MATHEMATICIANS' FIDELITY OF TASK IMPLEMENTATION ALONG TWO DIMENSIONS

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Fidelity of implementation is a crucial factor in understanding the impact of instructional interventions, yet little research has attended to fidelity of implementation in undergraduate mathematics settings. In this report, we investigate a fidelity of task implementation analysis of eleven instructors implementing group-worthy proof tasks developed collaboratively by mathematicians and mathematics education scholars. We found that instructors generally adhered to the mathematical storyline but demonstrated substantial variation in adherence to the pedagogical storyline, often modifying or omitting group-worthy features. These variations suggest additional support may be needed for implementing structured group work.

Keywords: Undergraduate Education, Instructional Activities and Practices.

A crucial component of understanding the impact of educational interventions is the "character of implementation and its fidelity to intended practice" (Schoenfeld, 2006, p. 17). Attention to fidelity of implementation has largely been studied in K-12 contexts as teachers adopted standards-based curriculum and student-centered pedagogies (Brown et al., 2009; O'Donnell, 2008). In this literature, scholars have taken a multitude of foci and approaches to examine fidelity of implementation to curricular material. Recently, scholars have made calls for attention to fidelity of implementation in discipline-based educational research (Stains & Vickrey, 2017). However, little work has been done to define and analyze fidelity of implementation in advanced mathematics beyond the K-12 setting. This can partially be explained by far fewer intervention-based studies as well as a much more limited research base on instructional practice. Yet there is good reason to anticipate that implementation fidelity might have a different character in advanced mathematics, where instructional knowledge, autonomy, and practices differ substantially from K-12 counterparts.

In this brief report, we share our approach to examining fidelity of implementation of tasks developed in a collaborative workshop setting with mathematics education scholars and mathematicians. These summer workshops emphasized group-worthy (Cohen & Lotan, 2014; Featherstone et al., 2011) task design for proof-based courses that included attention to elements of Complex Instruction and pre-existing templates to support students in proof comprehension, construction, and analysis. We consider content and pedagogical fidelity of the task implementations of eleven undergraduate mathematics instructors.

Professional Learning and Collaboration Between Mathematics Educators and Undergraduate Mathematics Instructors

In general, university mathematics instructors have received little education in pedagogy and little encouragement to engage with the results of mathematics education research (Nardi et al., 2005; Winsløw et al., 2018). Literature points to two images of how mathematicians and mathematics educators may co-exist: as divided by epistemology and practice (Darragh, 2022; Goldin, 2003) or as collaborators benefiting from different strengths (Bleiler, 2015). The divide has been well-traversed, with the overarching theme that mathematicians are guided primarily by concern for the quality and accuracy of mathematics content, while education research positions students first and may be dismissive of a positivist view on mathematics (e.g., Goldin, 2003; Schoenfeld et al., 2016). Sultan and Artz (2005) suggest that for mathematics education scholars and mathematicians to successfully collaborate around teaching, they must have "Motivation to collaborate," "acknowledgement of the strengths of each collaborator," "trust that the motives of each collaborator involve improving student learning," and "helpfulness of both collaborators in reaching mutual goals" (p. 53). Projects such as the PLATINUM project in Europe (Gómez Chacón et al., 2021), the DATUM project in New Zealand (Barton et al., 2015), and the TIMES project in the United States (Andrews-Larson et al., 2021) have attempted to foster collaboration and professional learning where undergraduate mathematics instructors engage with education scholars and mathematics education research. These projects point to the potential of professional learning to be productive and transformative in undergraduate settings under the right conditions. However, collaboration is likely to be shaped by differences in knowledge, beliefs, and epistemologies of the two fields of scientific inquiry.

The Professional Learning Model and Context

The professional learning workshops in our study involved three full days of collaboration between mathematicians and mathematics educators to design a series of group-worthy tasks related to mathematical proof (Cohan & Lotan, 2014; Featherstone, et al. 2011). By group-worthy, we mean that the tasks include attention to interdependence (a student could not solve the task independently), multiple abilities (there are multiple ways to be "smart" and contribute to the task), and open-endedness (there is not one well-defined path). To meet these aims, the instructors adapted three templates that focus on one of three proof activities respectively: construction, validation/analysis, and comprehension. These templates were originally created in the context of a design-based research project in abstract algebra (Melhuish et al., 2024). During the workshops, the mathematics instructors engaged in a group-worthy task themselves, were exposed to principles of Complex Instruction and instructional moves for facilitating group-worthy tasks and then worked in small groups to develop three tasks, each focused on a theorem and proof they intended to teach. The task templates included focus on both the type of mathematics to engage students in (e.g., example and representation exploration, focal proof activity) and corresponding social structures (e.g., group roles, partner exchanges, distributed information). A total of 18 instructors participated in a summer workshop in 2023 or 2024. Over the last three semesters, 11 instructors implemented at least one task developed during their workshop. These include two instructors each of abstract algebra, introduction to proof, analysis, linear algebra, topology, and one discrete math instructor.

