum Memory

Quantum memory is a key element in information processing in both quantum computing and quantum networking. Quantum states can lead to certain forms of space advantage for computations. Additionally, taking advantage of the size of the Hilbert space in which quantum states live, it is sometimes possible to computationally exploit the representation of an n-dimensional vector or n*n-dimensional density matrix as a log(n)-qubit quantum state. The preparation of such states is not efficient in general, but it is efficient if certain types of quantum storage are available. Mathematicians can help find efficient ways to encode useful data, in storage or via compact circuits. Another area for mathematicians to assist is in the access to individual elements in the data structure in which the data is saved (e.g., sorted data accessed as a binary tree). Analysis of what special/particular structures are needed for storage in quantum memory and extending the duration data can survive in quantum memory are areas where mathematics can provide insight.

Quantum Data Compression

Making quantum sensors widely accessible for educational and research purposes can significantly advance the field. These sensors detect, measure, and generate highly precise data, offering opportunities to explore the types and structures of data best suited for quantum computing. This accessibility would also support the formulation of quantum data standards to better advantage quantum computers. This would enable more mathematical researchers to investigate quantum data issues. Specifically—and possibly starting from or with analogs to classical data compression algorithms—researchers could then develop the mathematics for basic quantum compression algorithms and investigate quantum error correction that compresses data. Perhaps trading off some protection of an error-correcting code would provide insight and solutions to the data input/output issues in quantum computing. Creating opportunities to bring data science mathematicians together with quantum information science researchers to address quantum data issues would be another initial starting point.



Alex Dalzell of AWS Center for Quantum Computing gave an introduction and some perspectives on quantum machine learning. Photo Courtesy of SIAM.

Quantum Machine Learning with Classical Data

Significant progress has been made in implementing classical machine learning algorithms and techniques on quantum platforms or designing quantum algorithms for classical machine learning tasks. However, recent advances have led to substantial insights into quantum machine learning (QML), raising skepticism about its practicality when applied to classical data, particularly regarding whether quantum machine learning algorithms offer a genuine computational advantage over classical methods. This skepticism has been fueled by significant theoretical developments, including dequantization, lower bounds, the understanding of barren plateaus, and input/output challenges (see earlier commentary in 2015 [9] and a more recent one in 2022 [10]). Mathematical insights can play a crucial role in deepening our understanding of the potential and limitations of QML. Although exponential or superpolynomial speedups may not be expected, questions remain about whether significant end-to-end super-quadratic speedups are feasible, what practical applications could benefit from such improvements, and whether such performance gains justify the use of quantum computers.

Learning with Quantum Data

Tasks involving quantum data appear more promising, with a number of examples already demonstrating exponential improvements in sample complexity over their classical counterparts. The concept of “quantum for quantum” (learning from quantum experiments) is significant (see the broader discussion on this topic in the section on quantum learning and sensing theory). However, quantum data does not inherently address all the challenges associated with quantum neural networks or, more generally, variational quantum algorithms, as these may still be vulnerable issues such as barren plateaus. Notably, all known variational quantum algorithms that successfully avoid barren plateaus can also be efficiently simulated classically. While quantum learning and sensing can leverage quantum-enhanced strategies for improvement, these strategies often do not rely on quantum neural networks or variational quantum algorithms. How to use other mathematical ideas in data science to better learn from quantum systems in a nontrivial way remains an interesting open direction. Mathematical insights can contribute to identifying and analyzing quantum problems, deepening our understanding of the potential and limitations of learning tasks for quantum systems.

Scientific Computing and Optimization

Quantum Logic

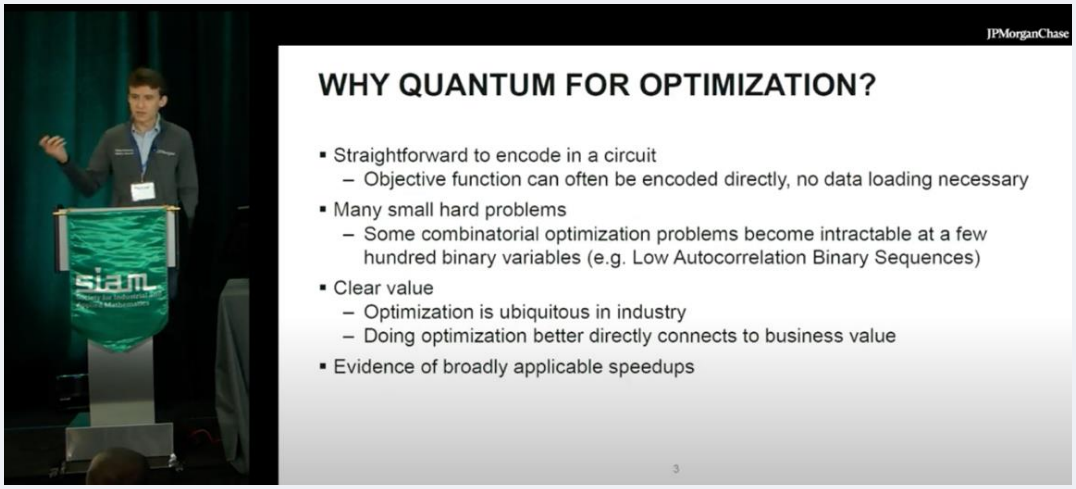
Support research in quantum logic/reasoning, including investigating the use of quantum to improve fully automated reasoning. Such research could build on results in SAT (Boolean satisfiability problem) and other logic problems that show promise for establishing connections between mathematical logic and quantum computing.

Scientific Computing

Mathematicians with expertise in differential equations, linear algebra, numerical optimization, optimal control, probability theory, dynamical systems, fine-grain complexity theory, and other areas can have impact in numerous areas of quantum computing such as addressing problems in quantum compilers, quantum networks, and developing new mathematical formulations that are amenable for quantum simulations. Initially, creating a public collection/repository of the various quantum and/or classical problems potentially advantaged by quantum that quantum and mathematical experts have access to will focus and accelerate mathematical-quantum collaboration on problems.

Optimization

Mathematical optimization methods are often used in various business settings such as mechanics, economics, and engineering to provide insight for decision making to address industrial and societal issues. Quantum Approximation Optimization Algorithm (QAOA) provides a framework to study the approximate solution of optimization problems with a NISQ-friendly algorithm. For some highly structured problems, it leads to useful characterization of solution quality. In addition to the QAOA, many areas of optimization might benefit from established quantum linear algebra routines. Mathematical exploration and research to develop quantum optimization algorithms, while of mathematical interest, would also create impact in business applications, accelerating the uptake of quantum computing.



Ruslan Shaydulin of JPMorgan Chase shared his expertise and perspectives on quantum optimization. Photo Courtesy of SIAM.

Mathematics to Advance Quantum Networks

Quantum networking has the potential for various critical applications, such as running algorithms like quantum key distribution over quantum devices that are spread far apart [11]. While connecting and exchanging information between quantum devices leverages, to some extent, ideas of classical networking, there is a host of new mathematical challenges associated with achieving large-scale, resilient quantum networks. Such challenges include:

Asymptomatic Analysis

Understanding asymptotic behavior of quantum networks (e.g., scaling properties) as the number of nodes and edges in a network go to infinity. Geometry/topological/physical function scaling are all different for quantum networks in comparison with classical communication networks, so there are theoretical questions to be answered.

Computability

Questions regarding computability in quantum information theory in the context of quantum networks. Computability has been investigated for quantum computers [12], but how does this change in the context of a quantum network?

Uncertainty Quantification

There is a lack of understanding of uncertainty quantification of loss degradation in networks, which is fundamentally different than in quantum computation. Building mathematical models for these scenarios would help inform resilient quantum network design.

Error Propagation

It is difficult to model the compounding of quantum computing uncertainty and quantum network uncertainty. In other words, in quantum networks, uncertainty comes not just from classical effects like loss degradation, but also inherent quantum behaviors like decoherence—further work must be done to incorporate these effects into uncertainty quantification frameworks.

Given research challenges like these, several steps will help enable mathematicians to push the boundaries of quantum networking:

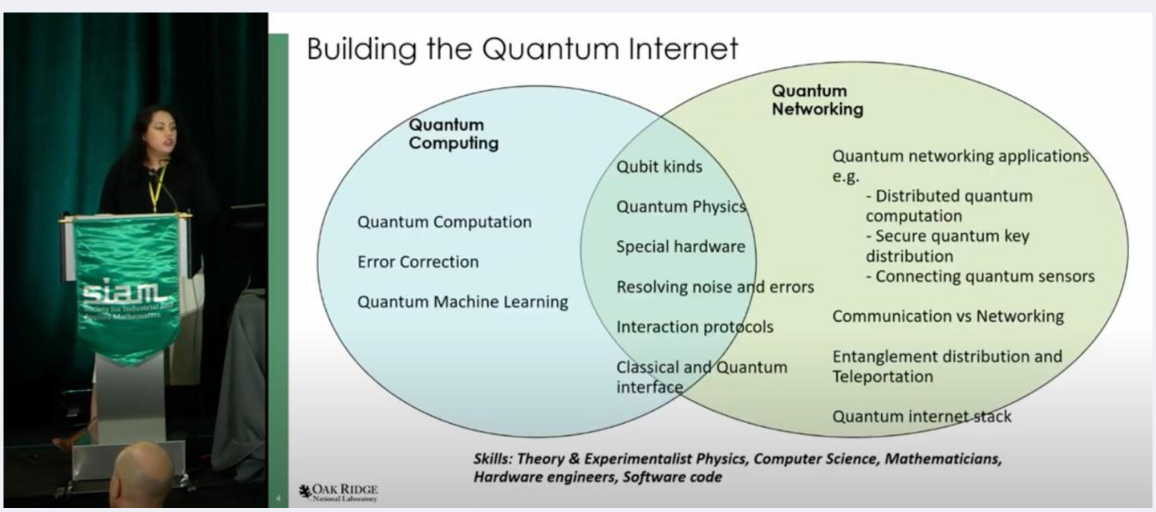
Quantum Analogs of Signal-to-Noise

Mathematicians can investigate definitions and models for an analog of the "signal-to-noise ratio" used in classical networking, which fails to capture the various effects seen in a quantum network.

