

COMPARING STAND-LEVEL VOLUME GROWTH AND YIELD PREDICTIONS FROM TIME-EXPLICIT, STATE-SPACE, AND SIMULTANEOUS APPROACHES ACROSS COMPLEX MIXED FOREST STANDS IN THE ACADIAN REGION OF NORTH AMERICA

CEN CHEN^{1,2}, BABURAM RIJAL³, AARON WEISKITTEL^{4*}, JOHN A. KERSHAW, JR.⁵

¹ Hainan University, School of Tropical Agriculture and Forestry, Danzhou, Hainan, China

² Alabama A&M University, Department of Natural Resources and Environmental Sciences, Normal, AL, USA

³ University of Quebec at Rimouski, Department of Biology, Chemistry and Geography, Rimouski, QC, Canada

⁴ University of Maine, Center for Research on Sustainable Forests, Orono, ME, USA

⁵ University of New Brunswick, Faculty of Forestry and Environmental Management, Fredericton, NB, Canada

*Corresponding Author

ABSTRACT. Stand-level growth and yield models are computationally simple and have moderate demands on input information, but their applications have largely been limited to single-species and even-aged stands. State-space and simultaneous approaches that are not explicitly functions of time potentially extend the applicability of stand-level growth and yield models to mixed-species and/or multi-cohort stands where age (and the period of stand growth that often is derived as the difference in ages) may not be an available and/or suitable predictor. The Acadian Forest, dominated by naturally-regenerated and mixed-species stands of primarily multi-cohort structure, offers an ideal framework for a comprehensive assessment of alternative approaches such as time-explicit, state-space, and simultaneous approaches' performance in stand-level growth and yield predictions across a wide range of complex forest stands. We conducted a comparative assessment utilizing an extensive database from this region. It was found that the three approaches were highly consistent in providing relatively accurate and largely unbiased stand-level growth and yield predictions. The time-explicit approach had a simplistic form but similar prediction performances comparable to the other two more complex modeling approaches. In comparison, the simultaneous approach, despite being path-invariant, was computationally challenging and offered limited improvements from the other approaches. Our findings showed potential applicability of stand-level growth and yield models beyond single-species and even-aged forests.

Keywords: Uneven-aged stands; mixed stands; growth and yield models; forest management; base-age-invariance; path-invariance.

1 INTRODUCTION

Stand-level growth and yield models are characterized by their computational simplicity, moderate demands on input information, robustness, and ease of application (Weiskittel et al., 2011a), particularly for mortality predictions (e.g., Garcia, 2009). This characteristic depends on the premise that trees in a stand are similar and their distribution is generally uniform. Conse-

quently, individual trees in a stand are assumed to vary little in growth and yield. At the same time, their collective dynamics can be realistically predicted as a whole by stand-level models without accounting for individual-tree differences and potential interactions. This simplicity makes stand-level growth and yield models a convenient tool for forest management, but may limit their application primarily to single-species and even-aged stands subject to standardized silvicultural treatments (e.g., Garcia, 1994; Navar et al., 2016).

Time has regularly been a primary predictor of stand-level growth and yield and has often been utilized in the form of age or a period of time as the difference in ages (e.g., Clutter [1963], Fang et al. [2001], Pienaar and Shiver [1986]). However, information on age is not always readily available, and can be difficult to determine in complex natural stands. In addition, understory and overstory trees as well as trees of different species, obviously have diverging paths of growth over time in mixed-species and/or multi-cohort stands. In addition, natural disturbances and selection cutting also cause changes in growing space and, consequently, the growth of trees (Chen et al. [2017a], Pamerleau et al. [2015]). Therefore, age and difference in ages may not always be optimal predictors of stand-level growth (Peng [2000]), and the validity of predicting stand-level growth explicitly as a function of time (i.e., by a time-explicit approach) needs to be further evaluated in mixed-species and/or multi-cohort stands.

Garcia [1994] described a state-space approach for stand-level growth and yield predictions. The current state of a stand is represented by a few state variables (e.g., basal area and dominant height), whose current values are predicted from their previous values by differential/difference equations (i.e., transition functions). Yield is subsequently predicted as a function of the current state by an output function. The most notable feature of a state-space approach is that it characterizes the state of the system in a particular moment of time and uses this information to project the future values of the principle stand variables (Garcia [1994]). Consequently, stands in the same state will have the same growth in the following period of time, regardless of the various temporal paths (ages) to the state. There are several examples of the state-space approach for single-species, even-aged stands (e.g., Cieszewski and Bella [1993], Nord-Larsen and Johannsen [2007], Tait et al. [1988], Tewari et al. [2014], Waldy et al. [2021]). A state-space approach potentially extends the applicability of stand-level growth and yield models to mixed-species and/or multi-cohort stands where age and hence the growth period between ages may not be an available and/or suitable predictor, which has been previously showcased by Garcia [2011]. A more comprehensive assessment of the state-space approach may be warranted as Garcia [2011] primarily focused on even-aged, mixed spruce (*Picea* spp.) stands in sub-boreal British Columbia.

An important limitation of the state-space approach comes from its two-piece design of a set of transition functions and an output function. Parameters optimized for the transition functions do not necessarily ensure the most accurate yield predictions from the output function. A potential advance from the state-space approach is a simultaneous approach. Sullivan and Clutter [1972]

proposed a simultaneous approach before the idea of a state-space approach was formalized. Basal area was a function of age and also a predictor for stand-level volume yield in this simultaneous approach. The function used to predict basal area was inserted into the volume yield function to estimate parameters in both functions simultaneously. Based on the idea of Sullivan and Clutter [1972], transition functions in a state-space approach can also be inserted into the output function (i.e., yield function) to obtain simultaneous estimations of all parameters that optimize yield and hence growth predictions. An advantage of this simultaneous approach is achieving path-invariance (and possibly base-age-invariance), which is sometimes considered a preferred model property that may improve analytic compatibility between growth and yield predictions and reduce errors in these predictions (Burkhardt and Tome [2012], Rose et al. [2003]). Beyond Sullivan and Clutter [1972], the simultaneous approach has seen limited applications for growth and yield predictions (e.g., Borders [1989], Borders and Bailey [1986]).

The state-space approach has seen increased applications in recent years (e.g., Diéguez-Aranda et al. [2006], Nord-Larsen and Johannsen [2007], Stankova [2015], Tewari and Singh [2018]). Comparative assessments of the three approaches are needed to test whether the theoretical advancement, such as path-invariance (e.g., Cieszewski [2021], Cieszewski and Bailey [2000], Cieszewski and Strub [2018]), translates into improved stand-level growth and yield predictions. However, this type of assessment is scarce in the literature. Waldy et al. [2021] recently compared the time-explicit and state-space approaches in hybrid *Eucalyptus* plantations. This comparison found that the time-explicit approach performed generally better than the state-space approach for single-species plantations, where age can be a primary driving factor. However, the analysis did not evaluate the potential advantage of the simultaneous approach, which is path-invariant contrary to the other two approaches. In addition, the applicability of these modeling approaches to mixed-species and/or multi-cohort stands remains unevaluated mainly, except for Garcia [2017].

The Acadian Forest, dominated by naturally-regenerated and mixed-species stands of primarily multi-cohort structure, offers an ideal environment for comprehensively assessing the three approaches' performance in stand-level growth and yield predictions across a wide range of complex forest stands. This comparison potentially provides early evidence on whether the applicability of stand-level models can be extended appropriately beyond single-species and even-aged stands by using the state-space and simultaneous approaches. In addition, stand-level equations, except for the recent mortality

models developed by Chen et al. (2023), remain relatively rare in this region and may offer key advantages in specific applications like constraining tree-level analyses or large-scale optimization efforts (Weiskittel et al. 2011a).