Methods and Analytic Framing

For each instructor, we had one to three members of the research team observe task implementation. Additionally, we audio and video recorded the class with a focus on groupwork and whole-class instruction (with one exception who did not consent to video). The videos, transcripts, and our field notes provide the data set for this analysis.

To explore the fidelity of task implementation, we consider two dimensions (adapted from Heck et al., 2012): the mathematical and pedagogical storylines (see Table 1). The mathematical storyline reflects the topics, order, and learning goals. In contrast, the pedagogical storyline captures how students are engaged with the content which can vary in terms of student interaction with each other and the instructor. We find this delineation useful because the existing literature frames content and pedagogy aspects as areas that can contribute to the divide between mathematicians and mathematics education scholars. For each implementation, we evaluate the degree to which the participating instructor adhered to the fidelity elements as specified by the original task templates (Table 1). For each instructor (taking the mode if they implemented multiple), if all elements are maintained in a dimension, the implementation is considered high fidelity in that dimension. If none of the elements are maintained, it is considered low fidelity. If one or two elements are maintained, the fidelity is considered at a medium level.

Table 1: Fidelity of Task Implementation Elements

Dimension	Fidelity Element	Description
Mathematical Storyline	Examples and Motivation	Focal theorem vocabulary unpacked and explored with planned examples
	Theorem and Proofs	Focal theorem and proof(s) as planned
	Ordering of Content	Ordering of mathematical elements as planned
Pedagogical Storyline	Multiple Abilities Launch	Task launched with a list of needed skills/knowledge and statement
	Student Activity	Students engage in proof comprehension, analysis, construction as planned
	Social Scaffolding	Implementation of planned structures such as group roles, partner exchanges, division of resources

Results

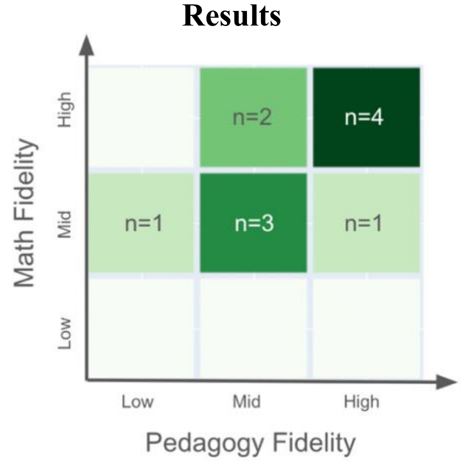

Figure 1: Frequency of Instructors According to Fidelity Dimensions

To better understand variation in implementation (see Figure 1), we reflect on the profiles of implementations found outside of the high-fidelity classification.

Fidelity to Mathematical Storyline

Every instructor preserved the focal theorems of the tasks they implemented. However, the mathematical storyline sometimes diverged from the task template in the launch or the ending.

Planned task launches included unpacking and documenting key mathematical terms and definitions in whole class. In several cases, the instructors jumped immediately into the primary task without a launch discussion. In other cases, the instructors encountered time constraints where students engaged in groupwork around the theorem and examples, but did not always arrive at an important aspect such as concluding the proofs. We saw this play out in three different ways. One way involved the instructor maintaining the activity as planned and extending the activity to a second class day. A second way involved the instructor presenting the proof in the next class, thus maintaining only the mathematical storyline. Finally, one instructor ultimately decided the exploration of theorem and conjecture was all that was critical for their students and never returned to a proof of the focal theorem. This last version diverges from the planned mathematical storyline.

Fidelity to Pedagogical Storyline

Fidelity to the pedagogical storyline varied substantially. In one case, the instructor repurposed an activity that was intended to be a proof comprehension task into the format: introduce the theorem; let students have time to try to prove in small groups. This was the most substantial divergence from the pedagogical storyline without any attention to the group-worthy features planned for during the summer workshops. In the mid cases, we identified three profiles: the handoff, the takeover, and the stripper. In the handoff case, the instructors printed the instructor task guide into a large packet, preserving the task as written, but did not facilitate the group work otherwise. In the takeover case, the instructor, usually due to time constraints, took over a portion of the task that was intended for small-group work. Finally, some instructors removed one element from the task as written. For example, an instructor did not include a multiple abilities treatment in an otherwise faithfully implemented task.

Discussion

In this brief report, our goal was to provide insight into variation in fidelity of task implementation by instructors of upper division undergraduate courses. This collaborative project involved co-planning a series of group-worthy proof tasks that engaged students through activities and incorporated Complex Instruction principles. We offer several general observations and conjectures. First, instructors rarely diverged greatly from the mathematical content of the tasks. Rather, it was the pedagogical content that encountered the most variation. This may be the result of differing orientations towards student outcomes and instruction more generally. This aligns with prior research that has identified the mathematics and its accuracy as a driving pedagogical feature amongst those trained as mathematicians. Second, instructors may need more substantial support on group work implementation depending on their prior experiences. In all cases, the instructors in this study had implemented unstructured group work in their classes previously—where students engage on, for example, a worksheet. Many did not have any prior training on group work with more attention to interdependence among the group. This may explain why some instructors provided the task guide as a handout and did not actively monitor the group work. Additional research and more extensive analysis of this data set can provide a more refined view of fidelity along these two dimensions.

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