Information Theory of Quantum Networks

Mathematicians can research information-theoretic frameworks for quantum networks in order to unlock more efficient computation (e.g., developing theory around the optimal way to perform domain decomposition on a quantum problem while retaining robustness across a noisy quantum network).

Implementing these recommendations will help lead to more secure, efficient, and practical quantum communication; better design of quantum networks; and greater theoretical understanding of the best ways in which to use quantum networks for mathematical and computational problems.



Mariam Kiran of Oak Ridge National Laboratory spoke about the mathematics, computer science, physics, and engineering challenges in quantum communications and networking. Photo Courtesy of SIAM.

Mathematics to Advance Quantum Sensing and Learning Theory

Quantum sensing and quantum metrology are fields dedicated to enhancing measurement precision by leveraging quantum mechanical principles such as superposition and entanglement (see earlier reviews, e.g., in 2011 [13] and 2017 [14]). These fields are closely connected to quantum learning theory, which designs algorithms—also known as experimental protocols—with rigorous theoretical guarantees to extract information from quantum systems. It is important to note that quantum learning typically has no connection to neural networks and should not be confused with quantum machine learning.

In quantum sensing and metrology, a major challenge is scalability, as quantum systems operate in exponentially large Hilbert spaces, and measurements are destructive, potentially altering or destroying the state upon measurement. Establishing scalability, sample complexity, error robustness, and other theoretical guarantees have become central to the field and are typically referred to as quantum learning theory. Quantum-enhanced techniques often surpass classical methods in detecting small signals or achieving high precision, making them valuable in fields such as physics, biology, and materials science. In quantum metrology, the concept of the *Heisenberg limit* serves as a key benchmark, defining the maximum attainable precision for parameter estimation using quantum resources, which outperforms the classical strategies that will typically result in the so-called *standard quantum limit*. Designing both algorithms and providing information-theoretic provable guarantees are important mathematical directions of the field, which are closely linked to applied mathematics, computational mathematics, and probability. In applied math terms, sensing problems are a quantum analog of an inverse problem.

Advancing Quantum Sensing with Applied Mathematics

Applied mathematicians can play a vital role in advancing the field of quantum sensing by contributing to the design/implementation of numerical methods for optimization, signal processing, and data analysis. In preparing maximally sensitive states, optimization techniques can help identify optimal control parameters for quantum systems that enhance their sensitivities to desired quantities. Mathematicians have critical expertise in developing noise models of various quantum effects.

Developing Robust Algorithms for Quantum Sensing

Algorithmic development is needed for protocols that are resilient and robust to noise effects. Contributions in signal processing and data analysis techniques within quantum sensing frameworks will allow for more efficient extraction of information from complex quantum systems. Further integration with ideas from inverse problem techniques, sensitivity analysis, and uncertainty quantification methods will advance the practical application of quantum-enhanced sensing; this will enable new applications of quantum sensing in diverse scientific domains.

Mathematical Foundations for Quantum Learning Theory

Quantum learning provides a theoretical framework for gaining information from quantum systems at a greater precision and efficiency than classical methods. Key research topics include classical shadows, shadow tomography, many-body Hamiltonian learning, and many other learning tasks [15] - [22]. Applied mathematicians can make valuable contributions to this area by developing measurement and learning protocols that provably approach the Heisenberg limit and enhancing error robustness in algorithmic protocols for various learning tasks, particularly with respect to state preparation and measurement (SPAM) errors. Classical shadow methods and shadow tomography enable the efficient estimation of quantum states or observables with minimal measurements, making it possible to reconstruct properties of quantum systems without exhaustive sampling. Applied mathematics can contribute to advancing these methods, as they require sophisticated probabilistic modeling, efficient sampling, and data analysis techniques to manage the complexity and potential noise in real quantum systems.

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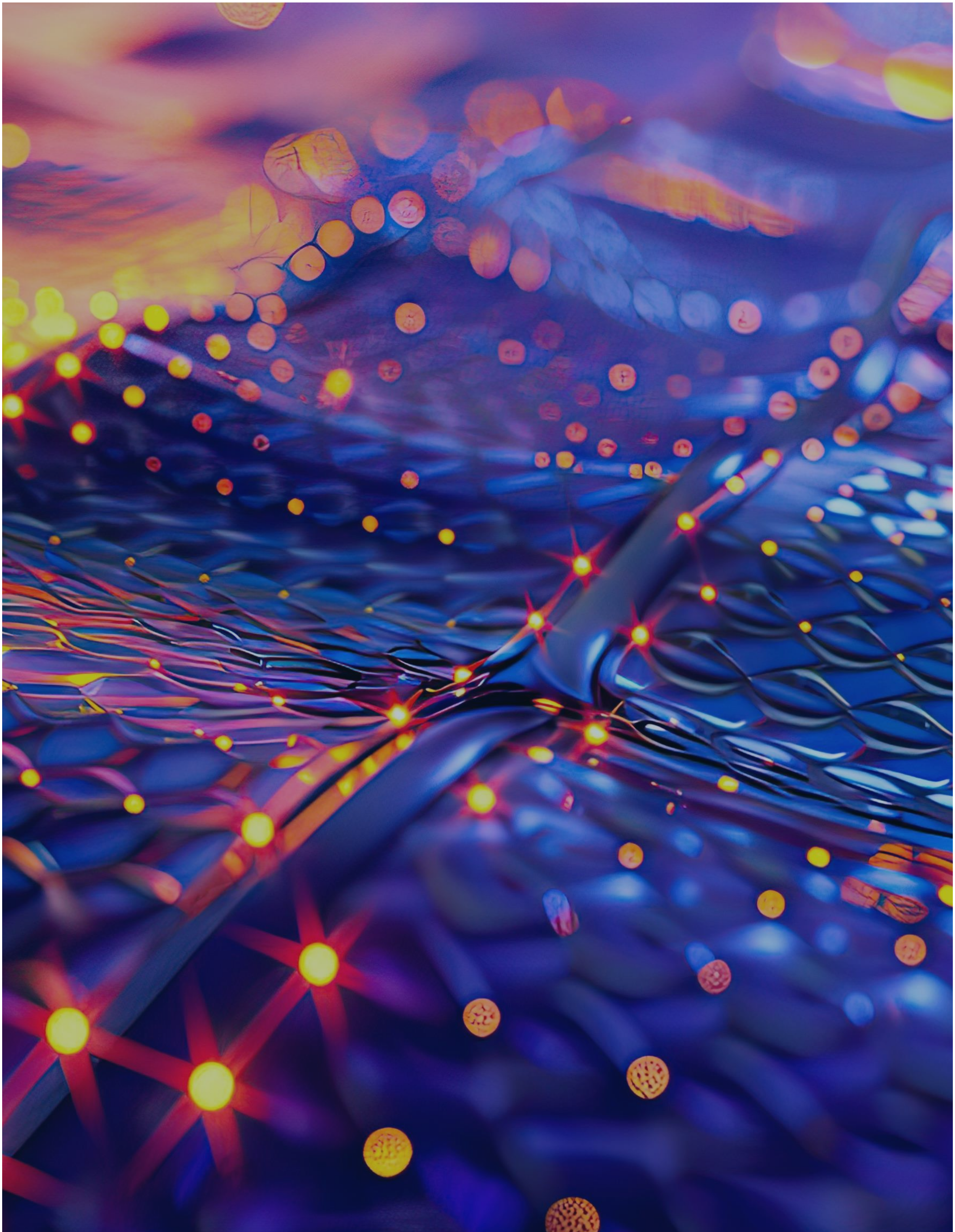


Opportunities to Increase the Involvement of Applied Mathematicians in QIS Research and Collaborations

As has been discussed in this report, there are numerous research topics where QIS would benefit from the involvement of mathematicians and the mathematics community. Many participants at the workshop noted that while these opportunities exist and some mathematics researchers are already engaged, a broad swath of the community is quantum-curious but without clear experience or pathways to gain that experience. Despite agency efforts over the last five years, QIS research is still highly concentrated in physics and at a relatively small number of universities. There are several critical issues to address: how to build capacity for individual mathematics researchers to conduct QIS research; better networking and engagement to connect mathematics researchers to the broader QIS community; mechanisms to encourage a broader set of universities to engage in QIS; and pathways and opportunities for mathematics researchers to get involved in larger QIS research collaborations. In this section, we consider a variety of mechanisms to address these issues. Funding opportunities should create clear pathways that engage faculty and students at different levels of quantum experience and aim to build clear communities and networks enabling deeper involvement of mathematicians in larger quantum research teams.