Specific objectives of this study were to: 1) develop comparable stand-level volume growth and yield models based on time-explicit, state-space, and simultaneous approaches, 2) apply these models across a wide range of mixed-species and/or multi-cohort stands in the Acadian Region, and 3) assess the performance of these models in stand-level volume growth and yield predictions. Expected findings were the general robustness of these various approaches with potential advantages of the state-space and simultaneous approaches being present as stand complexity increases.

2 METHODS

2.1 Study area

The Acadian Forest region resides in the transition zone between the softwood-dominant boreal forests to the north and the hardwood forests to the south (Rowe 1972). This region encompasses three Canadian Maritime Provinces (New Brunswick, Nova Scotia, and Prince Edward Island) along with southern portions of Québec and much of the US state of Maine. Across the region, climate estimates indicate that average annual precipitation is 1,130 mm with a range of 870 to 1,750 mm, while mean growing degree days (sum of temperatures $>5^{\circ}\text{C}$) is 1,625 days with a range of 726 to 2,292 days (Rehfeldt 2006). Glacial till is the principal soil parent material, with soil types ranging from well-drained loams and sandy loams on glacial till ridges to poorly and very poorly drained loams on flat areas between low-profile ridges.

The Acadian Forest is dominated by naturally-regenerated, mixed-species stands that primarily display multi-cohort stand structures. Among the over 60 tree species that are found in the region are coniferous evergreen species such as red spruce (*Picea rubens* Sarg.), balsam fir (*Abies balsamea* L. Mill.), eastern white pine (*Pinus strobus* L.), and eastern hemlock (*Tsuga canadensis* (L.) Carr.) as well as deciduous hardwood species such as red maple (*Acer rubrum* L.), yellow birch (*Betula alleghaniensis* Britton), sugar maple (*Acer saccharum* Marsh.), American beech (*Fagus grandifolia* Ehrh.), paper birch (*Betula papyrifera* Marsh.), and northern red oak (*Quercus rubra* L.). Common forest types consist of mixed conifers (*Picea-Abies*), mixed hardwood (*Populus-Betula*), mixed hardwood and conifer (*Quercus-Pinus*), and wetland forests (*Acer-Picea*).

2.2 Data

An extensive regional database of tree-level measurements has been compiled for the Acadian Forest from multiple sources (Weiskittel et al. 2010), which consists of over two million diameter at breast height (DBH) measurements from $\sim 10,000$ plots. The database also contains ~ 1.07 million height measurements from five sources, but missing values account for 12.6–99.9% of the observations across the sources. Excluding the sources with large numbers of missing height measurements, data from the following four sources were used in this study: 1) the Commercial Thinning Research Network from the Cooperative Forestry Research Unit of the University of Maine (CTRN; Seymour et al. 2014), 2) the Ecological Reserve Monitoring of Maine (ERM; Kuehne et al. 2018), 3) the US Forest Service Forest Inventory and Analysis in Maine (FIA; Bechtold and Patterson 2005), and 4) the network of permanent sample plots across Nova Scotia (NS PSP). An FIA plot comprised four 168 m² sub-plots. CTRN, ERM, and NB PSP plots were 810, 168, and 400 m², respectively. There were 49,546 plot-level observations from 4,201 plots available in this study (Table 1). Measurement intervals at these plots ranged 1–15 years, while the longest and average measurement histories were 40 and 13 years, respectively.

Missing height measurements from the four sources were imputed using a species- and plot-specific mixed effects model outlined by Robinson and Wykoff (2004). Individual-tree volumes were estimated using a species-specific taper function for the region, which uses DBH and height as input (Li et al. 2012; Weiskittel and Li 2012). In each plot, several attributes describing stand structure and composition were derived (Table 1). Relative density was computed using an equation proposed by Woodall et al. (2005) that predicts maximum stand density index using average species-specific gravity (USDA 2010). Site productivity was estimated through the dominant height and climate site index. The climate site index was based on the mean temperature of the warmest month, the temperature difference between the warmest and coldest month, and the ratio of precipitation during the growing season to total annual precipitation (Weiskittel et al. 2011b). The climate site index was considered more indicative of the mixed-species and multi-cohort Acadian Forest than the dominant height (Weiskittel et al. 2011b). The hardwood percentage in each plot was computed based on the initial basal area (Table 1).

2.3 Analysis

The primary goal of this study was to compare time-explicit, state-space, and simultaneous approaches' per-

Table 1: Summary statistics of initial measurements (mean, minimum, and maximum) by data source, where CTRN is the Commercial Thinning Research Network in Maine, ERM is the Ecological Reserve Monitoring of Maine, FIA is the US Forest Service Forest Inventory and Analysis in Maine, and NS PSP is the network of permanent sample plots across Nova Scotia.

Attribute	CTRN			ERM			FIA			NS PSP			Overall		
	mean	min	max	mean	min	max	mean	min	max	mean	min	max	mean	min	max
Number of plots	129	--	--	48	--	--	2,799	--	--	1,225	--	--	4,201	--	--
Number of observations	4,784	--	--	48	--	--	11,168	--	--	33,546	--	--	49,546	--	--
Climate site index	13.7	11.4	17.4	13.5	11.4	15.5	14.7	9.6	24.8	12.8	9.0	17.3	14.1	9.0	24.8
Stem density (tree ha ⁻¹)	1,787	432	7,604	2,538	357	12,243	2,908	59	33,403	716	25	2,825	2,230	25	33,403
Relative density	0.54	0.20	0.98	0.57	0.10	0.99	0.36	0.00	1.00	0.36	0.00	0.98	0.37	0.00	1.00
Basal area (m ² ha ⁻¹)	25.0	8.1	53.2	32.0	5.4	64.1	20.1	0.0	74.9	15.6	0.2	64.5	19.0	0.0	74.9
Volume (m ³ ha ⁻¹)	151.1	38.5	316.2	621.2	7.7	3,200.9	90.2	0.0	512.3	79.3	0.1	697.4	95.0	0.0	3,200.9
Mean DBH (cm)	14.1	7.9	20.2	24.8	13.4	35.1	15.4	2.5	44.5	15.5	9.1	60.2	15.5	2.5	60.2
Top height (m)	14.9	11.7	22.3	16.0	6.2	29.7	14.5	2.3	31.4	11.4	2.0	23.1	13.6	2.0	31.4
Hardwood (%)	2.7	0.0	66.2	46.4	0.0	100.0	46.3	0.0	100.0	32.3	0.0	100.0	40.9	0.0	100.0

formance on stand-level growth and yield predictions, where growth was defined as the difference in volume between two measurements. Other factors that may affect the approaches' performance have been controlled to ensure the comparison was focused on the various approaches. Specifically, the same data introduced above were used to develop the multiple approaches, and the same Gompertz yield function in the form of the equation below was applied to each of the approaches. Finally, the Gompertz function utilized the same set of predictors in different approaches.

$$y_t = \alpha \cdot \exp(-\beta \cdot \exp(-f(x_t))) \quad (1)$$

where y_t is yield in volume (m³ ha⁻¹) in any year t , $f(x_t) = p_1 \cdot ba_t + p_2 \cdot rd_t + p_3 \cdot hw$ is a linear function of predictors. ba_t and rd_t are basal area (m² ha⁻¹) and relative density, respectively, in year t , and hw is the percentage of hardwood in terms of initial basal area. α (the asymptote) and β (the rate of change) are primary parameters of Equation (1) in addition to the parameters of $p_1 - p_3$.

Basal area has long been found to be strongly correlated with volume (e.g., [Harry et al., 1964](#)), and how densely a site is occupied affects the forms and volumes of trees and their growth. Site occupancy was accounted for by basal area and relative density in Equation (1). The Acadian Forest is in a transition zone between the softwood-dominant boreal forests and the northern hardwood forests. The metric of the percentage of hardwood was used to represent this stand composition complexity.

