Challenges in Numerical Simulation of Frost Heave

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ABSTRACT

Frost heave of soil extensively exists in northern regions and poses a significant threat to infrastructure in cold regions. Despite over a century of research, challenges persist in numerically simulating frost heave. This study addresses two key issues: (1) What is the primary driving force for liquid water transfer during the freezing process? (2) How can we correctly represent unfrozen water content? Critical insights are derived from the theoretical analysis of coupled hydrothermal migration during soil freezing processes, followed by a case simulation using COMSOL Multiphysics. It concludes that of the water content gradient, suction gradient, and hydraulic gradient, only the hydraulic gradient is the fundamental driving force for liquid water flow. Moreover, unfrozen water content is an intrinsic property of frozen soil and should not be indirectly determined by assessing ice content. This study enhances our understanding of the frost heave mechanism and contributes to developing a unified model for frost heave.

Keywords: frost heave; water transfer; unfrozen water content, driving force

INTRODUCTION

Over 50% of the exposed land in the Northern Hemisphere undergoes seasonal freeze-thaw cycles. Frost heave poses a significant challenge in cold regions, causing substantial damage to infrastructure like pavements, railways, pipelines, and buildings (Palmer and Williams 2003; Teng et al. 2022). Frost heave is a coupled thermal-hydro-mechanical (THM) process involving heat transfer, water migration, phase change, and soil deformation (Huang and Swan 2017; Zhang and Michalowski 2015). As soil temperature drops, free water within larger soil pores transitions into ice, reducing the unfrozen water content (Spaans and Baker 1996). This process is analogous to soil drying, resulting in increased soil suction. Elevated suction drives water towards the freezing front, where it continues to freeze under favorable conditions such as low temperature and adequate water supply. Frost heave occurs when ice lenses form and expand in the soil, exerting upward pressure that displaces and deforms the soil surface.

Despite extensive research on frost heave, challenges and debates persist in modeling this phenomenon (Konrad 1994). The first debate concerns the driving force that controls liquid water migration through freezing soils. Various forms of driving forces, such as water content gradient, porosity gradient, temperature gradient, suction gradient, and hydraulic gradient, have been used to simulate frost heave (Dong and Yu 2017; Michalowski 1993; Wand and Ma 2021). The criteria for selecting the driving force should align with the physical process's internal mechanism to avoid significant errors in simulations. For instance, the water content gradient assumes liquid water flows from high to low water content zones, but differences in soil types may cause flow from low to high water content zones. Both porosity and temperature gradients are indirect factors affecting the liquid water flow and should not be directly used as the main driving force. The suction gradient is favored because it establishes a direct relationship with the soil water characteristic curve (SWCC). When the air pressure gradient is zero, the suction gradient is a more desirable driving force, which explains its widespread use. However, for cases where the air pressure gradient is not zero, the suction gradient should be used cautiously. Essentially, flow is best defined in terms of the hydraulic gradient of each phase. Thus, for liquid water in freezing soil, the hydraulic gradient of liquid water is most appropriate.

The second point of contention is how to represent the unfrozen water content. When the temperature drops below 0°C, some liquid water remains unfrozen due to the capillary and surface energy of soil particles (Zou et al. 2023). The unfrozen water serves as a medium for the ice-water phase transition and as pathways for water movement within soil systems, especially via unfrozen water in micro-pores and adsorbed water films (Watanabe and Osada 2016). The unfrozen water content is directly affected by suction and temperature and is a fundamental property of frozen soil (Zhang and Michalowski 2015). Although some pore ice forms from unfrozen water, there is no one-to-one correspondence between the two. In open systems, more pore ice results from the phase change of external water inflow to the freezing front, and only a small fraction is from in situ freezing. Therefore, back-calculating unfrozen water content from pore ice content is inappropriate and introduces potential errors in frost heave simulations.

This study investigates the debates on the forces driving liquid water flow and how to represent unfrozen water content to develop a clearer understanding. Simulations of coupled hydrothermal frost heave processes were conducted using COMSOL to illustrate the validity of the proposed views. This research provides a clearer theoretical basis for simulating frost heave processes in cold regions, contributing to the explanation of the hydrothermal coupled migration mechanism.