Building Capacity for Individual Mathematics Researchers

Federal agencies should consider how to get more researchers in the mathematics community to consider QIS challenges and build the skills necessary to conduct QIS research. A variety of mechanisms could be used to foster this capacity building.

Core Research Funding

Core research funding plays an important role in supporting individual researchers throughout the mathematics research community. Currently the funding programs do not clearly indicate where quantum challenges best fit and the community is not clear on whether QIS research would be welcomed and if so in which programs. QIS research challenges go across various subareas of mathematics and do not neatly belong in any one core program. It is essential that quantum interests are elucidated and supported in their appropriate programs. This could be accomplished through communications such as a Dear Colleague Letter or through a dedicated QIS core program. Funding for these individual investigator efforts would enable a broader group of faculty to engage in QIS research. NSF should especially consider key topics that are not well covered by other agencies. For instance, the Department of Energy (DOE) Advanced Scientific Computing Division (ASCR) specifically excludes the development of quantum algorithms and quantum cryptography.

Capacity Building Activities

NSF has for several years focused on capacity building in QIS through programs such as Expand QISE. These efforts have primarily focused on expanding the diversity of institutions involved in QIS research such as building capacity at Minority Serving Institutions or non-research-intensive universities. However, many mathematics researchers in research-intensive environments also need capacity building to be able to fully engage in QIS. Agencies should consider programs that focus on capacity building for the mathematics community at all types of institutions. NSF should create a special math track within ExpandQISE or a new program that specifically focuses on building capacity in mathematics or other computationally-focused departments. Eligibility should be based on the quantum funding by department or left open to include math departments at R1 institutions that are trying to build quantum expertise and do not yet have significant funding or efforts. Another model would be focused on individuals like the Directorate for Engineering TRAILBLAZER for Engineering Impact program that enables a mid-career faculty member to pivot to a new research area. The existing DMS Research Training Groups in the Mathematical Sciences (RTG) program would also be a potential model that could have a quantum focus or track. Capacity-building funding of this style would help focus mathematics and related departments toward quantum topics, potentially hiring more faculty in this area or encouraging faculty to pivot their research towards quantum from more mature areas of mathematics.

Institutional Diversity

While programs that address capacity building across the mathematics research ecosystem are needed, the workshop found that efforts to support institutional diversity are also critically important and should be continued. Agencies should also consider support for faculty at less-resourced institutions to participate in networking opportunities, summer schools, or other community activities.

Networking Mechanisms to Engage Mathematics Researchers in QIS Communities

The Quantum Intersections Convening revealed the power of bringing communities together to consider QIS topics and network. More community building is needed, and these challenges will not be solved by one convening. There are still many challenges that exist in understanding quantum challenges and building collaborative research teams. For example, QIS researchers from the physics community use different terms and speak about mathematics differently than the mathematics community; participants from different disciplines may have different incoming knowledge levels about QIS; and different communities have different goals and incentive structures around their engagement in research and collaboration. Workshops and trainings can help address these challenges.

Several kinds of workshops and trainings should be supported:

Initiate specialized workshops focusing on subtopics in more detail. Distill challenges into actionable math research efforts. Alternatively, organize breakout sessions that focus on foundational topics in each field. For instance, basic quantum mechanics for mathematicians and essential numerical methods for quantum computing researchers.

Bring physicists together with mathematicians to solve research problems. There could also be Ideas Labs or other sandbox-type efforts to encourage team building. To overcome different research priorities, design problem-focused workshops where both groups work together on challenges that require insights from both disciplines (e.g., improving quantum algorithms with new numerical methods).

Support training and summer institutes or other sessions focused on education in quantum topics for the math community.

Incorporate basic training for quantum-curious mathematics researchers ahead of collaborative workshops so that the mathematicians are on a more even playing field with physicists during collaborative discussions.

Create mentorship pairings or collaborative groups, where an expert from each field works together, learning from each other.

By addressing these challenges thoughtfully, these more targeted workshops can create meaningful collaboration opportunities and help bridge the gap between quantum computing and applied mathematics.

Collaborative Funding Mechanisms to Encourage QIS Collaboration

Currently, researchers-who are interested in QIS but are outside core quantum fields may find it challenging to know how to get engaged and work together with quantum experts. The above efforts will help the mathematics community grow their QIS expertise and engage in small research. However, once mathematicians are ready to work on more complex quantum challenges, there are limited funding opportunities for small or midsize teams to address QIS. Given the convergent nature of QIS challenges, there should be mechanisms to support these teams.

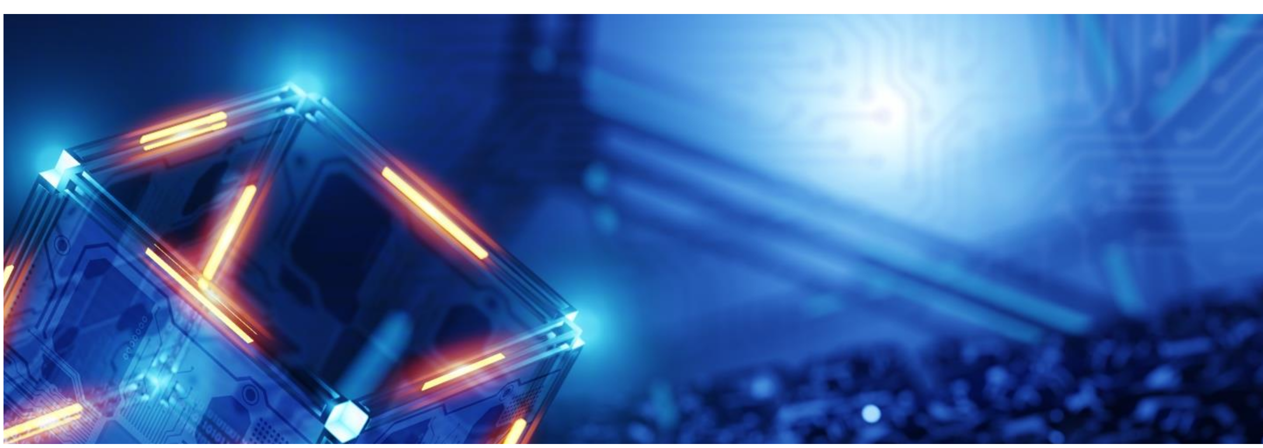
Small and Mid-Size Team Funding

NSF should invest in programs that bring together the math community with those in traditional quantum fields as well as computer scientists and engineers to work in small convergent teams on quantum computing and other quantum information challenges. Major funding such as large institutes or centers requires an institution to already be a national leader in quantum technologies. Meanwhile many problems could be worked on by smaller teams and more mid-sized funding opportunities would broaden quantum activities to many more universities. DMS involvement in programs in partnership with other NSF divisions would bring more awareness to the math community about quantum opportunities and more awareness to those other communities about the importance of including mathematicians in their collaborations. The Transformational Advances in Quantum Systems (TAQS) program could be a useful model but it is currently archived and has recently been focused on sensing challenges that are not as relevant to the math community.

Models for potential programs could be Accelerating Computing-Enabled Scientific Discovery, TRIPODS, or the recent Mathematical Foundations of Digital Twins and Foundations for Digital Twins as Catalyzers of Biomedical Technological Innovation programs. NSF could also consider adding new tracks to programs already focused on quantum such as TAQS or existing mid-sized programs such as the Emerging Frontiers in Research and Innovation (EFRI), Foundations of Emerging Technologies, or the Convergence Accelerator. It is essential that the math community can participate in these opportunities and are incentivized to join teams by calling out relevant research challenges in solicitations or specifically requiring or encouraging math participation. These efforts are ripe for interagency partnerships to address specific QIS challenges of relevance to the Department of Defense, Department of Energy, or National Institutes of Health.

Cross-Sector Partnerships

As agencies consider team-building and convergent efforts, they should also consider how to incentivize partnerships in QIS across the research ecosystem and with different types of organizations such as industry and national laboratories. These efforts would provide better workforce pathways for the mathematics community and aid in translational efforts as QIS discoveries increasingly become ready for commercialization. Agencies could specifically encourage cross-sector team building or engagement in QIS funding opportunities. Prize challenges or sabbatical programs modeled on the Civic Innovation Challenge (CIVIC), Convergence Accelerator, or Grant Opportunities for Academic Liaison with Industry (GOALI) would directly support these collaborations. Agencies could also consider incorporating these partnerships into new translational efforts such as Quantum Testbeds or major funding programs such as the Quantum Leap Challenge Institutes.



