EXPLICATING THE RETROSPECTIVE ANALYSIS PROCESS FOR CHARACTERIZING STUDENTS' THINKING IN A GRAPHING TASK

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Modeling student thinking is a complex and arduous task. Readers do not typically see the often-messy analytical nuances and paths that must occur within analysis cycles to produce such models. In this paper, we elaborate on the complex undertaking of analyzing task-based clinical interview data from a single task with 14 middle grades students.

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Models of students' thinking can shed light on students' cognitive resources and thus support effective teaching and curriculum design (Carpenter et al., 1998; Hackenberg, 2014). For example, such models can provide teachers with insights into strategies students might use, enabling them to orchestrate discussions that foster connections and understanding (Smith & Stein, 2018). However, the process of modeling student thinking is complex and arduous. Task-based clinical interviews (hereafter CI; Clement, 2000; Goldin, 2000) and teaching experiments (hereafter TE; Steffe & Thompson, 2000) have been widely used in mathematics education research as methodologies to examine and model students' thinking. After collecting data through CI/TEs, researchers retrospectively analyze the data corpus, which involves revisiting the interviews/episodes and refining preliminary findings from ongoing analyses.

While readers of qualitative papers have an idea of what generally occurs in the analysis process and know that it results in the narratives presented, what they do not typically see is the often-messy data analysis spirals (Creswell & Poth, 2018) that must occur to produce such stories. In this paper, we share our retrospective analysis journey to create models of students' graph thinking to illustrate the complexity of this process and to reflect on and operationalize what retrospective analysis *could* look like.

Theoretical Orientation and Underpinnings

We define a graphical representation (GR) as a spatial depiction of quantities (Thompson, 2011) to mathematize some phenomena. We assume a GR does not represent by itself; it must be created and interpreted by an individual (von Glasersfeld, 1987). Therefore, we take GRs as resulting from a cognizing subject's goal-directed activity and use TE/CI methodologies to produce explanatory accounts of such activity. We view a GR consisting of three layers: reference frames (RFs), a coordinate system (CS), and a graph. RFs are mental structures used to gauge the relative extents of various attributes in the phenomenon being depicted (Joshua et al., 2015; Lee, 2017; Levinson, 2003). Thinking within RFs entails attending to and establishing reference points, directionality, and having an idea of what and how to measure the quantities being depicted (Joshua et al., 2015; Lee et al., 2019). CSs are the geometric representation of the RFs (e.g., axes) that allow an individual to systematically express and coordinate RFs. Finally, a

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graph is a collection of points depicted upon the underlying CS. Attending to all three layers of a graphical representation has been a critical component undergirding our analysis.

Methods

We analyzed data from the Family Frenzy—The Sun Family (FF) task (Figure 1), which was designed to examine how students construct a graph for the given situation. See Zolt et al. (2024) for additional detail about the task. Our focus was not to evaluate how close students' representations are to conventional graphs; instead, we observed the constructive resources students drew upon when creating *their* GRs. The data comes from hour-long, individual CIs with 14 middle grades students (grades 5–8). Video, audio, and students' written work were collected. We used both ongoing and retrospective analysis methods (Steffe & Thompson, 2000). Ongoing analysis involved documenting and analyzing students' activities during and immediately after each interview to identify patterns and make initial hypotheses about students' cognitive strategies. In the next section, we elaborate on the retrospective analysis process.

Figure 1: Family Frenzy – The Sun Family

Explicating the Retrospective Analysis Process

Broadly, our retrospective analysis followed the phases outlined in Figure 2 above. We next elaborate on each phase with an emphasis that the process was cyclic rather than linear.

Creating Thick Descriptions and Identifying Critical Instances

Specific to FF, we first watched several students solving the FF to observe students' graphing strategies and resituate ourselves back in the data. We then constructed thick descriptions (hereafter TDs; Geertz, 1973) to create written records of each students' FF activity. TDs included students' verbal utterances (sometimes paraphrased, sometimes quoted verbatim), their drawings and writings, gestures or motions, and any facial expressions or pauses. When relevant, we also noted the interviewers' comments. One research member created a first draft and a second member or the whole group read and revised the draft while watching the video to finalize each TD. For every 30 minutes of video, the first draft took around 4–6 hours and the second draft took around an hour to make. During the creation of TDs, comments and questions such as noticeable actions, recurring patterns, or unclear utterances, were left in the document and discussed during weekly research meetings.

Abstracting Patterns and Shifts—Developing a Guiding Framework

As we finalized TDs, we sought to develop an analytic framework to systematically model student thinking. Table 1 summarizes the evolution of this framework. By sharing the process and reasoning behind key shifts, we aim to support others in their analytic considerations.