GOVERNING EQUATIONS FOR COUPLED HYDRO-THERMAL MIGRATION

Water Transfer Equation

The driving force for water migration during frost heave includes water pressure gradient and positional gradient. The unfrozen water content has been expressed in two forms: volumetric unfrozen water content and mass unfrozen water content, where we choose the latter. This is because its denominator is the mass of the soil particles, a constant value, making comparisons uniform. Based on Richard's equation, the controlling equation for moisture migration is:

$$\nabla \cdot \left(\frac{k_\mathrm{w}}{\gamma_\mathrm{w}} \nabla u_\mathrm{w} \right) + \frac{\partial k_\mathrm{w}}{\partial z} = \frac{G_\mathrm{s}}{1+e_0} \left(\frac{\partial w_\mathrm{u}}{\partial t} + \frac{\partial w_\mathrm{i}}{\partial t} \right) \tag{1}$$

where, k_w is the hydraulic conductivity of freezing soil (m/s); γ_w is the specific weight of water (N/m^3); u_w is the water pressure (kPa); z is the position in the direction of gravity (m); G_s is the specific gravity of soil; e_0 is the initial void ratio; w_u is the unfrozen water content; w_i is the ice content.

Heat Transfer Equation

According to Fourier's law of heat conduction and the principle of conservation of energy, and considering the two-dimensional hydrothermal coupling problem with phase change heat, the differential equation controlling the thermal conductivity are:

$$\nabla\cdot\left(\lambda\nabla T\right)=\frac{\rho_{\mathrm{s}}}{1+e_0}\left(C_{\mathrm{eq}}\frac{\partial T}{\partial t}-L\frac{\partial w_{\mathrm{i}}}{\partial t}\right) \tag{2}$$

where, λ is the thermal conductivity of freezing soil (W/(m·K)); ρ_s is the density of soil (kg/m^3); C_{eq} is the equivalent of specific heat capacity (J/(kg·K)); L is the latent heat of fusion (J/kg).

MODEL IMPLEMENTATION

The proposed model considers both frozen and unfrozen zones. In THE unfrozen zone, the ice content is zero, and moisture migration is equivalent to unsaturated soil, with heat migration not coupled to moisture migration. In the frozen zone, heat and moisture migration are coupled, and liquid water migrating to the freezing front and undergoing a phase change, resulting in frost heave.

As previously analyzed, unfrozen water content is an intrinsic property of freezing soil and does not have a one-to-one correspondence with ice content. Therefore, this study advocates that unfrozen water content should be determined by suction and temperature. For simplicity, a linear equation is presented to determine the unfrozen water content:

$$w_{\mathrm{u}}=\begin{cases}w_{\mathrm{s}}+\dfrac{u_w}{10000}\left(w_{\mathrm{s}}-w_{\mathrm{r}}\right)+\dfrac{T-T_{\mathrm{f}}}{10}\left(w_{\mathrm{s}}-w_{\mathrm{r}}\right), & T\le T_{\mathrm{f}}\\ w_{\mathrm{s}}+\dfrac{u_w}{10000}\left(w_{\mathrm{s}}-w_{\mathrm{r}}\right), & T>T_{\mathrm{f}}\end{cases} \tag{3}$$

where, w_s is the saturated water content; w_r is the residual water content; T_f is the freezing temperature (°C). The hydraulic conductivity is closely related to the unfrozen water content and can be expressed as:

$$k_{\mathrm{w}}=k_{\mathrm{s}}\left(\frac{w_{\mathrm{u}}-w_{\mathrm{r}}}{w_{\mathrm{s}}-w_{\mathrm{r}}}\right)^8 \tag{4}$$

where, k_s is the saturated hydraulic conductivity (m/s).

In the freezing zone, there are two equations, equations (1) and (2), but three unknowns: temperature, water pressure and ice content. Thus, a third equation is needed. Considering that when thermodynamic equilibrium is established, there is a relationship between temperature and water pressure, such as the Clapeyron equation. In this study, we assume that this relationship is satisfied in the freezing zone: $u_w = 100\ T$.

In this study, COMSOL Multiphysics software is used for the simulation of frost heave. The calculations are performed using the customizable weak-form PDE module in COMSOL, which is needed to transform the above equations (1) and (2) into weak form. The calculations are carried out in two steps: the steady-state process under gravity (Stationary) when no sub-zero temperature is applied, and the freezing process with temperature changes and frost heave (Time Dependent).

The geometrical condition of the computational example is 10 cm in length and 6 cm in width. The mesh division is shown in Figure 1, with finer meshing at the boundaries. The boundary conditions are: (1) water pressure at the bottom is set to 0 to satisfy the unpressurized water replenishment; (2) temperature at the top is 1℃; (3) temperature at the bottom is -5℃. The simulation runs for 120 hours, starting with an encrypted computational time step of the first 1 hour, followed by a computational time step of 5 hours. Table 1 lists the core parameters used in the simulation.