Infrastructure for Research in QIS

The above efforts will help incentivize and expand QIS research among the mathematics community. However an additional barrier needs to be addressed, which is that QIS infrastructure is very expensive and difficult to access for researchers not already located at a well quantum-resourced institution or lab. Existing resources can be hard to find or access for mathematics researchers and there are limited national resources that are specialized for mathematics challenges in QIS. Agencies should consider several mechanisms to improve access to current infrastructure and add new resources.

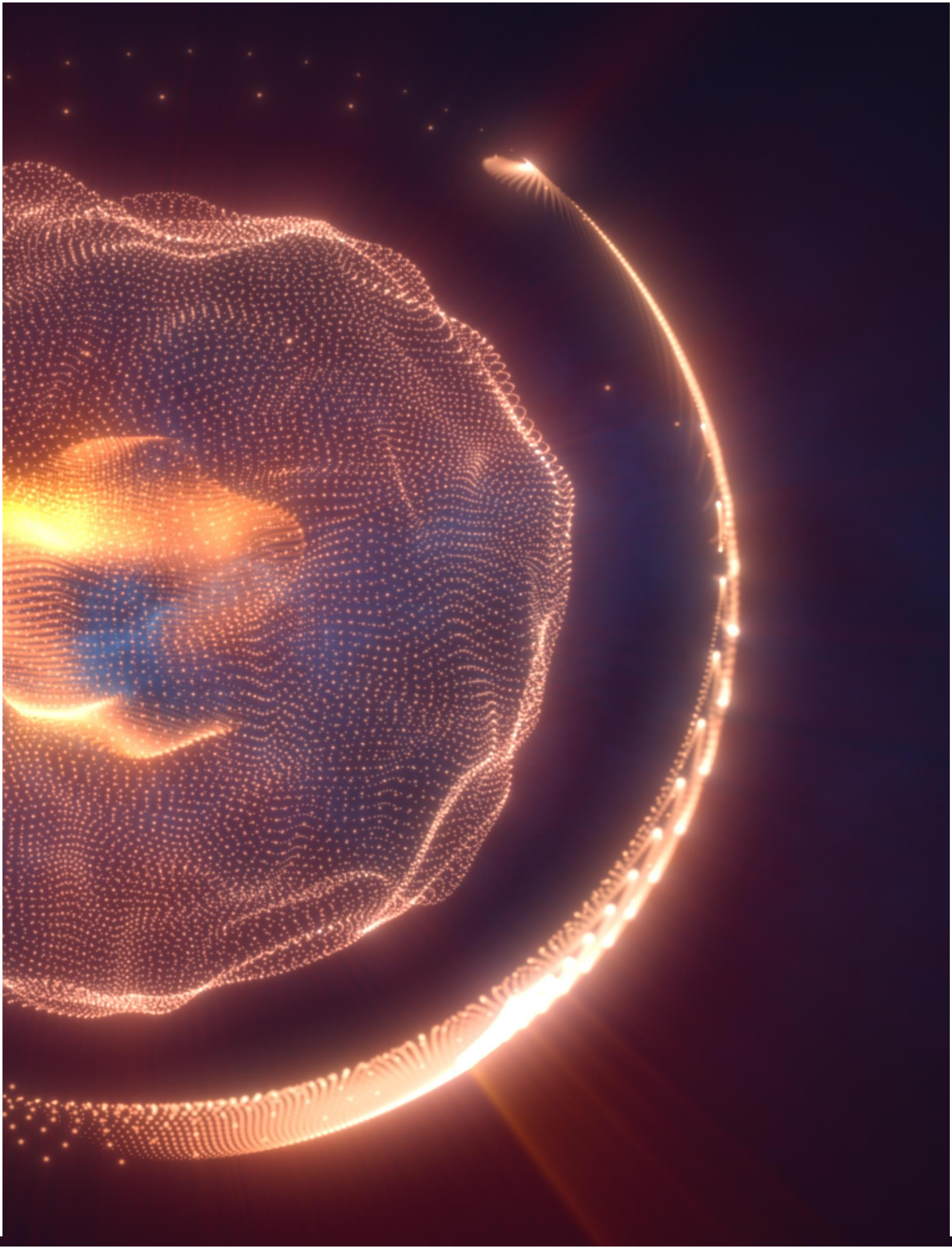
Make a centralized database, website, Dear Colleague letter, or other information source that better explains which resources are already available and creates a central repository on how to access these resources.

Consider partnering with quantum companies to provide credits for math researchers to use industry-based assets similar to the current NSF model around cloud computing credits.

Invest in additional resources that scale the available access and democratize research through a distributed network such as the National Quantum Virtual Laboratory or quantum-appropriate resources within the ACCESS computing infrastructure network.

In the long term, agencies should continue efforts to expand quantum networks and build the quantum internet which will increasingly be needed as quantum technologies advance. This is primarily an effort of DOE. We recommend implementing a funding mechanism or program that explicitly calls for and supports mathematicians (whether graduate students, postdoctoral scholars, or faculty) to investigate pure and applied mathematical aspects of pertinent quantum networking problems.

These efforts would ensure current assets are better utilized and new assets address evolving QIS research needs. Better infrastructure will increase the involvement of mathematicians and other researchers in quantum efforts and enable researchers from varied disciplines and institutions to participate. Infrastructure access will also help students to learn and provide more opportunities for hands-on engagement.



Educational Resources and Workforce Development in Quantum Information Science

As rapid advancements in quantum information science and technology continue to push the frontiers of innovation, it is imperative to cultivate a well-prepared workforce that is equipped to tackle the challenges and opportunities of this emerging field. Working groups at the QIC identified several critical areas that require strategic action to empower mathematics educators and students alike, and emphasized the need to lower the barriers for teaching quantum computing topics, foster hands-on engagement, and create a robust support system for educators and learners.

Photo Courtesy of SIAM.

A primary focus is the development of resources to make QIS accessible to educators and students at all levels, from K-12 to graduate programs. There is a need for a centralized hub where educators can access materials and share best practices and students can access resources around workforce and career pathways and internship opportunities. Students must also have access to meaningful, hands-on learning experiences to bridge the gap between theoretical mathematics knowledge and practical applications. Opportunities for internships, collaborative projects with industry, and real-world quantum computing experiences are essential to prepare students in the mathematical sciences for careers in this rapidly evolving domain.

Educational Resources for Quantum Topics

Educators have limited time and resources to add aspects of quantum science to their classes. As such, there is a need to develop frameworks and disseminate materials that ease the incorporation of quantum topics into existing mathematics coursework and encourage experiential learning.

Steering committee member Bashir Mohammed of Intel moderated an engaging panel about educational issues in quantum with Emily Edwards of Duke University and Said Rayyan of IBM. Photo Courtesy of SIAM.

K-12 Curriculum

The incorporation of QIS concepts into K-12 education represents a significant opportunity to inspire the next generation of scientists and engineers. Students, particularly those from backgrounds that are underrepresented in the sciences, tend to start self-selecting out of STEM in middle school; so, educational intervention is necessary at that level. While existing structures, such as the QIS K-12 Framework, already align public education standards with emerging topics in QIS, practical implementation faces challenges such as the limited time and resources of K-12 educators, equity and access concerns given the high knowledge threshold, low student engagement, and institutional priorities that may not yet align with the goals of QIS education.

A multifaceted approach could help overcome these barriers. Based on the Q-12 Framework and other standards, educators can intersperse quantum concepts into existing science and mathematics curriculum, avoiding additional content burdens by reprioritizing current material. Collaborations between QIS experts and math pedagogy specialists should focus on identifying parallel competencies—such as coding, data analysis, and foundational computational thinking—that can be introduced to younger students (K-6) as foundational skills, preparing them for future quantum-specific learning. To catalyze these efforts, a convening of K-12 math education decision-makers could explore actionable strategies, drive the development of accessible educational resources that reduce teacher workloads and build consensus around QIS integration.

Undergraduate Curriculum

Developing a robust undergraduate curriculum for QIS poses unique challenges, as QIS inherently spans multiple disciplines and thus requires a thoughtful educational approach that is both accessible and effective for students from diverse academic backgrounds. Moreover, quantum topics often rely on advanced mathematical concepts, like complex variables, that are not typically covered in standard math courses such as linear algebra.

To address these challenges, institutions could foster **interdisciplinary collaboration in QIS education** by forming faculty teams from mathematics, physics, computer science, and/or engineering departments to coordinate and co-teach quantum courses. Strong cooperation among faculty members would ensure a cohesive and comprehensive learning experience for students. As a preparatory step, institutions should consider hosting interdisciplinary seminars to establish foundational knowledge and ensure alignment among faculty before launching new courses.

There is a specific opportunity to incorporate QIS modules within **undergraduate linear algebra courses**, which could demonstrate the real-world relevance of mathematical concepts. Linear algebra underpins many foundational ideas in quantum computing—such as vector spaces, eigenvalues, and unitary transformations—yet its connection to cutting-edge technologies often remains opaque to students. By explicitly linking these topics to quantum computing, educators can motivate students to delve deeper into linear algebra while bridging the disciplinary language gap between mathematics and physics. The early introduction of quantum computing notation alongside traditional linear algebra topics could be complemented by illustrative examples from quantum physics that contextualize abstract mathematical concepts.