2.3.1 Time-explicit approach

The time-explicit approach adds a growth component $g(x)$ to Equation (1), and the resulting model of volume yield (m³ ha⁻¹) in year t_2 is in the following form.

$$y_{t_2} = \alpha \cdot \exp(-\beta \cdot \exp(-(f(x_{t_1}) + g(x_{t_2})))) \quad (2)$$

where $g(x_{t_2}) = (p_4 + p_5 \cdot ba_{t_1} + p_6 \cdot rd_{t_1} + p_7 \cdot hw + p_8 \cdot CSI) \cdot (t_2 - t_1)$ is a function of the growth period between the years of t_1 and t_2 , which often are obtained from stand ages. CSI is the climate site index (used as a metric of site potential productivity in place of site index/dominant height, both of which are derived from tree heights and difficult to accurately measure in the field and often unavailable), and $p_4 - p_8$ are parameters to be estimated. Equation (2) is the yield function of Equation (1) when $t_2 - t_1 = 0$.

2.3.2 State-space approach

The state-space approach contains a set of transition functions:

$$\begin{cases} \frac{dba}{dt} = r_t \cdot ba \cdot \left(1 - \frac{ba}{k}\right) \\ \frac{drd}{dt} = q_3 \end{cases} \quad (3)$$

where $r_t = (q_0 + q_1 \cdot rd_t + q_2 \cdot CSI)$, and $q_0 - q_3$ and k are parameters to be estimated. This set of transition functions of Equation (3) predict increments of basal area and relative density (i.e., state variables) at any year t while assuming the climate site index is time-invariant. Predictions using Equation (3) is annualized following the method outlined by [Chen et al., 2017b](#), e.g., $ba_{t_1+1} = ba_{t_1} + \frac{dba_{t_1}}{dt}$, $ba_{t_1+2} = ba_{t_1+1} + \frac{dba_{t_1+1}}{dt}$, and so on. Both ba and rd are recursively predicted in this way at an annual step until t_2 , the year of yield prediction, is reached. Predicted values of ba and rd in year t_2 will then be fed into equation (1), the output function, to predict volume yield. t_1 and t_2 need not to be known in this approach, and the only information required is the length of prediction, i.e., how many years the recursive transition functions will move forward.

Integrals of Equation (3) show that basal area is a logistic function of relative density, climate site index, and initial basal area and has a sigmoidal pattern (i.e., $ba_t = \frac{ba_0 \cdot k}{ba_0 + (k - ba_0) \cdot \exp(-r_t \cdot t)}$). Consequently, basal area

increment follows a parabolic trajectory. In addition, relative density is a linear function of time, and its increments are constant (i.e., $rd_{t_2} = rd_{t_1} + q_3 \cdot (t_2 - t_1)$). Both functional types meet biological expectations of the growth process.

Equations (1) and (3) were separately fitted in the state-space approach, i.e., basal area and relative density were first predicted using Equation (3), and their predicted values were used as predictors to fit Equation (1). All available pairs of observations over time were used in the model-fitting process. For example, if a plot was measured in years of t_1 , t_2 , and t_3 , both Equation (1) and (3) were fitted from t_1 to t_2 , from t_2 to t_3 , and from t_1 to t_3 . The purpose was to improve model validity by utilizing measurements of various intervals to increase sample size.

2.3.3 Simultaneous approach

The simultaneous approach has the same form as the state-space approach but uses a different model-fitting technique. In the state-space approach, transition functions of Equation (3) are optimized to make the best predictions of basal area and relative density, which are subsequently inserted into Equation (1) as predictors to optimize the output/yield function. Values of the two predictors are fixed when they are inserted into Equation (1) because the fitting of the transition functions of Equation (3) has already been completed. In contrast, Equations (1) and (3) are simultaneously fitted in one step in the simultaneous approach. Basal area and relative density predicted by Equation (3) are used as predictors to fit Equation (1). If Equation (1) is not optimized, the fitting of Equation (3) will start over again to generate new predictions of basal area and relative density, which will again be used as predictors to fit Equation (1). This process will iterate until Equation (1) is optimized. Parameter estimates in both Equations (1) and (3) are simultaneously returned at this point.

In summary, the state-space approach consists of two sub-models (the transition functions and the output/yield function) fitted in two separate steps, and the output of the optimized transition functions of Equation (3) does not necessarily optimize the yield function of Equation (1). The simultaneous approach combines the two sub-models into one overarching model, and all parameters in the overarching model are simultaneously optimized to make the best predictions of volume yield regardless of the intermediate estimates of basal area and relative density in transition functions of Equation (3). As highlighted above, the simultaneous approach is the only approach that ensures path-invariance (e.g., Cieszewski, 2021; Cieszewski and Strub, 2018), which is often critical for stand-level models (Burkhardt and Tomé, 2012).

2.3.4 Model fitting, goodness-of-fit, and comparison

The time-explicit, state-space, and simultaneous approaches introduced above were fitted with nonlinear mixed-effects models using the lme4 package (Bates et al., 2015) in R v4.4.0 (R Core Team, 2024). Random effects were first tested on both primary parameters α and β of the output/yield function (Equation 1). Random effects on α (in terms of variance) were found to be approximately 10^5 the size of those on β . Random effects on α were further tested on levels of data source and plot, and the size of the former was about twice the size of the latter. The final models comprised random effects on α across data sources and plots.

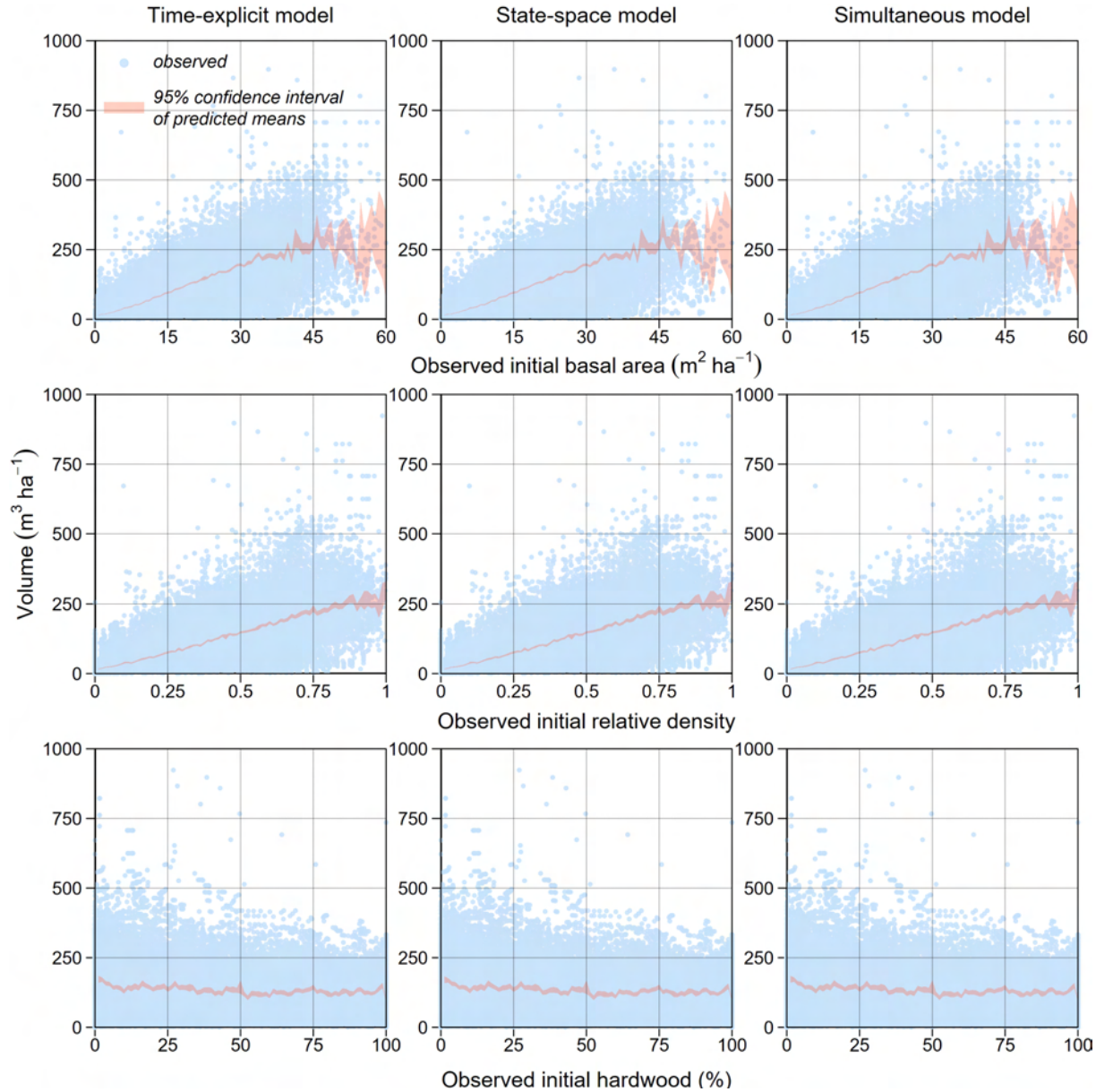
The goodness-of-fit of the models was summarized and used in model comparison from four perspectives: 1) the significance of parameter estimates, and whether the estimated values meet biological expectations, 2) the Akaike Information Criterion (AIC), 3) mean biases, and 4) root mean square errors (RMSE). In addition, predicted values of volume and their 95% confidence bands were compared with observed values. The confidence bands were empirically obtained based on the mean and standard error of predicted values in each of the 100 bins evenly divided across the ranges of the predictors (basal area, relative density, and hardwood percentage) in the output/yield function (Equation 1).