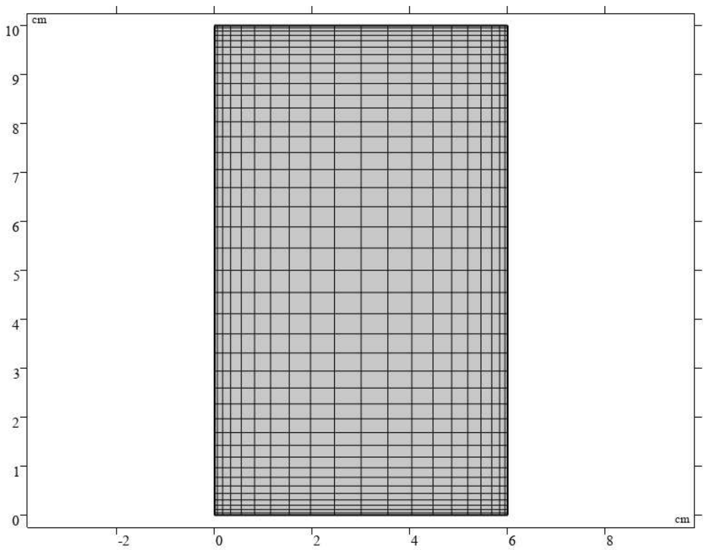


Figure 1. Meshing of soil columns.

Table 1. Soil parameters used for simulation.

Parameter	Value	Unit	Description
k_s	1×10^9	m/s	Saturated hydraulic conductivity
γ_w	9800	N/m^3	Specific weight of water
G_s	2.7	/	Specific gravity of soil
L	3.34×10^5	J/kg	Latent heat of fusion
w_s	1	/	Saturated water content
w_r	0	/	Residual water content
T_f	-0.5	℃	Freezing temperature
ρ_s	2700	kg/m^3	Density of soil
ρ_w	1000	kg/m^3	Density of water
ρ_i	916	kg/m^3	Density of ice
λ_s	1.2	W/(m·K)	Thermal conductivity of soil
λ_w	0.58	W/(m·K)	Thermal conductivity of water
λ_i	2.22	W/(m·K)	Thermal conductivity of ice
C_s	5800	J/(kg·K)	Specific heat capacity of soil
C_w	4200	J/(kg·K)	Specific heat capacity of water
C_i	2100	J/(kg·K)	Specific heat capacity of ice

SIMULATION RESULTS

Figure 2 shows the numerical simulation results of the temperature change over time at different heights of the soil column. Within the first 5 hours of freezing, the temperature changes drastically; then it changes slowly, and after 30 hours, it remains almost constant. The turning point in each curve indicates the phase change, where a large amount of heat is released, causing slow temperature change in the freezing zone, consistent with the physical law.

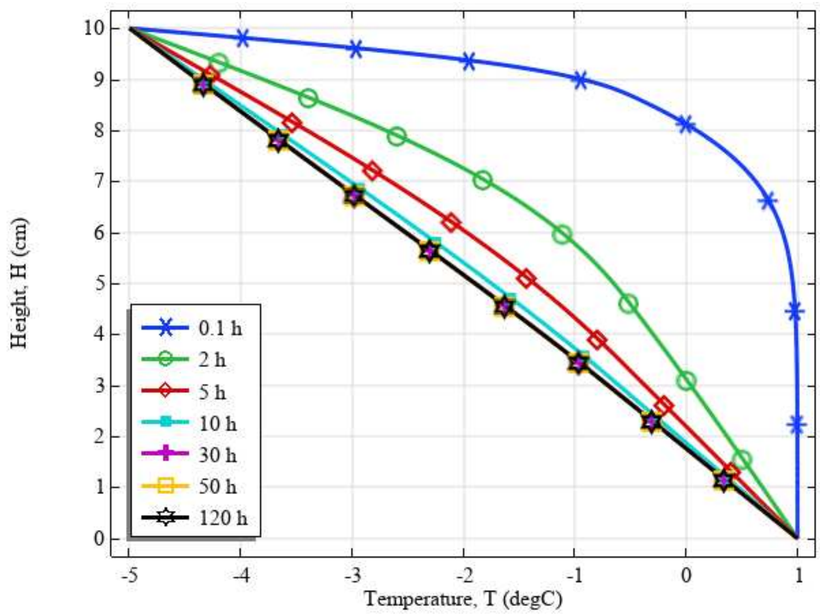


Figure 2. Distribution of soil temperature over time.