Expanding **experiential learning** opportunities is critical to equip math students with the necessary skills to apply theoretical concepts to practical problems in QIS. While there exist numerous online platforms to practically engage with quantum computing, these are often too advanced or poorly tailored for undergraduate students. Bridging this gap requires synthesizing and simplifying materials to create a structured progression of learning experiences that are available in a central repository. These resources should encompass a range of activities—including in-class group work, labs, homework problems, games, programming exercises on platforms like Qiskit, small-scale projects, capstone experiences, and internship projects—to provide students with a scaffolded approach to mastering quantum concepts. Ideally, curated content should have clearly specified prerequisites and difficulty levels to ensure accessibility for students and educators. Additionally, fostering collaborative spaces for experiential learning, such as weekly gatherings for students to work through problems together, would create a supportive environment for exploring quantum computing challenges.

Several kinds of workshops and trainings should be supported:

Develop a **comprehensive book of quantum computing activities and modules** that could serve as a cornerstone resource for educators. Such a book could help define what a quantum computing course should look like at the undergraduate level, providing clarity and consistency across institutions.

Support **faculty workshops** to collaboratively develop activities and modules. These workshops could serve as a platform for educators across disciplines to exchange ideas, refine teaching practices, and create a cohesive set of quantum-enriched resources.

Build pipelines between four-year institutions and **community colleges** to prepare students for advanced quantum topics. This collaboration would both strengthen foundational learning and broaden access to QIS education for students from a wide range of backgrounds.

By integrating these strategies, institutions can improve educational opportunities for more students, foster greater communication and resource sharing among educators, and establish a strong foundation for interdisciplinary teaching and learning in QIS.

A National Center for Quantum Education

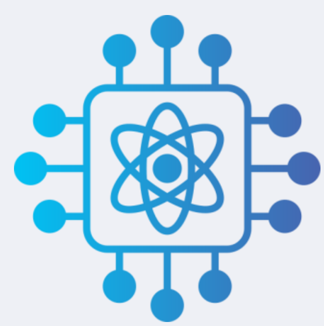
The establishment of a national center for quantum education would serve as a transformative step towards unifying and amplifying quantum educational efforts across the U.S. Currently, resources and expertise are fragmented and spread across industry, academia, and government initiatives, who often differ in their objectives. This fragmentation leads to the duplication of efforts, gaps in resource accessibility, and challenges for educators who are seeking to implement quantum curricula or find tailored support. A national center could address these challenges by consolidating efforts into a single, cohesive hub that is designed to support and scale quantum education and workforce development initiatives.

The center would provide a space for activities such as the design, testing, and dissemination of educational materials and programs. A cornerstone of its mission could be the introduction of a fellowship program aimed at educators who are interested in offering quantum curricula for the first time. Such a program would equip participants with the resources, training, and mentorship needed to overcome barriers to teaching quantum computing while fostering a national community to share insights and best practices. Furthermore, the center could develop an online learning community that allows faculty to connect with other educators undertaking similar instructional efforts.

By centralizing resources, expertise, and support, the national center would lower the barriers to entry for instructors, thereby increasing the accessibility and scalability of quantum education. This initiative would catalyze the development of a robust talent pipeline to ensure that the U.S. remains a global leader in quantum science and technology.

Workforce Opportunities for Students

Preparing the next generation of the quantum workforce requires more than academic education. Public engagement, internships, and opportunities to connect with industry are all crucial for opening pathways into quantum fields, which may seem impenetrable to the uninitiated. Broadening these efforts could create more pathways for math students to gain real-world experience in quantum science and ultimately open doors to rewarding careers in this interdisciplinary field.



Internships and Work Training for Students

Expanding internship and training opportunities is essential for preparing math students to enter the rapidly growing field of QIS and providing pathways into careers in industry and the national laboratories. Currently, information about available internships—especially those outside of academia—can be difficult to find, and many students are unaware of or underprepared for these opportunities. Addressing these gaps requires a coordinated effort to ignite student interest, improve access to experiential learning opportunities, and equip educators with the tools to effectively guide their students.

A key step is fostering students' passion for QIS, as enthusiasm and engagement are critical for securing competitive internships. Encouraging participation in experiences with a low barrier to entry, such as some internships at the national laboratories, can help students explore the field while building their skills and strengthening their resumes.

Several targeted strategies could facilitate these efforts:

- Increase faculty awareness of existing student research programs
- Create an accessible venue to publicize low-barrier-to-entry experiences
- Enhance the visibility of agency-funded opportunities, particularly at non-R1 universities.

Additionally, strengthening research programs at primarily undergraduate institutions could broaden access to internships and help build a more diverse quantum workforce. Ensuring the long-term success of these efforts will require dedicated attention from faculty and sustainable funding to maintain a centralized database of opportunities.

Preparation for Quantum Careers During Graduate Education

Graduate programs in mathematics could be strategically adapted to equip master's and Ph.D. students for careers in QIS and streamline their entry into the workforce. Currently, students whose advisors do not already happen to have QIS connections must expend significant effort to independently identify relevant resources and opportunities. This lack of systematic support creates barriers that hinder the growth of a well-prepared quantum workforce.

Mathematics graduate divisions should incorporate domain-specific general education requirements, such as programming and statistics, to ensure that students develop foundational skills applicable to quantum research. Initiatives inspired by the National Science Foundation's Research Training Groups in the Mathematical Sciences program could place a further emphasis on the alignment of graduate mathematics research with QIS. Furthermore, faculty should stay informed about emerging quantum opportunities and actively share them with students, while industry representatives should be encouraged to recruit directly within mathematics graduate departments.

Establishing formal partnerships between academia, industry, and the national laboratories is another critical step to enhance preparation for quantum careers at the graduate level. Collaborations could include team-taught courses by academic faculty and external researchers, as well as a formalized process for appointing co-advisors from outside of academia. Increased funding from the National Science Foundation and other agencies for research collaborations between industry and academia would further strengthen these ties, providing additional avenues for mathematics graduate students to get involved with real-world quantum problems.

Industry Engagement in Education and Workforce Initiatives

Strengthening industry engagement in mathematics education is critical to preparing K-12, undergraduate, and graduate students for careers in the quantum workforce. However, current systems for industry participation in education are underdeveloped, limiting the opportunities for meaningful collaboration between companies and educational institutions. Large and small companies often differ in their motivations and capacities to contribute to education, necessitating tailored strategies to foster their involvement.

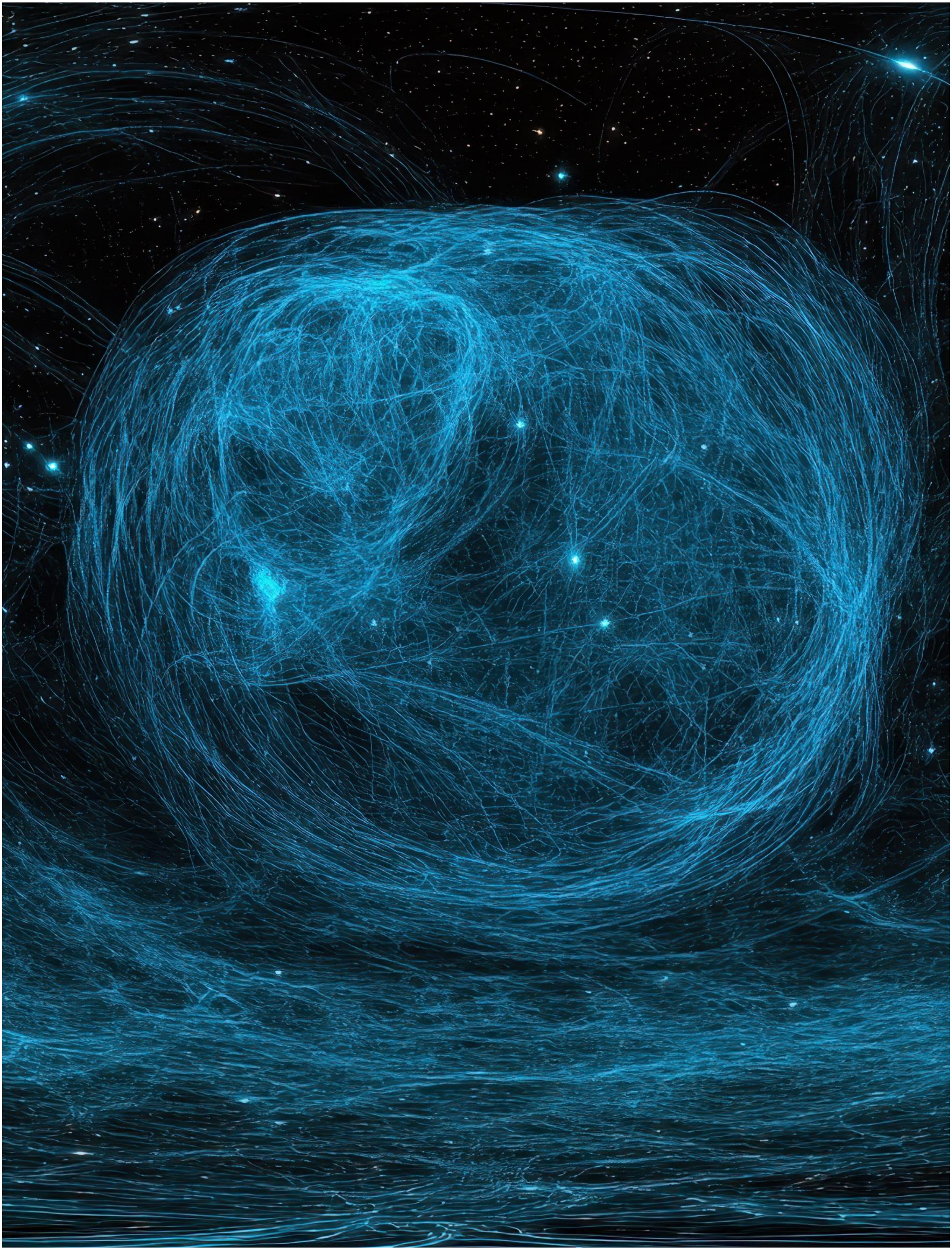
To address this gap, organizations could build on successful initiatives such as the National Q-12 Education Partnership, the Quantum Economic Development Consortium, and prize challenges from industry leaders like IBM and Google. Expanding these efforts to further involve underrepresented groups, such as rural communities, can increase the geographic and demographic diversity of students entering the quantum field.

