CONSIDERING THE ALIGNMENT BETWEEN TEACHERS' CONCEPTIONS OF JUSTIFICATION AND THEIR VISIONS FOR EQUITABLE INSTRUCTION

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While proving, and more broadly conceived "reasoning and sense-making," have received a great deal of attention in mathematics education research over the past three decades, recently scholars have argued for the importance of justification as a learning and teaching practice. As teachers work toward realizing goals for more equitable classroom environments, little is known about whether teachers' conceptions about mathematical practices, such as justification, reflect an understanding of how students' engagement in those practices can support more than just mathematical achievement. In this paper, we present findings from our analysis of interviews with 10 secondary mathematics teachers engaged in participatory action research to explore connections, and potential disconnections, between teachers' conceptions of justification and their visions for equitable instruction.

Keywords: Teacher Beliefs; Reasoning and Proof; Professional Development; Equity

Strand: Mathematical Processes and Practices

Developing a deep understanding of mathematics is a core principle of equitable mathematics teaching (Horn, 2012). Scholars have examined, both theoretically and empirically, the role that opportunities to justify play in advancing equitable learning outcomes in K-12 classrooms (Bartell et al., 2017; Boaler & Staples, 2007). Some existing research has offered insights about whether opportunities that teachers provide students to engage in sensemaking and justification are robust, meaningful opportunities (Bieda, 2010; Henningsen & Stein, 1997), and other studies have provided insight into how classroom norms can influence students' access to and participation in argumentation practices (Klosterman, 2016; Staples, 2007; Yackel & Cobb, 1996). However, little is known about whether teachers' conceptualizations of justification and its role in school mathematics aligns with teachers' views about equitable learning outcomes. We argue that how teachers conceive of what justification is, and its role in teaching and learning mathematics, influences how they utilize justification opportunities as a tool to advance equitable learning outcomes. This paper explores the question, "In what ways do teachers' conceptions of justification align with, or deter from, their visions for equitable classrooms?"

Background

Teachers' Conceptions of Justification

Although the role of proof in the discipline of mathematics has been well-documented, there is less understanding of the role of justification and its relationship to proof and proving and

other mathematical reasoning processes essential to learning mathematics (Ellis et al., 2021). However, teachers may see a bigger role of justification in school classrooms, as they often question the role that formal proof should play in students' learning of mathematics and what students are expected to learn and be able to do (Knuth, 2002). In interviews with seventeen high school mathematics teachers, Knuth found that fourteen teachers did not feel proof should play a central role in learning mathematics, whereas all of the teachers indicated that informal proofs – such as explanations and being able to justify one's reasoning – were an important part of learning mathematics. Given the importance that teachers place on justification to support students' mathematical learning, understanding how they define justification may offer windows into the ways that they incorporate this process into the teaching of mathematics.

The Role of Justification in Teaching and Learning Mathematics

Teachers' conceptions of justification, and the role it plays in learning and teaching mathematics, influences the opportunities teachers provide for students to engage in justification (Gonzalez Thompson, 1984). The work of Staples, Bartlo, and Thanheiser (2012) specifically explored ways that teachers came to understand the role of justification through a professional development with 12 middle school teachers (grades 6 - 8) designed to co-inquire with teachers to-understand justification at the middle school level and its importance for learning mathematics. Staples and colleagues discovered that teachers aligned the purposes of justification much more with its role as a teaching practice and a learning practice than its usefulness in establishing the validity of mathematical results. Specifically, teachers discussed its value in students' mathematics learning, particularly promoting conceptual understanding, fostering valued mathematical skills and dispositions. Additionally, teachers discussed justification's value to support teaching, such as gathering information about what students know, supporting students' engagement with other students and enabling more student-student interactions, and supporting students' sense of agency in and outside of the classroom.