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Figure 2: Retrospective Analysis Process

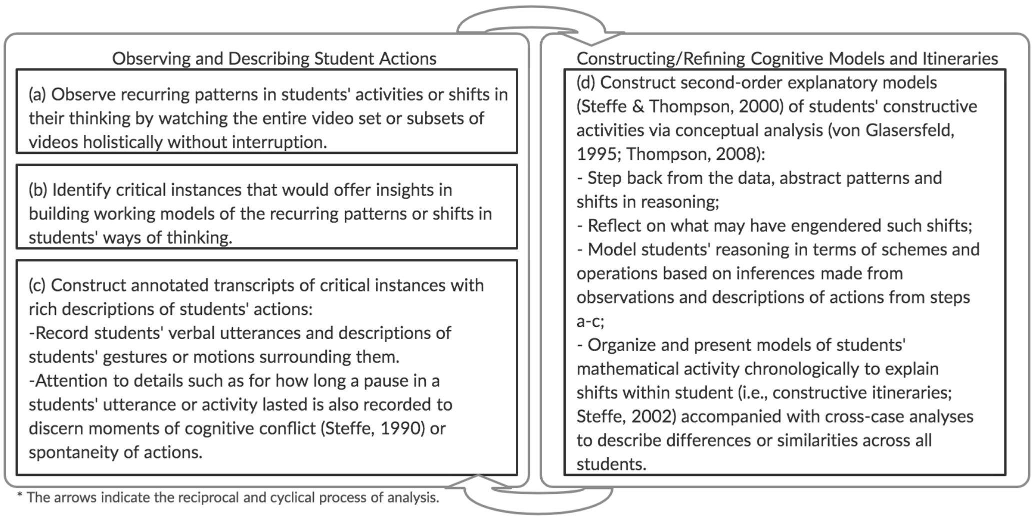

Table 1. Progression of Frameworks

First framework	Other frameworks	Final framework
Set of categories and codes related to students' RFs, CS, graphs to select for each category. Allowed us to capture some characteristics of students' strategies but missed some nuances.	Set of guiding open-ended questions related to students' RFs, CS, and graph. Too broad/specific and/or redundant.	Set of guiding questions focusing on students' actions to extract the quantities, represent and coordinate them, with some options to select from, but also open to extensions.

First framework. Our first framework emerged from open and axial coding (Corbin & Strauss, 2015) of student activity to capture patterns in student thinking in the FF. Two members created codes describing student strategies by rewatching videos and using the TDs. Next, a third member examined the codes and restructured them. As a result, the first framework offered codes for how students placed each quantity (height & age), ordered each quantity in their placement, and coordinated the quantities together. After team discussion, we made minor edits, clarifying terminology and adding examples. Using the updated version, two members outside of the initial coding team analyzed four randomly selected students' FF. From their coding, we found the framework as stabilized; this framework was previously presented in Zolt et al. (2024).

Reflecting on the first framework and developing other versions. Reflecting on our models of students' graphing activity using the first framework led to some concerns. For example, while the framework captured what the student plotted in the graph space, it did not capture their activity towards the photo of the family. We observed students deciding how to gauge height/age in unique ways that often impacted their overall GR, and we wanted to make more explicit connections to students' RFs and CSs in the framework. These thoughts launched several iterations of revisions, which served us in creating a more detailed list of guiding questions to use in the initial open coding phase with attention to RFs and CSs. Once these questions were stabilized, we analyzed several students using the revised framework. We found some questions redundant while some others too specific to the FF that we questioned the usefulness of this framework for others. We also had difficulty answering some questions due to a lack of evidence, and we found the fine detail distracting us from the bigger picture.

We returned to the data and literature for insights. After another cycle of attempting a different approach without resolution, we recognized a need to step back again. Instead of starting with the RF, CS, and graph distinction and trying to fit codes within each of those categories, we created memos on broad, big picture noticings of students' activities while watching videos/TDs and then thinking about how to unpack those in detail, discussing items that could be involved in each process. We then each watched 10 student videos, taking notes on major categories we noticed in students' activities. As we each created categories, we added notes on who may fit into it and what questions we may think about within those categories. We compiled our thoughts, highlighting categories that were similar or could easily be combined and worked together to do so. When we noticed less "big picture" categories emerging, such as nuances within RFs, we kept these notes in mind and thought about sub-questions we could add under our bigger categories that would capture these observations. Once we stabilized this combined document, we separated again to try these questions out with two students who we previously noted as unique, contrasting examples. We used the categories that emerged under the questions as a guide, but we selected multiple or added new subcategories and extra notes as needed. We then met to watch videos and compared our answers to the new questions. Adding clarity as necessary, we came to a consensus.

The final framework. The above process culminated in our final version: Final Framework. Compared to the first framework, this framework had more categories to capture patterns in students' graphing. While our first framework attended to how students plotted height and ages and coordinated the two (like items 3–5 in our final framework), we wanted to capture more than that. We found our final framework did that for us: it helped us attend to additional activities, such as how students made sense of each quantity in the photo and abstracted the quantity, how students interacted with the drawn axis lines, and the kind of objects they chose to make their graphs. These additions helped us gain a better understanding of RFs and CSs students were working with. We also found that our final framework provided enough specificity to capture a particular student's activity while remaining general enough to capture all students' activities. We found it useful to have several sub-categories with examples to choose from. At the same time, we appreciated the flexibility to write narratives about how each category applied to the student's strategy and add notes on the nuances of the solution as necessary.

Analytical frameworks don't fall from the sky. As we have briefly illustrated, the journey to our framework was a long and arduous one. The conception of this framework would not have been possible without all the seemingly trial-and-error process we went through. There are pre-existing frameworks that researchers can use for retrospective analysis. Even then, any analytical framework needs to be modified and adjusted to work for the data, research questions, and the researchers' theoretical assumptions. Although we call the latest framework our "final" one, we don't mean it to be final in the absolute sense. Frameworks, and more broadly, models of students' thinking evolve over time and our goal is to build superseding models as we continue to learn from students and their graphing activity (Steffe & Thompson, 2000). We ask other researchers to join us in reflecting on their journeys in building models of student thinking.

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