Industry involvement should also extend to the development of math educational resources. The establishment of a comprehensive list of relevant skills and competencies would ensure alignment with industry needs. Hosting professional development sessions and offering co-op or internship programs that provide students with hands-on experiences and clear pathways to quantum careers would further strengthen the connection between industry and education. Finally, companies could be incentivized to partner with educational institutions using metrics that recognize their contributions. By integrating industry expertise and resources into education, these collaborations will help align academic preparation with real-world workforce needs and ensure that students are well-equipped to contribute to the future of quantum science and technology.

Inspiring General Audiences With Popular Science

Developing the quantum workforce begins with educating the public about the primary principles of quantum science, the potential impacts of this emerging technology on their lives, and ways to get involved in the field. Disseminating information about quantum careers outside of the typical formal channels could grow the workforce pipeline.

Technical experts should be involved in the creation of authentic, scientifically informed popular media—such as art, videos, posters, and museum exhibits—that avoid sensationalization while building interest. Cross-cutting student internships that bridge technical, artistic, and communication skills could also focus on the creation of quantum-related media, and research institutions could increase their recruitment of talented individuals who will develop creative new outlets for popular quantum communications.



Creating a Math-Centered Quantum Community

Throughout this report we have made recommendations for federal agencies to support QIS research, education, and workforce. The mathematics community also stands ready to address these challenges. Convening participants had many suggestions for ways the mathematics community and societies can support QIS awareness and community building. At SIAM, we have already started to address these goals and in this chapter we include some of SIAM's initial efforts along with further recommendations. Federal agencies should encourage and support these activities and ensure the mathematics community can participate in emerging society programs.

Bridging the gap between the Quantum Information Science (QIS) and mathematics communities is crucial for fostering interdisciplinary collaboration and advancing quantum research. However, many mathematicians remain unaware of the National Quantum Initiative Act, key quantum research challenges, or how their expertise could address these issues. The absence of clear entry points into the quantum field limits their involvement, emphasizing the need for targeted networking opportunities and resources to integrate mathematicians into the quantum ecosystem.

Applied and computational mathematicians play a critical role in advancing QIS by applying existing mathematical techniques and algorithms and developing new mathematics and approaches inspired by quantum problems. QIS challenges not only leverage established mathematical tools but also stimulate the creation of novel methods. Building and supporting a multidisciplinary, mathematically-focused community of quantum-engaged and quantum-curious researchers and students is essential. To achieve this, platforms for communicating current QIS research challenges and dedicated outlets for publishing math-centered quantum science research must be established.

Quantum information science requires the engagement of experts from many scientific domains. However, the different domain languages and formal approaches used by the various scientific research communities may present barriers to effective communication and collaborative problem-solving. Bridging these divides is essential for fostering interdisciplinary dialogue. Mathematicians must be willing to learn the languages of other disciplines, while non-mathematicians may face challenges in extracting and representing the core mathematical aspects of complex QIS problems. Addressing these challenges through targeted strategies will be vital for building a cohesive and productive math-centered quantum information science community.

It is important to build **math-centered yet multidisciplinary quantum science communities**. As a start, participants at the convening recommended the formation of a SIAM Activity Group (SIAG) on QIS. Activity groups provide an intellectual forum for SIAM members interested in exploring a particular area of applied mathematics, computational science, or cross-disciplinary applications. Members exchange ideas, collaborate on research, and develop activities related to their specialty including conferences, prizes and awards, and virtual webinar series that are broadly accessible. A virtual quantum colloquium series, modeled after successful initiatives like the Simons Institute's Quantum Colloquium, could feature interdisciplinary talks followed by panels discussing open questions where mathematicians could contribute. Recorded sessions made available on platforms like YouTube would further broaden access and impact.

There are a few established SIAGs where some activity in quantum is already present, e.g. Computational Science and Engineering, Supercomputing, Linear Algebra, Optimization, and Mathematical Aspects of Material Science, but there is a need for a solid, focused home dedicated to the "new" field of Quantum Information Science. Establishing such a group would provide a dedicated space to build connections, share findings, and publicize relevant mathematical challenges, bridging gaps between theoretical research and real-world application.

Each SIAM Activity Group has an online community platform where members can network and share information about upcoming meetings, career prospects, research, and more. Beyond SIAM membership, we propose creating a stack exchange-style forum, a dedicated Discord channel, or a community-driven wiki where individuals can post questions and open problems. This platform would allow users to collaboratively refine and reformulate these issues into mathematical terms, facilitating deeper understanding and potential solutions. To make navigation easier, the platform should be organized by specific quantum topic areas and feature tools like hashtags to allow quick searches within mathematical subfields.

These community-oriented points of contact should be linked to established resources, such as the quantum domain of the Error Correction Zoo. These touchpoints would leverage existing audiences to foster greater participation, knowledge sharing, and scientific language translation within a broader network. These dynamic platforms might encourage more researchers to contribute, both in terms of formulating questions and offering solutions, without the barriers of writing and publishing an entire paper.

The NSF-funded Mathematical Sciences Institutes must play a part in bringing researchers physically together via week-long workshops and semester-long thematic programs. For example, the Institute for Mathematical and Statistical Innovation (IMSI) ran the long program "Statistical Methods and Mathematical Analysis for Quantum Information Science" in fall 2024, the Institute for Pure and Applied Mathematics (IPAM) held the long program "Mathematical and Computational Challenges in Quantum Computing" in fall 2023, and the American Institute of Mathematics (AIM) held a one-week workshop on "Post-quantum group-based cryptography" in spring 2024. Expanding such opportunities is essential for fostering innovation and collaboration in this rapidly evolving field.

Math-Centered, Quantum Science Journal

To increase visibility and involvement of mathematicians and statisticians in quantum science research, it would be beneficial to have a peer-reviewed mathematics journal dedicated to QIS research. There are a very limited number of math journals that publish quantum-related submissions. Over the past 4 years, in the suite of 18 SIAM journals, quantum is mentioned in the abstracts of publications in several journals including SIAM J. on Computing (17), SIAM J. on Scientific Computing (16), SIAM J. on Control and Optimization (9), SIAM J. on Matrix Analysis and Applications (8), SIAM J. on Mathematical Analysis (6), and SIAM J. on Optimization (5). For comparison, SIAM published over 5000 articles in this timeframe. The SIAM Journal on Mathematical Analysis is the only one that explicitly has the word "quantum" in its *About the Journal* section.

Researchers need a dedicated mathematics journal to submit their work, so they do not need to send their articles to physics or computer science journals which are not ideal fits. For example, quantum computing papers published in physics journals often hide most of the mathematics; if at all, the mathematics may be explained in the supplementary material and relies on physical intuition which mathematicians may lack.

To enable journals to become more open to considering quantum research submissions, an immediately actionable recommendation is for editorial boards of mathematics journals to diversify their editorial boards to include quantum-informed scientists. This change would allow for a broader review and consideration of quantum-related work, fostering a more inclusive publication environment.

A more ambitious but potentially transformative recommendation is for a group of mathematicians with expertise in QIS to establish a new mathematics journal dedicated to quantum science. Such a journal would serve as a specialized platform for publishing mathematical research related to quantum topics. It was expressed that this journal would be best situated in the SIAM family of journals as SIAM is a multidisciplinary community.

Recent SIAM Activity in Quantum Science

SIAM News published its first-ever Special Issue on Quantum Computing in two parts in 2024. This double feature consisted of two introductory pieces by David Hyde and Alex Pothen who solicited and curated seven other technical articles. The resulting quantum computing content was well received by the SIAM community, as evidenced by its exceptionally high viewership numbers. David and Alex's "Introduction to Quantum Computing and Applied Mathematics" was the most read *SIAM News* article in 2024 followed by Casey Dowdle and James Whitfield's "Practical Introduction to Quantum Computing"; and Giacomo Nannicini's "What Can Quantum Computers Do for Applied Mathematicians" is one of the top ten. Recent *SIAM News* quantum material and more is available at on the *SIAM News* website and from the second part of the introduction to the special issue.