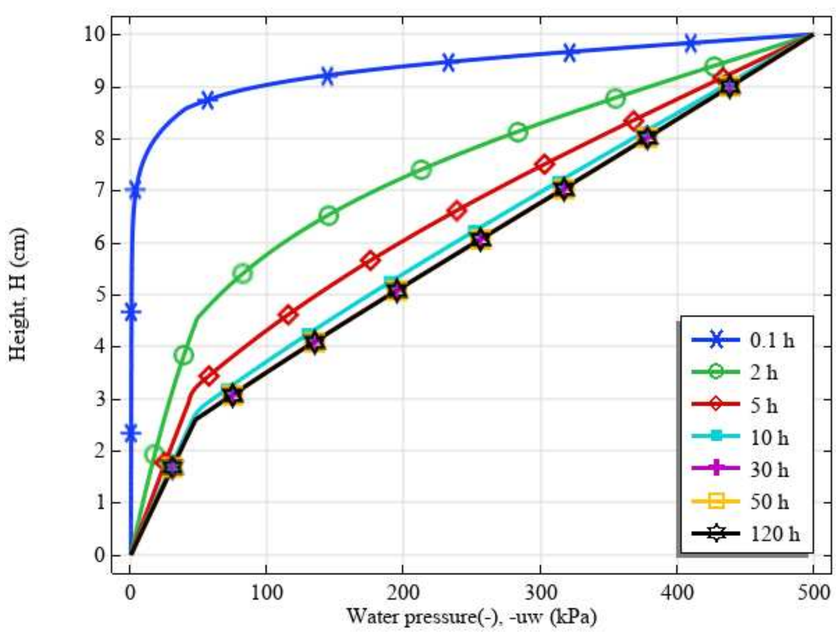
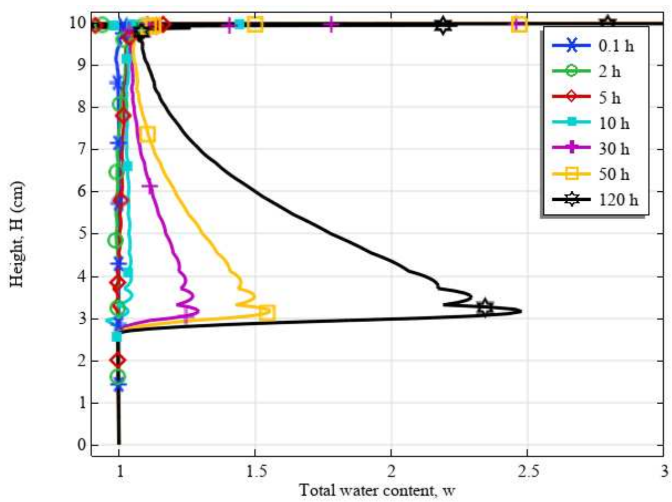
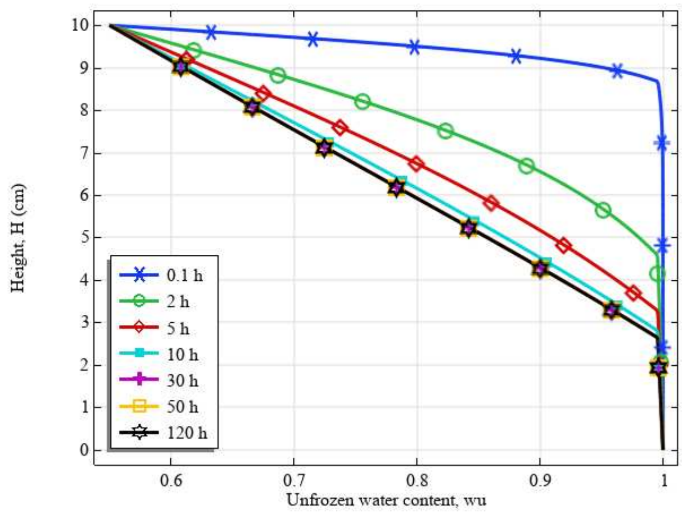