The entire dataset was used for model construction and comparison because data-splitting and, hence, cross-validation rarely is adequate for relatively large datasets such as the one used in this study (Kozak and Kozak, 2003). In addition, parameters estimated from the entire dataset generally are more precise than those derived from a portion of the data (Hirsch, 1991). Cross-validation based on 50-50 simple random sampling with replacements (permutation) was performed ten times, within which parameter estimates were consistent, and all RMSE statistics were within $\pm 1 \text{ m}^3 \text{ ha}^{-1}$ of the final models.

3 RESULTS

All parameter estimates, except p_3 from the time-explicit approach and q_0 from the simultaneous approach, were significantly different from zero at the 0.05 significance level (Table 2). Estimates of the primary parameters α and β of the output/yield function (Equation 1) were generally consistent across the approaches, where α ranged 431-477 $\text{m}^3 \text{ ha}^{-1}$, while β was 1.8-2.4 $\text{m}^3 \text{ ha}^{-1} \text{ yr}^{-1}$ (Table 2). Observed volumes exhibited significant amounts of variation (Figure 1). Observed and predicted volumes were positively related to basal area and relative density (Table 2 and Figure 1), while higher hardwood percentages resulted in slightly lower volumes in all approaches (Figure 1). The 95% confi-

Figure 1: Observed volumes and 95% confidence bands of predicted volumes from different modeling approaches over the ranges of the predictors (basal area, relative density, and hardwood percentage) in the yield function.



dence bands of predicted volumes were centered in observations and generally linear across the range of observed basal area, relative density, and hardwood percentage in all approaches (Figure 1).

The relationship between observed and predicted volumes was highly consistent across the approaches and data sources. Overestimations were at very small scales in the lower ranges of observed volume, while underestimations became increasingly noticeable after observed volume surpassed $200 \text{ m}^3 \text{ ha}^{-1}$, where the number of observations decreased (Figure 2). Mean biases of pre-

dicted volumes were close to zero across the approaches (Table 2). RMSE was $22.6 \text{ m}^3 \text{ ha}^{-1}$ in the time-explicit approach, $23.6 \text{ m}^3 \text{ ha}^{-1}$ in the state-space approach, and $22.5 \text{ m}^3 \text{ ha}^{-1}$ in the simultaneous approach (Table 2). These values accounted for $<20\%$ of mean observed final volume (i.e., equivalent to a R^2 of >0.96 over an average 13-year growth period). Mean biases were consistently small across various levels of projection length, relative density, and climate site index, while RMSE was much smaller when projection length was shorter than five years in all approaches (Table 3). Prediction errors

Table 2: Parameter estimates and statistics of the time-explicit, state-space, and simultaneous modeling approaches (random effects across the 4,201 plots are not reported here).

Parameter	Time-explicit			State-space			Simultaneous		
	parameter estimate	standard error	p-value	parameter estimate	standard error	p-value	parameter estimate	standard error	p-value
Fixed effects									
α	431.88	184.06	0.02	477.39	202.04	0.02	473.61	203.99	0.02
β	2.4938	0.0457	<0.01	2.4442	0.0369	<0.01	1.8060	0.0281	< 0.01
p_1 (basal area)	0.0586	0.0016	<0.01	0.0550	0.0008	<0.01	0.0348	0.0011	<0.01
p_2 (relative density)	0.7133	0.0733	<0.01	0.6128	0.0163	<0.01	1.1923	0.0530	<0.01
p_3 (% hardwood)	0.0111	0.0104	0.28	-0.1307	0.0080	<0.01	-0.1390	0.0076	<0.01
p_4 (intercept-year)	-0.0125	0.0019	<0.01	---	---	---	---	---	---
p_5 (basal area-year)	0.0003	0.0001	0.02	---	---	---	---	---	---
p_6 (relative density-year)	0.0355	0.0050	<0.01	---	---	---	---	---	---
p_7 (% hardwood-year)	-0.0082	0.0005	<0.01	---	---	---	---	---	---
p_8 (climate site index-year)	0.0022	0.0001	<0.01	---	---	---	---	---	---
q_0 (intercept)	---	---	---	0.1160	0.0004	<0.01	0.0026	0.0049	0.59
q_1 (relative density)	---	---	---	-0.1073	0.0010	<0.01	0.0455	0.0049	<0.01
q_2 (climate site index)	---	---	---	-0.0020	0.0001	<0.01	0.0042	0.0003	<0.01
q_3 (relative density)	---	---	---	0.0159	0.0004	<0.01	-0.0107	0.0015	<0.01
k	---	---	---	50.248	0.5642	<0.01	59.366	1.6907	<0.01
Random effects									
α (CTRN)	-169.34			-184.66			-196.25		
α (ERM)	634.99			696.70			704.09		
α (FIA)	-224.42			-240.95			-240.95		
α (NS PSP)	-241.23			-271.09			-266.90		
AIC	471,820			476,164			471,445		
Mean observed final volume ($\text{m}^3 \text{ ha}^{-1}$)	122.7			122.7			122.7		
RMSE ($\text{m}^3 \text{ ha}^{-1}$)	22.6			23.6			22.5		
Mean bias ($\text{m}^3 \text{ ha}^{-1}$)	0.0			-0.1			0.0		

Table 3: Mean biases and root mean square errors (RMSE) of volumes predicted by different modeling approaches at various levels of projection length, relative density, and climate site index (percentages are of observed final volumes).