The SIAM Activity Groups (SIAGs) support collaboration across various disciplines in applied mathematics, computational science, and engineering.

Quantum computing is an emerging focus across several groups:

Computational Science and Engineering (CSE): Promotes computational technologies for scientific problem-solving, highlighting quantum algorithms at the 2023 CSE Conference.

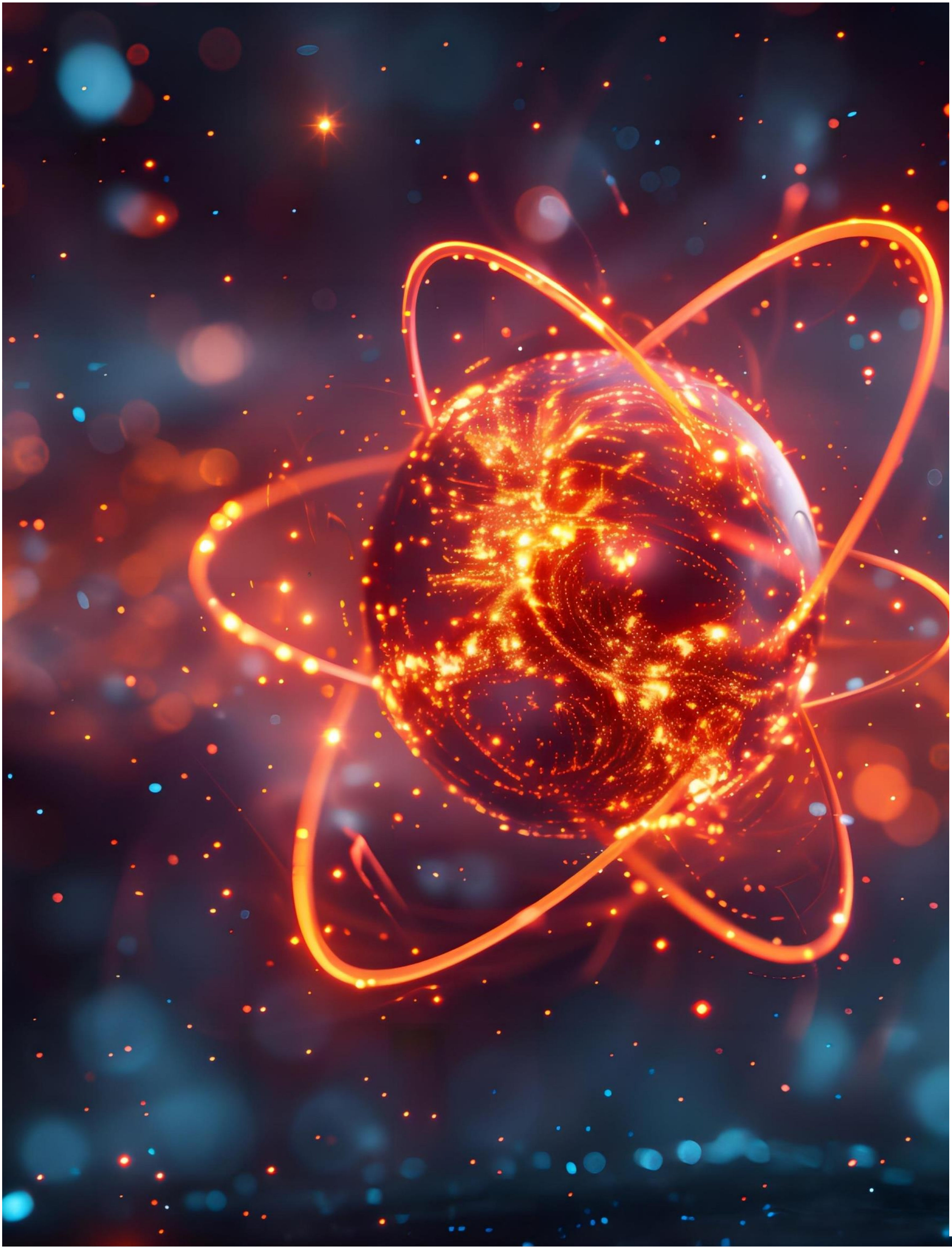
Supercomputing: Facilitates discussions on high-performance computing, featuring quantum-related sessions at the 2024 Parallel Processing Conference.

Linear Algebra: Advances linear algebra research, with quantum computing as a key theme at its 2024 triennial conference in Paris.

Optimization: Focuses on optimization theory and software, addressing quantum computing applications at its 2023 conference.

Mathematical Aspects of Material Science: Bridges mathematics and material science, discussing quantum chemistry and electronic structure at the 2024 Materials Science Conference [MS24].

Additionally, the 2023 Gene Golub SIAM Summer School at Lehigh University centered on quantum computing and optimization, fostering interdisciplinary learning among students and experts. SIAM is excited to build upon these activities to foster a strong mathematics community around QIS research and education challenges and looks forward to working with NSF to advance the recommendations in this report.



Appendix 1: Convening Schedule

Below is the agenda and text that was provided to the SIAM Quantum Intersections Convening participants.

The workshop will include highly interactive, hands-on activities paired with each of the speaking segments. Expect to dive in and engage with your fellow participants in iterative, collaborative activities. Expect the facilitation team to be responsive and flexible in designing each day to the needs of the group and the goals of the event. and bring a sweater in case the room gets cold.

Below is a flexible agenda overview for your reference. We thank you and look forward to working with you.

Day 1 Agenda Overview

Please note that each speaker segment includes a presentation followed by an interactive, collaborative discussion activity.

Time	Activity
8:00 am	**Coffee & Breakfast** Conference Room C
8:30 am	**Registration & Doors Open** Conference Room AB (Main Plenary)
9:00 am	**Workshop Kick-Off, Welcome & Call to Action**
9:40 am	**Broad Overview of Quantum Science** Bert de Jong (Lawrence Berkeley National Laboratory) (followed by interactive discussion activity)
10:45 am	**Break**
11:00 am	**The National Quantum Initiative** Jacob Taylor (NIST)
11:20 am	**Federal Outlook and Opportunities for Quantum Research and Workforce Development** Miriam Quintal (Lewis-Burke Associates LLC) (followed by interactive discussion activity)
12:00 pm	**Lunch** Conference Room C
1:00 pm	**Quantum Algorithms Teaser** Jeffrey Larson (Argonne National Laboratory)

Time	Activity
1:10 pm	**Quantum Error Corrections** Michael Perlin (JPMorgan Chase) (followed by interactive discussion activity)
2:05 pm	**Quantum Cryptography** Qipeng Liu (University of California, San Diego) (followed by interactive discussion activity)
2:55 pm	**Break**
3:10 pm	**Post Quantum Cryptography** Angela Robinson (NIST) (followed by interactive discussion activity)
4:15 pm	**Quantum Science – Hope, Hype, Hamiltonians, and Homology Panel Discussion** Lior Horesh (Moderator, IBM) Di Fang (Duke University) Marcos Crichigno (Phasecraft) Alex Dalzell (AWS Center for Quantum Computing) (followed by interactive discussion activity)
5:10 pm	**Closing Remarks & Next Steps**
5:20 pm	**Adjourn**
6:00 pm	**Dinner** Conference Room C (Meal Room)