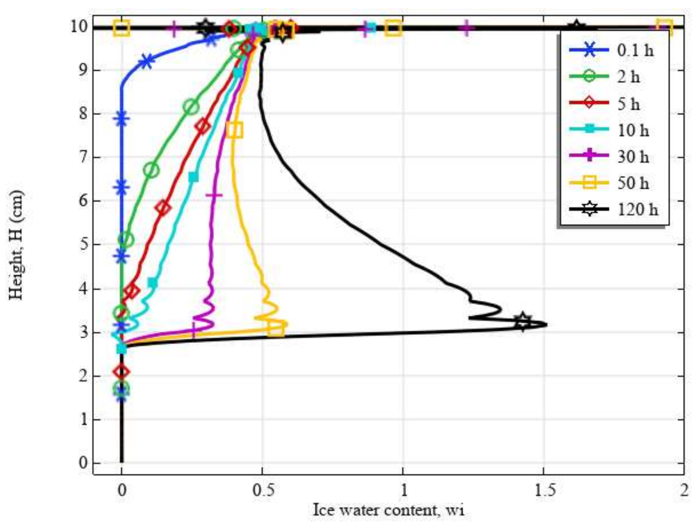
Figure 3. Distribution of water pressure over time.



(a) Total water content



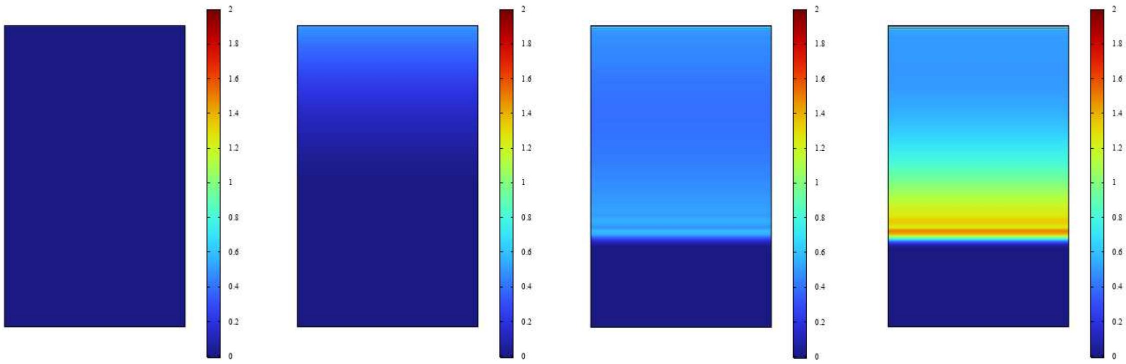
(b) Unfrozen water content



(c) Ice content

Figure 4. Distributions of water in soil columns over time.

Figure 3 shows the numerical simulation results of the water pressure variation over time at different heights of the soil column. Water pressure changes differently in the frozen and unfrozen areas. In the frozen area, water pressure changes are consistent with temperature changes, whereas in the unfrozen area, water pressure changes independently of temperature. The absolute water pressure increases with height, facilitating the upward migration of liquid water and causing soil in the unfrozen area to transition from saturated to unsaturated.



(a) t = 0 h (b) t = 2 h (c) t = 50 h (d) t = 120 h

Figure 5. 2D distribution of ice content in soil columns over time.

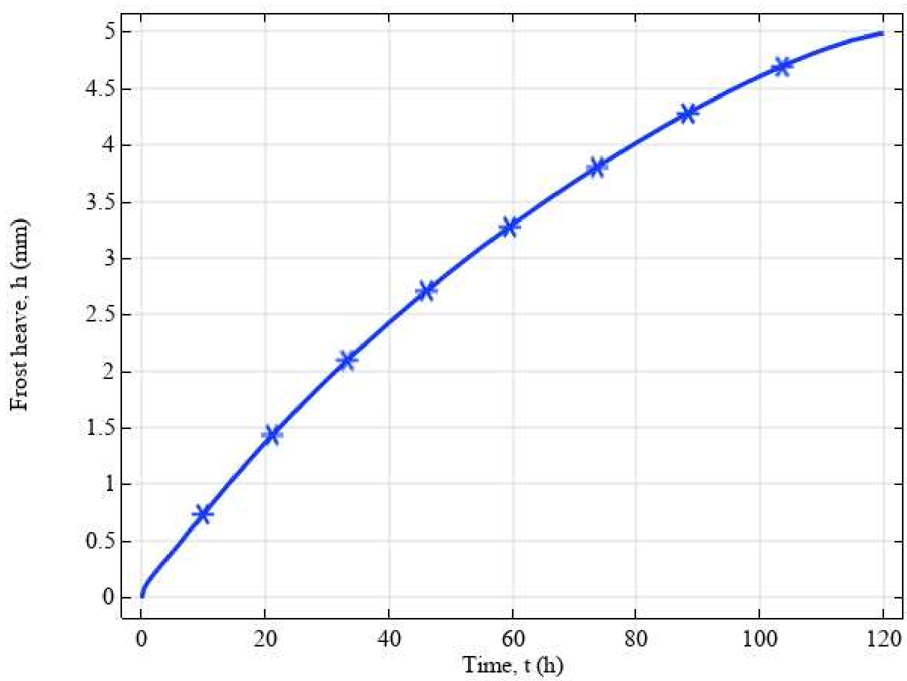


Figure 6. Variation of frost heave over time.