The study reported in this paper builds upon the work of Staples, Bartlo and Thanheiser (2012) to explore how teachers see justification as playing a role in teachers' efforts to create more equitable learning environments. Specifically, we explore conceptually the connections between ways justification is defined and the dominant and critical dimensions of equity as conceptualized by Gutiérrez (2012). Further, we illustrate, based on data from 11 high school mathematics teachers participating in a study group, alignments and disconnections between teachers' conceptions of justification and their descriptions of equitable learning environments.

Theoretical Framework

Gutiérrez (2012) conceptualized equity in mathematics education as comprising dimensions that reflect a dominant perspective (*access* and *achievement*) as well as a critical one *(identity* and *power)*. The components of the dominant perspective have been the traditional focus of gaps-oriented equity work, namely the opportunities each and every student has to learn rigorous and meaningful mathematics (access) as well as how well they can demonstrate what they have learned as a gateway for academic success (achievement). In the past two decades, elements of the critical dimension have gained more prominence in mathematics education research, due in part to their role in rewriting the script of how mathematics should be taught and learned. Gutiérrez frames *identity* using a window/mirror metaphor; "students need to have opportunities to see themselves in the curriculum (mirror), as well as have a view onto a broader world (window)" (2012, p. 19-20). As students are able to use mathematics to make sense of the world, they need agency to engage in social transformation as a result of their mathematics learning.

This component, *power*, prompts us to consider how we are promoting students with the mathematical power they need to change systems they deem unjust.

In our work, we draw upon this frame to think about how possible conceptions and purposes of justification may work to support these aspects of equity. For example, if we consider justification as "the process of supporting your mathematical claims and choices when solving problems or explaining why your claim or answer makes sense" (Bieda & Staples, 2020, p. 103), the role of justification in teaching and learning mathematics goes beyond explaining the veracity of a claim. By also seeing moments where students explain their problem-solving *choices* as the mathematical practice of justification, teachers implicitly communicate that students have agency to not only argue why they have a correct answer, but also why their solution strategy is valid (even if it is different from other strategies). This conception not only supports students' achievement (dominant axis), but also students' identity (critical axis), in that *their* ways of doing mathematics have a space to be legitimized through justification.

Building on our data, we argue first that teachers' conceptions of justification matter for how justification can be employed in their classrooms to advance equity goals. We further argue that various conceptions and purposes of justification align with components of Gutiérrez's (2012) frame on equity more principally than other components. We might imagine these conceptions imposed on the dominant and critical axes of Gutiérrez's framing of equity, with their placement highlighting how particular conceptions frame justification relative to dominant and critical mathematics within school mathematics. For instance, Figure 1 shows the placement of Bieda and Staples' (2020) conception along the axes.

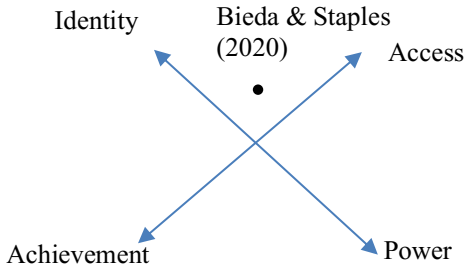

Figure 1: Plotting conceptions of justification along dimensions of equity

Methods

Research Context

To better understand how teachers conceptualize justification and its relationship to equity, we have been working with ten teachers and one teacher candidate from two high schools in two different states (one in the Midwest and one in the Northeast) to conduct participatory action research over the course of the 2022-2023 academic year. These teachers volunteered to participate in this collaboration given their interests and commitments to advance equity in their classrooms. The majority of the participants are white, and the majority of their students come from minority backgrounds and homes with low income. The teaching experience of the in-service teachers ranges 3-33 years and averages 12.4 years.

Prior to their participation in the study group, participants engaged in two activities. First, we conducted a pre-interview with each participant to gather background information and experiences, including their initial thoughts about justification and equity. Second, during summer 2022, we held a workshop at each school (12-15 hours) with the goal of working toward

shared understanding of justification and equity within each group. Since the start of the school year, we have held 1.5-hour study group meetings with the participants twice per month at each site. During the meetings, teachers report about the successes and challenges they are experiencing in supporting equitable learning opportunities and in providing students justification opportunities. Teachers' thoughts about the relationship between these two are also discussed.