Number of observations			Time-explicit				State-space				Simultaneous			
			Mean bias		RMSE		Mean bias		RMSE		Mean bias		RMSE	
			$\text{m}^3 \text{ ha}^{-1}$	%	$\text{m}^3 \text{ ha}^{-1}$	%	$\text{m}^3 \text{ ha}^{-1}$	%	$\text{m}^3 \text{ ha}^{-1}$	%	$\text{m}^3 \text{ ha}^{-1}$	%	$\text{m}^3 \text{ ha}^{-1}$	%
Projection length (year)	<5	2,944	3.4	2.0	20.9	12.2	2.9	1.7	20.9	12.2	3.6	2.1	21.0	12.2
	5-10	27,283	-2.2	1.8	23.1	18.9	-4.0	3.3	23.7	19.3	-1.9	1.6	23.1	18.9
	>10	19,319	2.6	2.3	22.1	19.1	4.9	4.2	23.9	20.7	2.2	1.9	21.9	19.0
Relative density	<0.3	17,401	0.3	0.6	15.4	31.3	2.3	4.7	17.8	36.2	0.1	0.2	15.4	31.3
	0.3-0.6	21,436	0.3	0.2	23.1	17.0	-1.1	0.8	23.8	17.5	0.4	0.3	23.1	17.0
	>0.6	10,709	-1.1	0.5	30.1	13.9	-2.1	1.0	30.7	14.2	-1.1	0.5	29.9	13.8
Climate site index	<10	645	0.2	0.2	23.0	20.5	0.6	0.5	26.6	23.7	0.4	0.4	23.2	20.7
	10-14	30,454	0.2	0.2	23.6	18.5	0.1	0.1	24.8	19.4	0.2	0.2	23.5	18.4
	>14	18,447	-0.3	0.3	20.8	18.2	-0.6	0.5	21.4	18.7	-0.3	0.3	20.8	18.2

appeared unbiased across the ranges of observed initial basal area, relative density, and hardwood percentage in all approaches (Figure 3).

4 DISCUSSION

The time-explicit, state-space, and simultaneous approaches consistently predicted stand-level volume growth and yield in the predominantly mixed-species and multi-cohort Acadian Forest. All three models yielded accurate and largely unbiased predictions across various complex stands. Annual rate of volume growth

of $1.8\text{--}2.4 \text{ m}^3 \text{ ha}^{-1} \text{ yr}^{-1}$ (indicated by the parameter estimate of β) was consistent with a previous large-scale study conducted in the same region (Chen et al., 2017a). Predicted relationships between growth and yield and stand/site attributes of basal area, relative density, and hardwood percentage were highly comparable across the models. The time-explicit approach had a simplistic form, but similar prediction performance compared to the other two more complicated methods. In comparison, the simultaneous approach, despite being path-invariant, was computationally challenging in simultaneously fitting several nonlinear functions, which is a concern expressed among modelers. Meanwhile, predic-

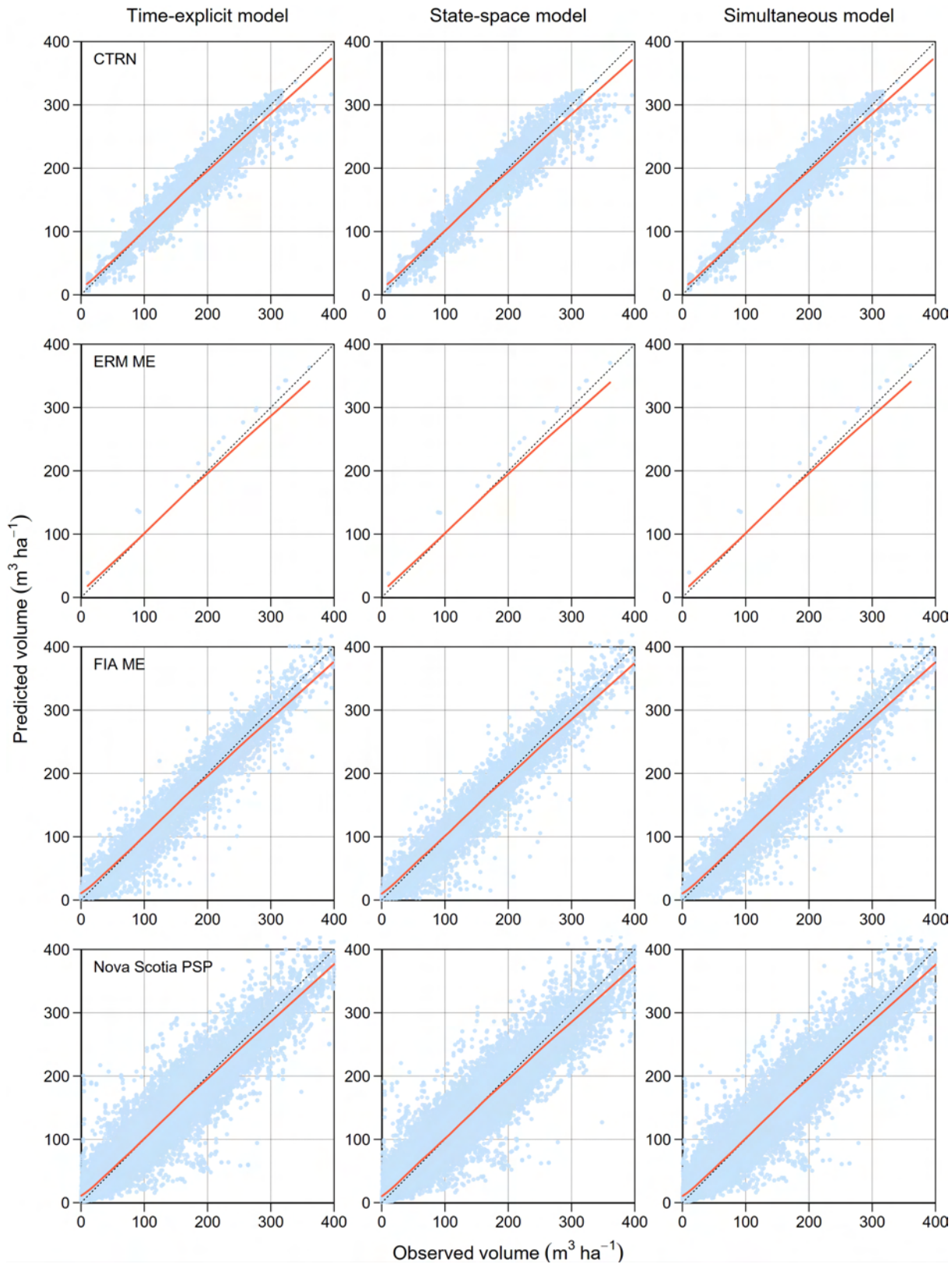


Figure 2: Observed and predicted volumes by different modeling approaches and data sources, where solid lines are loess smooth functions of predicted volumes over observed volumes.