Figure 4 shows the distribution of total water content, unfrozen water content, and ice content in the soil at different times. In Figure 4(a), during the initial freezing stage (0.1h, 2h), liquid water migrates to the top under the hydraulic gradient, increasing the total water content there. As freezing continues, the freezing front moves downward, and the peak of total water content shifts accordingly. After 30 hours, the temperature stabilizes, and the peak of total water content stabilizes at about 3 cm. Without additional constraints, the total water content increases over time. In Figure 4(b), the area below 3 cm remains unfrozen, and the unfrozen water content redistributes under gravity. Figure 4(c) shows that ice content increases over time, and in the frozen zone, the lower the position, the greater the ice content, consistent with the ice lens distribution.

Figure 5 shows the 2D distribution of ice content in the soil over time, illustrating the thickening of ice lenses near 3 cm. Figure 6 shows the change in soil frost heave over time, confirming the continuous increase of ice content at the stable freezing front.

CONCLUSION

This study primarily analyzes the challenges in simulating frost heave. It concludes that among the water content gradient, suction gradient, and hydraulic gradient, only the hydraulic gradient is the fundamental driving force for liquid water flow and is applicable in the widest range of scenarios. Unfrozen water content is an inherent property of frozen soil, determined by temperature and suction, and does not necessarily correlate directly with ice content. Considering these factors, a more accurate coupled thermal-hydro equation to describe the frost heave process is proposed. A frost heave case was simulated using COMSOL, with results consistent with the natural frost heave behavior of soil, demonstrating the model's rationality and effectiveness.

REFERENCES

Dong, S., and Yu, X. (2017). "Microstructure-based random finite element simulation of thermal and hydraulic conduction processes in unsaturated frozen Soils." In *Geotechnical Frontiers 2017*, 781-790.

Huang, Y., and Swan, C. (2017). "Evaluating the behavior of a cohesive soil undergoing one cycle of freeze-thaw." In *Geotechnical Frontiers 2017*, 643-651.

Konrad, J. M. (1994). "Sixteenth Canadian Geotechnical Colloquium: Frost heave in soils: concepts and engineering." *Canadian Geotechnical Journal*, 31(2), 223-245.

Michalowski, R. L. (1993). "A constitutive model of saturated soils for frost heave simulations." *Cold regions science and technology*, 22(1), 47-63.

Palmer, A. C., and Williams, P. J. (2003). "Frost heave and pipeline upheaval buckling." *Canadian Geotechnical Journal*, 40(5), 1033-1038.

Spaans, E. J., and Baker, J. M. (1996). "The soil freezing characteristic: Its measurement and similarity to the soil moisture characteristic." *Soil Science Society of America Journal*, 60(1), 13-19.

Teng, J., Liu, J., Zhang, S., and Sheng, D. (2022). "Frost heave in coarse-grained soils: experimental evidence and numerical modelling." *Géotechnique*, 73(12), 1100-1111.

Wang, C., and Ma, Z. (2021). "Mathematical model and numerical simulation of hydrothermal coupling for unsaturated soil subgrade in the seasonal frozen zone." In *IOP Conference Series: Earth and Environmental Science*, 719(3), 032042.

Watanabe, K., and Osada, Y. (2016). "Comparison of hydraulic conductivity in frozen saturated and unfrozen unsaturated soils." *Vadose Zone Journal*, 15(5), 1-7.

Zhang, L., Ma, W., Yang, C., Wen, Z., and Dong, S. (2016). "An investigation of pore water pressure and consolidation phenomenon in the unfrozen zone during soil freezing." *Cold Regions Science and Technology*, 130, 21-32.

Zou, Y., Jiang, H., Wang, E., Liu, X., and Du, S. (2023). "Variation and prediction of unfrozen water content in different soils at extremely low temperature conditions." *Journal of Hydrology*, 624, 129900.