Data Collection

This paper presents findings from our pre-interviews with each teacher. The pre-interviews gathered each teacher's initial conceptions of justification and equity. Teachers were asked, "How would you define justification in mathematics?", "What is the relationship you see between equity and justification?", and "When you think about creating equitable opportunities to learn, what does that mean to you?" Interviews were conducted by two members of the project team on Zoom, recorded, and transcribed.

Data Analysis

To understand the alignment between teachers' conceptions of justification and equity, we analyzed responses to each question separately and then looked across analyses to synthesize findings. To analyze responses to the questions, "How would you define justification in mathematics?" and "What is the relationship you see between equity and justification?", we first conducted open coding using a thematic analysis (Braun & Clarke, 2006) approach to teachers' definitions of justification. To analyze responses to the question, "When you think about creating equitable opportunities to learn, what does that mean to you?," we identified utterances corresponding to specific dimensions of Gutiérrez's (2012) framework for equity.

Findings

In this section, we present findings from our analysis of three cases that represent the range of responses provided by participants about their definitions of justification. In sharing these cases, we will juxtapose their definitions of justification with their thoughts about equitable opportunities to learn, to explore the alignments, and potential contradictions, between these conceptions.

Case 1: Justification as Revealing Students' Thinking

The first case, William, is an 9th-grade mathematics teacher in his fifth year of teaching. When asked to define justification, he responded:

> *"I mean, as simple terms, it's their thought process. How they got their answer. Getting it on paper. Because your goal as a teacher is to kind of provide those skill sets and for them to, you know, understand the process and the math skills that are needed to answer that, to breakdown a question. And so that justification piece is really, you know… show me your work. But it's more than that. But like, you're critical, like what was your thought process? How did you know that? What did you do? Why did you do it? And just getting that on paper because that's, end of the day, is like when they take those tests you know they have to be able to get it on paper. So that's that justification piece for me"* (William, Pre-Interview)

William's conception of justification is focused on assessing student's understanding, or "thought process." He discusses that justification involves how students arrived at their answer and why they solved the problem in the way they did. We also noted that his definition focuses more on individual understanding, rather than the collective understanding of the class.

We found that all of William's statements about equitable learning opportunities were coded on the dominant axis. For him, equity was about providing each individual student (*access*) what they need to be successful (*achievement*). He mentioned, "Success looks different for every

student and I think giving students you know opportunities to improve their practice is crucial, but what you provide each student to be successful is going to be different... and so the equity piece is just giving students opportunities to improve in different ways at the level they're at." Thus, if justification is a means for assessing students' thought processes, it is a tool for supporting both the access and achievement elements of Guttiérez's (2012) dimensions of equity.

Case 2: Justification as Explaining and Convincing Other People

Lynda is a high school mathematics teacher (grades 10-12) in her fourth year of teaching. Lynda described justification as:

> *"basically explaining your thinking and being able to prove whatever it is that you want to claim in a way that makes sense to other people. So I think a lot of times you can justify something in your own mind, but then actually coming up with the words or the logic or the picture or whatever the case may be to prove it to somebody else."* (Lynda, Pre-Interview)

In contrast to William's definition, Lynda describes justification as going beyond explaining your thinking to be able to "prove whatever it is that you want to claim in a way that makes sense to other people." Lynda's definition emphasizes how justification is a collective activity; coming up with a justification involves considering what will be convincing to somebody else.