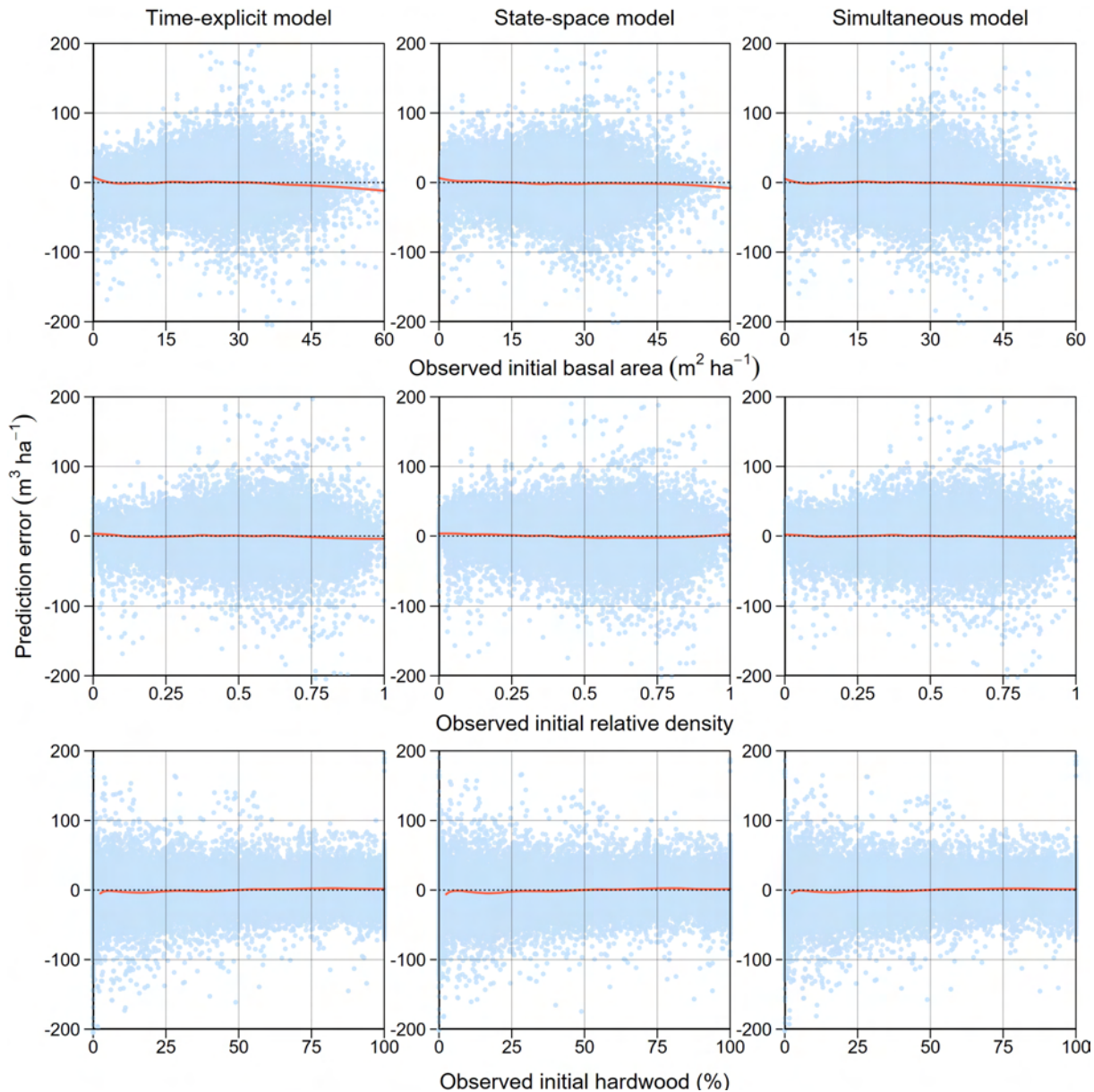


Figure 3: Prediction errors (predicted – observed) from different approaches over the ranges of the predictors (basal area, relative density, and hardwood percentage) in the yield function, where solid lines are loess smooth functions of the errors.

tion improvements from the simultaneous approach were not significant.

Complex state-space approach has been developed in previous studies (e.g., Nord-Larsen and Johannsen, 2007; Stankova, 2015; Walde et al., 2021), which may have limited the approach's adoption and application in forest management and research. This study selected rather simplistic equations for the different approaches because the primary objective of this study was to compare time-explicit, state-space, and simultaneous ap-

proaches' performance in stand-level volume growth and yield predictions. The simplistic equations were deliberately developed to focus the comparison on the different approaches and avoid complications from various modeling strategies such as selections of state variables, predictors, and specific transition functions.

State variables varied in previous models based on the state-space approach (Castedo et al., 2007; Diéguez-Aranda et al., 2006; García, 1994; Stankova, 2015), where a measure of stand occupancy, e.g., basal area,

usually was a primary state variable. This measure was frequently augmented by variables such as stem density (i.e. trees per ha) to indicate the stage of stand development/average tree size (e.g., Diéguez-Aranda et al., 2006; García, 1994; Stankova, 2015). Stem density has been considered to decrease with increasing tree size, but this inverse relationship between stem density and tree size may only apply to pure even-aged stands (Vanclay, 2009). In this study, stem density increased while trees grew larger in the mixed-species stands, meanwhile, stem density decreased with increases in tree size in the generally softwood stands in CTRN, consistent with prior findings (e.g., Wagle et al., 2022). These observations agree with Vanclay (2009) and suggest that the inverse relationship between stem density and tree size probably should not be considered as a general rule for transition functions in the state-space approach. Relative density replaced stem density to be a state variable in this study, similar to Waldy (2021). Relative density potentially is a better alternative to stem density because it is bounded between zero and one, hence comparable across stands and biologically consistent across species (Chivhenge et al., 2024). Our preliminary analysis also showed that RMSE in basal area predictions was $0.48 \text{ m}^2 \text{ ha}^{-1}$ lower when using relative density instead of stem density as a predictor. The relationship between relative density and tree size also were consistent across data sources such that they both increased over time, and a transition function based on this relationship likely was representative of the Acadian Forest and in agreement with the trend of increasing relative density of US forests reported by Woodall and Weiskittel (2021).

Site quality has been considered the most crucial factor in determining the potential productivity and, hence, growth and yield of a site (Skovsgaard and Vanclay, 2013). Dominant height is commonly used as an indicator of site quality in stand-level growth and yield models (e.g., Diéguez-Aranda et al., 2006; Fang et al., 2001; Nord-Larsen and Johannsen, 2007). Dominant height is derived from tree heights, which are difficult to measure in the field and often unavailable accurately. In addition, site quality is time-invariant in that it does not change within a relatively long time frame. However, dominant height (and metrics derived from it) changes dramatically over time. Consequently, a transition function has to be developed to accommodate changes in dominant height. This adds unnecessary complexity to the state-space approach. A climate site index derived from climate variables (Weiskittel et al., 2011b) replaced dominant height in this study. The climate site index is time-invariant, relieving the need for a transition function in model development and simplifying the state-space approach.

A notable advantage of the state-space and simultaneous approaches is the relief of the growth and yield model's dependence on the information of time (often in the form of the difference in ages). The use of dominant height to represent site quality in the state-space approach contradicts this advantage (e.g., Stankova, 2015; Waldy et al., 2021), where dominant height has been modeled as a function of age. In addition, age has been used to predict dominant height in an inverse form (i.e., $1/\text{age}$), which means age will not be differentiated/differenced away in a transition function and potentially affects the state-space approach's applicability to mixed-species and/or multi-cohort stands, where information on age is not always readily available or even straightforward to determine. Some dominant height models utilized ages in the form of their difference as input (e.g., Diéguez-Aranda et al., 2005). This age difference can be directly measured as the time interval of two measurements without the burden of directly knowing ages (e.g., the difference in age is five years if we know the measurement interval is five years, and there is no need to use the second age, say 45 to minus the first age, say 40). Our time-explicit approach (Equation 2), which explicitly used time interval as a predictor, generally performed well in this study.

Individual-tree models have been considered the new standard for forest growth and yield modeling (Weiskittel et al., 2011a) and thought to be advantageous in complex mixed-species and/or multi-cohort stands compared to stand-level models (García, 1994), but has not been widely tested. This study showed the simplicity of building and applying stand-level growth and yield models. These models also achieved relatively accurate and largely unbiased volume yield predictions across complex stands, comparable to those obtained using the individual-tree level Forest Vegetation Simulator Acadian Variant (Chen et al., 2018). This shows potential and warrants further investigation of the applicability of stand-level growth and yield models beyond single-species and even-aged stands. Finally, taking advantage of stand-level models' relatively high accuracies in predicting stand-level summary statistics (e.g., basal area and volume) to adjust and improve individual-tree level growth and yield predictions in a multistage modeling approach (e.g., Chen et al., 2023; Fridman and Ståhl, 2001) will be another direction to utilize the potentials of stand-level growth and yield models fully. Finally, the continued exploration and development of robust approaches for ensuring path-invariance and base-age-invariance as well as addressing data hierarchies is critical for further stand-level growth and yield model refinement.