Day 2 Agenda Overview

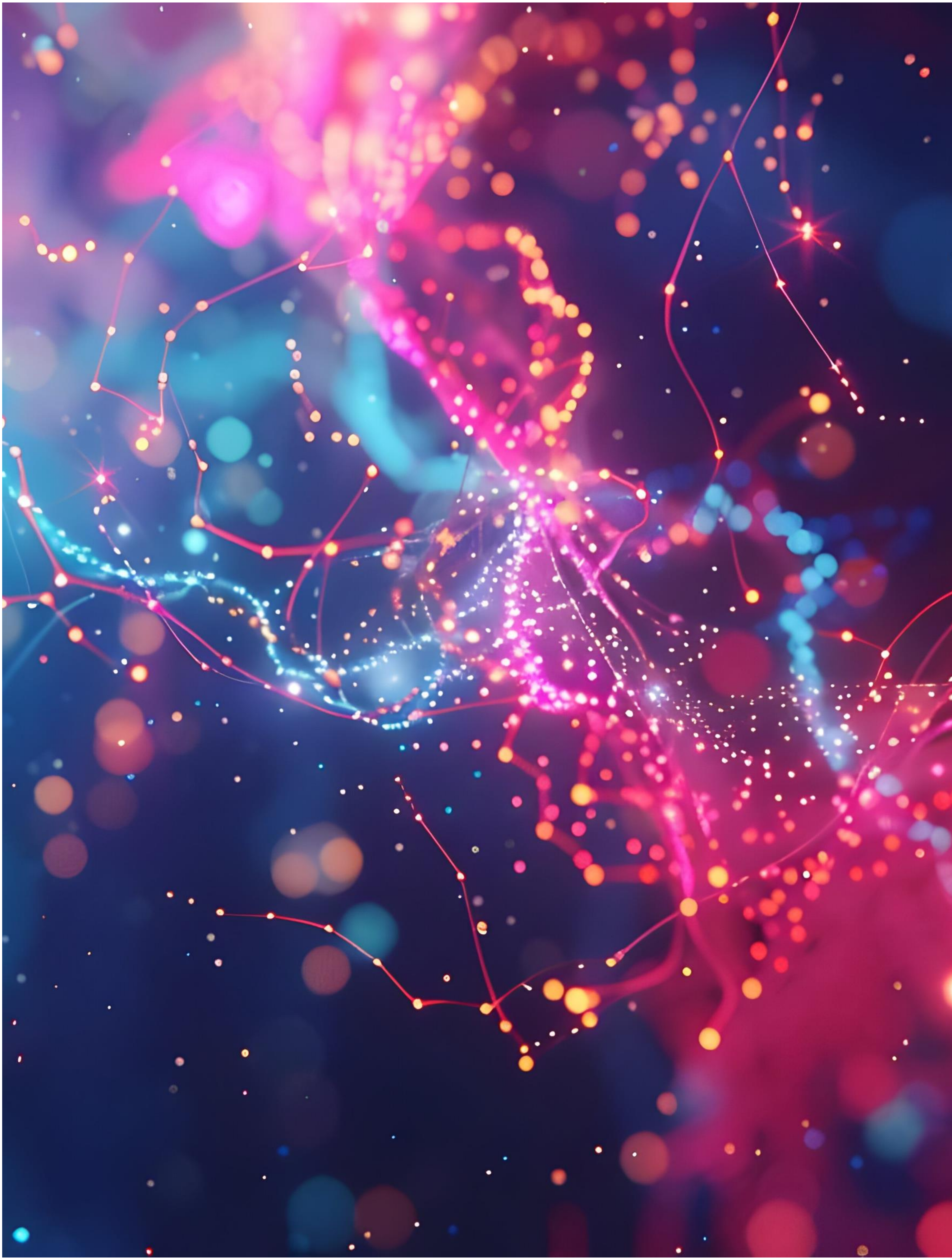
Please note that each speaker segment includes a presentation followed by an interactive, collaborative discussion activity.

Time	Activity
8:00 am	**Coffee & Breakfast** Conference Room C (Meal Room)
8:30 am	**Doors Open** Conference Room AB (Main Plenary)
9:00 am	**Welcome & Kick-off**
9:10 am	**Quantum Machine Learning** Alex Dalzell (AWS Center for Quantum Computing) (followed by interactive discussion activity)
10:05 am	**Break**
10:20 am	**Quantum Chemistry** James Whitfield (Dartmouth College, AWS) (followed by interactive discussion activity)
11:25 am	**Quantum Optimization** Ruslan Shaydulin (JPMorgan Chase) (followed by interactive discussion activity)
12:00 pm	**Lunch** Conference Room C
1:00 pm	**Industry Panel** Kirk Jordan (Moderator, IBM) Marcos Crichigno (Phasecraft) Mariam Kiran (Oak Ridge National Lab) Ruslan Shaydulin (JPMorgan Chase) (followed by interactive discussion activity)
2:20 pm	**Break**
2:35 pm	**Quantum Education Panel** Bashir Mohammed (Moderator, Intel) Emily Edwards (Duke University) Saif Rayyan (IBM) (followed by interactive discussion activity)
3:40 pm	**Recommendation Work**
5:05 pm	**Closing Remarks & Next Steps**
5:15 pm	**Adjourn**
6:00 pm	**Dinner** Conference Room C

Day 3 Agenda Overview

Please note that each speaker segment includes a presentation followed by an interactive, collaborative discussion activity.

Time	Activity
8:00 am	**Coffee & Breakfast** Conference Room C
8:30 am	**Doors Open** Conference Room AB (Main Plenary)
9:00 am	**Welcome & Kick-off**
9:10 am	Quantum Software & Compiler Kate Smith (Northwestern University) (followed by interactive discussion activity)
10:00 am	**Break**
10:15 am	**Quantum Communications & Networking** Mariam Kiran (Oak Ridge National Laboratory) (followed by interactive discussion activity)
11:10 am	**Recommendation Work**
12:00 pm	**Lunch** Conference Room C
12:50 pm	Concurrent Sessions: **Qiskit Demo** with David Hyde And **Finalize Recommendations** (participants will divide between the two concurrent sessions and then switch)
2:20 pm	**Final Recommendations Work & Team Feedback**
2:55 pm	**Closing Remarks & Next Steps**
3:00 pm	**Adjourn**



Appendix 2: Convening Participants

First Name	Last Name	Affiliation
Jimmie	Adriazola	Southern Methodist University
Angelynn	Alvarez	Embry-Riddle Aeronautical University, Prescott
Gaik	Ambartsoumian	University of Texas at Arlington
Kossi Pierre	Amenoagbadji	Columbia University
Daniel	Appelo	Virginia Tech
Anthony	Bloch	University of Michigan
Henry	Boateng	San Francisco State University
Ron	Buckmire	Marist College
Jan	Cameron	National Science Foundation
Sarah	Chehade	Oak Ridge National Laboratory
Brian	Choi	United States Military Academy
Giuseppe	Cotardo	Virginia Tech
Christopher	Cox	University of Tennessee at Chattanooga
Marcos	Crichigno	Phasecraft US
Alex	Dalzell	AWS Center for Quantum Computing
Wibe	de Jong	Lawrence Berkeley National Laboratory
Zijian	Diao	Ohio University
Baboucarr	Dibba	University of Texas Rio Grande Valley
Wandi	Ding	Middle Tennessee State University
Yulong	Dong	University of California, Berkeley
Emily	Edwards	Duke University
Alperen	Ergur	University of Texas San Antonio
Fariba	Fahroo	Air Force Office of Scientific Research
Di	Fang	Duke University
Zachary	Flores	Two Six Technologies
Dmitry	Golovaty	National Science Foundation
Stefanie	Guenther	Lawrence Livermore National Laboratory
Kaytlin	Harrison	Argonne National Laboratory
Cameron	Hogan	Cornell University
Lior	Horesh	IBM Research
Andrew	Horning	Rensselaer Polytechnic Institute
Yuxin	Huang	University of Southern California
David	Hyde	Vanderbilt University
Erika	Jones	NASA (contractor)
Kirk	Jordan	IBM Research Emeritus
Mariam	Kiran	Oak Ridge National Laboratory
Joseph	Kraisler	Amherst College
Jillian	Kunze	Society for Industrial and Applied Mathematics
S. Janani	Lakshmanan	University of Hawai`i at Mānoa
Jeffrey	Larson	Argonne National Laboratory
Rich	Lehoucq	Sandia National Labs
Fengyan	Li	Rensselaer Polytechnic Institute
Chi-Kwong	Li	College of William & Mary
Qipeng	Liu	University of California, San Diego
Diyi	Liu	University of Minnesota, Twin Cities
Ziwei	Ma	University of Tennessee at Chattanooga
Fenglou	Mao	National Institutes of Health
Gretchen	Matthews	Virginia Tech

First Name	Last Name	Affiliation
Anastasiia	Minenkova	University of Hartford
Atefeh	Mohajeri	Bell Labs
Muzamil	Mohamed	University of Hawaii at Manoa
Bashir	Mohammed	Intel Corporation
Jose	Morales Escalante	University of Texas at San Antonio
Ceferino	Obcemea	National Cancer Institute
Tomoki	Ohsawa	University of Texas at Dallas
Iván	Ojeda-Ruiz	Texas State University
Michael	Perlin	JPMorgan Chase
Brian	Pigott	Wofford College
Petr	Plechac	University of Delaware
Alex	Pothen	Purdue University
Miriam	Quintal	Lewis-Burke Associates LLC
Jorge	Ramirez	Oak Ridge National Laboratory
Saif	Rayyan	IBM
Reece	Robertson	University of Maryland, Baltimore County
Angela	Robinson	National Institute of Standards and Technology
Alexandra	Seceleanu	University of Nebraska-Lincoln
Ignacio	Segovia-Dominguez	West Virginia University
Julia	Shapiro	Virginia Tech
Ruslan	Shaydulin	JPMorgan Chase
Ashmeet	Singh	Whitman College
Kaitlin	Smith	Northwestern University
Mark	Squillante	Mathematical Sciences Department, IBM Research
Francis	Su	Harvey Mudd College
Jacob	Taylor	National Institute of Standards and Technology
Roel	Van Beeumen	Lawrence Berkeley National Laboratory
Christelle	Vincent	University of Vermont
Ryan	Vogt	U.S. Department of Defense
Suzanne	Weekes	Society for Industrial and Applied Mathematics
James	Whitfield	Dartmouth College, AWS
Elaine	Wong	Oak Ridge National Laboratory
Yang	Yang	Michigan State University
Xueyu	Zhu	University of Iowa
Ludmil	Zikatanov	National Science Foundation