When asked to explain about creating equitable learning opportunities, Lynda emphasized *access* and *identity*:

> "*I think [the basic idea is] that everybody has access. But to expand on that, right, that everybody would be able to, like, assess the problem from the beginning, but also, like remain in the problem the entire time. So I think like some lessons are built so that you know, there's like the opener that everybody can access, but then things step up, and maybe you lose people on the way. So to create like a whole equitable lesson some way that everybody can get in. And if you kind of fall out that there's a way back in. And I don't know, just being mindful of people's different like backgrounds in terms of like background knowledge for the math needed background in terms of just like cultural experiences, that may or may not play a part in the lesson. And then also just different, like accessibility needs if people have like, different impairments that might need to be adjusted for.*" (Lynda, Pre-Interview)

Lynda's attention to how lessons need to provide access for each student, and her awareness of differences in students' mathematics and cultural backgrounds, reflects a commitment to ensuring that all students can participate in her classroom mathematical community. By defining justification in a way that is not just convincing to oneself but also to others, it positions justification as building a classroom where students' access to the mathematical tasks is important and students' varied backgrounds are embraced as part of their mathematical identities.

Case 3: Justification as Knowing Why

Emma is a 9th- and 10th-grade mathematics teacher in her sixth year of teaching. When asked to define justification, Emma stated:

> *"I think it's being able to like explain to somebody why you did something rather than just what you did. I think a big issue is that we tend to teach processes and how to memorize processes, and we don't spend as much time focusing on like why are we actually doing this, what does it connect to. And so I think that that mathematical justification is just being able to explain the why rather than just the what."* (Emma, Pre-Interview)

We argue that Emma's response is somewhat of a blend of the ideas from William's and Lynda's responses. Emma's focus on justification promotes understanding the "why" behind the processes echoes William's attention to justification going beyond a student explaining how they

got an answer. Yet, like Lynda, Emma also mentions that justification involves explaining to others.

When asked about what it means to create equitable learning opportunities in her classroom, Emma's responses reflected attention to *access* in a couple of ways. She indicated:

> "*And so I try to think about like not just creating learning experiences based off of what like I would be able to successfully engage in, but thinking about the different things that might hold somebody back or make them experience it differently."* (Emma, Pre-Interview)

She continued: "*... it starts with just like getting to know your students and figuring out what they need."* Her attention to knowing students and their lives outside of class was predominant in her responses to questions related to creating equitable opportunities to learn, reflecting her concern for supporting students' identities, and using knowledge about their out-of-class identities to inform her instruction.

When considering if Emma's conception of justification aligns with her vision for creating equitable opportunities to learn, it is less clear whether her emphasis on being able to "explain to somebody why you did something" is supportive of her efforts to create a learning environment that connects with who they are and how they learn within and outside of the mathematics classroom.

Holistic Results

Overall, the majority of teachers (8 of 10) describe justification as an individual activity that involved some kind of written record, and provided a detailed accounting of what they students knew or how they had solved a problem. Only 2 of 10 teachers suggested justification was a *practice* where students engaged one another and potentially built knowledge. Additionally, when discussing what it means to create equitable learning opportunities, the majority of teachers' responses reflected aspects of the dominant axis of equity (Gutiérrez, 2012) and only 3 of the 10 teachers discussed aspects related to supporting the critical axis. Although a noticeable minority, the responses from those three teachers reflected a vision of supporting the *identity* dimension of the critical axis (Gutiérrez).

Discussion

Although scholars have argued how justification plays a role in promoting equity (Bieda & Staples, 2020; Boaler & Staples, 2008), little empirical work has been done to show that teachers' conceptualizations of justification, and therefore the nature of the justifications and justification activity they expect from students, reflect and align with their goals for creating equitable learning opportunities. In our findings, we discovered that most of the participating teachers conceive of justification as an activity that can promote students' access to deeper understanding of mathematics, but tend to focus less on how justification can support students' mathematical identities and become a tool for exercising power in changing their worlds. Moreover, the tendency to focus on elements of access and achievement were evident in their vision for creating equitable learning opportunities in their classrooms. What these findings suggest is that more work is needed to help teachers go beyond recognizing justification as a means for showing deeper understanding and explaining why, but to also conceptualize justification as an activity that builds mathematical identities needed for students to both step into and exercising agency with advocating for change in society.

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