ACKNOWLEDGMENT

We thank Mike Strub and prior reviewers for providing feedback on previous drafts of this paper. This study was supported by US NSF Center for Advanced Forestry Systems (Award #1915078), US NSF OIA EPSCoR RII Track-2 (Award #1920908), US NSF OIA EPSCoR RII E-RISE (Award #2416915), and USDA NIFA Sustainable Agriculture Systems program (Award: #2023-68012-38992).

REFERENCES

- Bates, D.; M. Maechler; B. Bolker; S. Walker. 2015. Fitting linear mixed-effects models using lme4. *Journal of Statistical Software* 67(1): 1–48. doi: <https://doi.org/10.18637/jss.v067.i01>
- Bechtold, W.A.; P.L. Patterson (Eds). 2005. The enhanced forest inventory and analysis program—national sampling design and estimation procedures. General Technical Report SRS-GTR-80. USDA, Forest Service, Southern Research Station. doi: <https://doi.org/10.2737/SRS-GTR-80>
- Borders, B.E. 1989. Systems of equations in forest stand modeling. *Forest Science* 35: 548–556. doi: <https://doi.org/10.1093/forestscience/35.2.548>
- Borders, B.E.; R.L. Bailey. 1986. A compatible system of growth and yield equations for slash pine fitted with restricted three-stage least squares. *Forest Science* 32: 185–201. doi: <https://doi.org/10.1093/forestscience/32.1.185>
- Burkhart, H.E.; M. Tomé. 2012. Modeling forest trees and stands. Springer Science & Business Media, Dordrecht. 458pp.
- Castedo-Dorado, F.; U. Diéguez-Aranda; J.G. Álvarez-González. 2007. A growth model for *Pinus radiata* D. Don stands in north-western Spain. *Annals of Forest Science* 64: 453–465. doi: <https://doi.org/10.1051/forest:2007023>
- Chen, C.; A. Weiskittel; M. Bataineh; D.A. MacLean. 2017a. Even low levels of spruce budworm defoliation affect mortality and ingrowth but net growth is more driven by competition. *Canadian Journal of Forest Research* 47: 1546–1556. doi: <https://doi.org/10.1139/cjfr-2017-0012>
- Chen, C.; A. Weiskittel; M. Bataineh; D.A. MacLean. 2017b. Evaluating the influence of varying levels of spruce budworm defoliation on annualized individual tree growth and mortality in Maine, USA and New Brunswick, Canada. *Forest Ecology and Management* 396: 184–194. doi: <https://doi.org/10.1016/j.foreco.2017.03.026>
- Chen, C.; A. Weiskittel; M. Bataineh; D.A. MacLean. 2018. Refining the Forest Vegetation Simulator for projecting the effects of spruce budworm defoliation in the Acadian Region of North America. *Forestry Chronicle* 94: 240–253. doi: <https://doi.org/10.5558/tfc2018-037>
- Chen, C.; J. Kershaw Jr.; A. Weiskittel; E. McGarrigle. 2023. Can a multistage approach improve individual tree mortality predictions across the complex mixed-species and managed forests of eastern North America? *Forest Ecosystems* 10: 100086. doi: <https://doi.org/10.1016/j.fecs.2023.100086>
- Chivhenge, E.; D.G. Ray; A.R. Weiskittel; C.W. Woodall; A.W. D’Amato. 2024. Evaluating the development and application of stand density index for the management of complex and adaptive forests. *Current Forestry Reports* 10: 133–152. doi: <https://doi.org/10.1007/s40725-024-00212-w>
- Clutter, J.L. 1963. Compatible growth and yield models for loblolly pine. *Forest Science* 9: 354–371. doi: <https://doi.org/10.1093/forestscience/9.3.354>
- Cieszewski, C.J. 2021. UTADA: Unified Theory of the Algebraic Differences Approaches—Derivation of Dynamic Site Equations from Yield-Site Relationships. *Mathematical and Computational Forestry & Natural Resource Sciences* 13: 36–43.
- Cieszewski, C.J.; R.L. Bailey. 2000. Generalized algebraic difference approach: theory based derivation of dynamic site equations with polymorphism and variable asymptotes. *Forest Science* 46: 116–126. doi: <https://doi.org/10.1093/forestscience/46.1.116>
- Cieszewski, C.J.; I.E. Bella. 1993. Modeling density-related lodgepole pine height growth, using Czarnowski’s stand dynamics theory. *Canadian Journal of Forest Research* 23: 2499–2506. doi: <https://doi.org/10.1139/x93-311>
- Cieszewski, C.J.; M. Strub. 2018. Comparing properties of self-referencing models based on nonlinear-fixed-effects versus nonlinear-mixed-effects modeling approaches. *Mathematical and Computational Forestry & Natural Resource Sciences* 10: 46–57.
- Diéguez-Aranda, U.; J.G. Álvarez González; M Barrio Anta; A. Rojo Alboreca. 2005. Site quality equations for *Pinus sylvestris* L. plantations in Galicia (North-Western Spain). *Annals of Forest Science* 62: 143–152. doi: <https://doi.org/10.1051/forest:2005006>

- Diéguez-Aranda, U.; F.C. Dorado; J.G.A. González; A.R. Alboreca. 2006. Dynamic growth model for Scots pine (*Pinus sylvestris* L.) plantations in Galicia (north-western Spain). *Ecological Modelling* 191: 225–242. doi: <https://doi.org/10.1016/j.ecolmodel.2005.04.026>
- Fang, Z.; R.L. Bailey; B.D. Shiver. 2001. A multivariate simultaneous prediction system for stand growth and yield with fixed and random effects. *Forest Science* 47: 550–562. doi: <https://doi.org/10.1093/forestscience/47.4.550>
- Fridman, J; G. Ståhl. 2001. A three-step approach for modeling tree mortality in Swedish forests. *Scandinavian Journal of Forest Research* 16: 455–466. doi: <https://doi.org/10.1080/02827580152632856>
- Garcia, O. 1994. The state-space approach in growth modeling. *Canadian Journal of Forest Research* 24: 1894–1903. doi: <https://doi.org/10.1139/x94-244>
- Garcia, O. 2009. A simple and effective forest stand mortality model. *Mathematical and Computational Forestry & Natural-Resource Sciences* 1: 1–9.
- Garcia, O. 2011. A parsimonious dynamic stand model for interior spruce in British Columbia. *Forest Science* 57: 265–280. doi: <https://doi.org/10.1093/forestscience/57.4.265>
- Garcia, O. 2017. Cohort aggregation modelling for complex forest stands: Spruce-aspen mixtures in British Columbia. *Ecological Modelling* 343: 109–122. doi: <https://doi.org/10.1016/j.ecolmodel.2016.10.020>
- Harry, J.; G. Smith; R.E. Breadon. 1964. Combined variable equations and volume-basal area ratios for total cubic foot volumes of the commercial trees of BC. *Forestry Chronicle* 40: 258–261. doi: <https://doi.org/10.5558/tfc40258-2>
- Hirsch, R. 1991. Validation samples. *Biometrics* 47: 1193–1194.
- Kozak, A.; R. Kozak. 2003. Does cross validation provide additional information in the evaluation of regression models? *Canadian Journal of Forest Research* 33: 976–987. doi: <https://doi.org/10.1139/x03-022>
- Kuehne, C.; J.J. Puhlick; A.R. Weiskittel. 2018. Ecological Reserves in Maine: Initial results of long-term monitoring. University of Maine. Center for Research on Sustainable Forests. Available online at: <http://www.nefismembers.org/documents/ecological-reserves-in-maine-initial-results-of-long-term-monitoring>. Last accessed Aug. 31, 2022.
- Li, R.; A. Weiskittel; A.R. Dick; J.A. Kershaw Jr.; R.S. Seymour. 2012. Regional stem taper equations for eleven conifer species in the Acadian region of North America: Development and assessment. *North-eastern Journal of Applied Forestry* 29: 5–14. doi: <https://doi.org/10.5849/njaf.10-037>
- Návar, J.; P.A. Dominguez-Calleros; F.J. Rodriguez-Flores; L. Lizárraga-Mendiola; R. de Hoogh; T.J. Synnott. 2016. A stand growth and yield model for northern reforested stands of Mexico. *Mathematical and Computational Forestry & Natural Resource Sciences* 8: 25–34.
- Nord-Larsen, T.; V.K. Johannsen. 2007. A state-space approach to stand growth modelling of European beech. *Annals of Forest Science* 64: 365–374. doi: <https://doi.org/10.1051/forest:2007013>
- Pamerleau-Couture, E.; Krause, C.; Pothier, D.; A. Weiskittel. 2015. Effect of three partial cutting practices on stand structure and growth of residual black spruce trees in north-eastern Quebec. *Forestry* 88(4): 471–483. doi: <https://doi.org/10.1093/forestry/cpv017>
- Peng, C. 2000. Growth and yield models for uneven-aged stands: past, present and future. *Forest Ecology and Management* 132: 259–279. doi: [https://doi.org/10.1016/S0378-1127\(99\)00229-7](https://doi.org/10.1016/S0378-1127(99)00229-7)
- Pienaar, L.V.; B.D. Shiver. 1986. Basal area prediction and projection equations for pine plantations. *Forest Science* 32: 626–633. doi: <https://doi.org/10.1093/forestscience/32.3.626>
- R Core Team. 2019. R: a language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. Available online at: <http://www.r-project.org>. Last accessed Oct. 24, 2024.
- Rehfeldt, G.E. 2006. A spline model of climate for the western United States. General Technical Report RMRS-165. USDA, Forest Service, Rocky Mountain Research Station. doi: <https://doi.org/10.2737/RMRS-GTR-165>
- Robinson, A.P.; W.R. Wykoff. 2004. Imputing missing height measures using a mixed-effects modeling strategy. *Canadian Journal of Forest Research* 34: 2492–2500. doi: <https://doi.org/10.1139/x04-137>
- Rose, C.E.; C.J. Cieszewski; W.H. Carmean. 2003. Three methods for avoiding the impacts of incompatible site index and height prediction models demonstrated on jack pine curves for Ontario. *Forestry*

- Chronicle 79: 928–935. doi: <https://doi.org/10.5558/tfc79928-5>
- Rowe, J.S. 1972. Forest regions of Canada. Canadian Forestry Service publication No. 1300. Department of the Environment, Canadian Forestry Service.
- Seymour, R.S.; S.R. Meyer; R.G. Wagner. 2014. The Cooperative Forestry Research Unit Commercial Thinning Research Network: 9-year results. In: Kenefic, L.S.; J.C. Brissette (Comps). Penobscot Experimental Forest: 60 years of research and demonstration in Maine, 1950-2010. General Technical Report NRS-P-123. USDA, Forest Service, Northern Research Station.
- Skovsgaard, J.P.; J.K. Vanclay. 2013. Forest site productivity: a review of spatial and temporal variability in natural site conditions. *Forestry* 86(3): 305–315. doi: <https://doi.org/10.1093/forestry/cpt010>
- Stankova, T.V. 2015. A dynamic whole-stand growth model, derived from allometric relationships. *Silva Fennica* 50: 1406. doi: <https://doi.org/10.14214/sf.1406>
- Stankova, T.; U. Diéguez-Aranda. 2012. A tentative dynamic site index model for Scots pine (*Pinus sylvestris* L.) plantations in Bulgaria. *Silva Balcanica* 13(1): 5–19.
- Sullivan, A.D.; J.L. Clutter. 1972. A simultaneous growth and yield model for loblolly pine. *Forest Science* 18: 76–86. doi: <https://doi.org/10.1093/forestscience/18.1.76>
- Tait, D.E.; C.J. Cieszewski; I.E. Bella. 1988. The stand dynamics of lodgepole pine. *Canadian Journal of Forest Research* 18: 1255–1260. doi: <https://doi.org/10.1139/x88-193>
- Tewari, V.P.; B. Singh. 2018. A first-approximation simple dynamic growth model for forest teak plantations in Gujarat State of India. *Southern Forests* 80: 59–65. doi: <https://doi.org/10.2989/20702620.2016.1277644>
- Tewari, V.P.; J.G. Álvarez-González; O. García. 2014. Developing a dynamic growth model for teak plantations in India. *Forest Ecosystems* 1: 9. doi: <https://doi.org/10.1186/2197-5620-1-9>
- USDA. 2010. Wood handbook: Wood as an engineering material. General Technical Report FPL-GTR-190. USDA, Forest Service, Forest Products Laboratory.
- Vanclay, J.K. 2009. Tree diameter, height and stocking in even-aged forests. *Annals of Forest Science* 66: 702–709. doi: <https://doi.org/10.1051/forest/2009063>
- Wagle, B.H.; A.R. Weiskittel; A.R. Kizha; J.P. Berrill; A.W. D’Amato; D. Marshall. 2022. Long-term influence of commercial thinning on stand structure and yield with/without pre-commercial thinning of spruce-fir in northern Maine, USA. *Forest Ecology and Management* 522: 120453. doi: <https://doi.org/10.1016/j.foreco.2022.120453>
- Waldy, J.; J.A. Kershaw Jr.; A. Weiskittel; M.J. Ducey. 2021. Comparison of time-based versus state-space stand growth models for tropical hybrid *Eucalyptus* clonal plantations in Sumatera, Indonesia. *Canadian Journal of Forest Research* 51: 1178–1187. doi: <https://doi.org/10.1139/cjfr-2020-0499>
- Weiskittel, A.R.; R.G. Wagner; R.S. Seymour. 2010. Refinement of the Forest Vegetation Simulator, north-eastern variant growth and yield model: Phase 1. In: S.R. Meyer (Ed). 2009 Cooperative Forestry Research Unit Annual Report. Univ. Maine, Orono, ME.
- Weiskittel, A.R.; D.W. Hann; J.A. Kershaw Jr.; J. Vanclay. 2011c. Forest growth and yield: Concepts and applications. Wiley-Blackwell, New York. 430pp.
- Weiskittel, A.R.; R.G. Wagner; R.S. Seymour. 2011b. Refinement of the forest vegetation simulator, north-eastern variant growth and yield model: Phase 2. In: Mercier, W.J.; A.S. Nelson (Eds). Cooperative Forestry Research Unit. 2010 Annual Report. Univ. Maine. Orono, ME.
- Weiskittel, A.; R. Li. 2012. Development of regional taper and volume equations: hardwood species. In: B.E. Roth (Ed). Cooperative Forestry Research Unit: 2011 annual report. University of Maine. Orono, ME.
- Woodall, C.W.; P.D. Miles; J.S. Vissage. 2005. Determining maximum stand density index in mixed species stands for strategic-scale stocking assessments. *Forest Ecology and Management* 216: 367–377. doi: <https://doi.org/10.1016/j.foreco.2005.05.050>
- Woodall, C.W.; A.R. Weiskittel. 2021. Relative density of United States forests has shifted to higher levels over last two decades with important implications for future dynamics. *Scientific Reports* 11: 18848. doi: <https://doi.org/10.1038/s41598-021-98244-w>