Analytical Streamer Model to Explain Very High Frequency (VHF) Radiation From Narrow Bipolar Events

Zaid Pervez, Reza Janalizadeh, and Victor P. Pasko

Abstract – Narrow bipolar events (NBEs) are highly impulsive and powerful sources of very high frequency (VHF) radiation in the Earth's natural environment, often preceding lightning discharges in thunderclouds. The VHF radiation is believed to be emitted by a system of streamers around the NBE source. In this article, an analytical model for a double-headed exponentially growing streamer is developed with parameters informed from results of past studies. Results from the model indicate streamers of growing strength and spatial scales with increasing altitude. This effect is explained by the suppression of electron losses due to three-body attachment with reducing air pressure. Further, the current moment and radiated electric field for an NBE are reproduced, assuming the NBE source current is entirely composed of streamers, although there might be other processes contributing to the low-frequency components, such as the relativistic photoelectric feedback process. The spectrum of the radiated field is reported and discussed.

1. Introduction

Lightning precursor discharges are highly impulsive sources of electromagnetic radiation on the Earth, occurring inside thunderclouds during the early stages of lightning development. They are most commonly identified by the very low frequency-low frequency (VLF-LF) band radio emissions, also called *sferics*, observed from the ground as bipolar pulses of tens of microseconds duration. These include initial bipolar pulses [1], energetic in-cloud pulses [2], and narrow bipolar events (NBEs) [3]. NBEs are the sferics associated with a type of discharge known as *compact intracloud discharge* (CID). CIDs have a spatial extent of a few hundred meters. NBE refers to the *observed sferic*, and CID refers to the *source discharge* [4]. However, the two terms are often interchangeably used in the literature, with the term *NBE* being used to refer to both the sferic and the source discharge. We similarly have used the term NBE to refer to both. NBEs are of particular interest because they are the strongest source of terrestrial very high frequency (VHF) radiation, and some of them are known to be the initiating event of lightning discharges [3, 4].

Although most commonly identified by their VLF-LF sferics, NBEs have several other signatures. Recent observations have identified strong emissions at 337 nm emerging from cloud tops as the optical signature of CIDs [5, 6]. There is also growing evidence that they are associated with a type of transient luminous event referred to as *elve* [5, 7]. Energetic in-cloud pulses, with higher peak currents and sferics of longer duration than NBEs [2], have been shown to be associated with terrestrial gamma ray flashes [2, 8], and NBEs are often observed to follow gamma ray emissions within several milliseconds [9]. Finally, as previously mentioned, NBEs are accompanied by strong VHF radiation. The source mechanism of NBEs is not well understood. There are several works that estimate the source current generating the sferic [1, 10, 11]. Recent work has reported a mechanism on the basis of photoelectric feedback in relativistic electron avalanches to explain the production of this source current [12]. The VHF radiation is proposed to be caused by a rapidly propagating system of streamers in a predominantly vertical direction termed as *fast breakdown* [3, 13]. Recent work gained further insight into the source mechanism on the basis of this picture of a streamer-produced HF-VHF spectrum, highlighting the difference from the radio spectra of other lightning processes, such as return strokes that roll off above ∼10 MHz [14, 15].

To further understand the streamer-produced HF-VHF spectra of NBEs, this work reports on a novel and simple analytical streamer current model. Although developed for the study of NBEs, the model can be applied to the study of streamer-related phenomena in industry or the Earth's atmosphere. Results from the reported streamer model highlight changes in streamer characteristics, such as dipole and current moments and spatial and temporal scales with altitude. Finally, an ensemble of these streamers is used to model an NBE source, the resulting time domain sferic, and its frequency spectrum.

2. Model Formulation

We model a double-headed streamer as a vertical linear current source centered at height z_0 above the ground. The positive and negative heads of the streamer initiate at z_0 and propagate with identical speeds in

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opposite directions along the z-axis. The specific direction of each head will depend on the polarity of the main discharge under consideration. For example, for a positive NBE that moves positive charge down, the positive streamer head will move in the $-z$ direction and the negative head in the $+z$ direction. For the purpose of calculating radiated electric field from this current source, the point of observation is placed on the ground at a horizontal distance d away from the source. Because several quantities in gas discharge phenomena scale with altitude as a function of atmospheric density [16], known as *scaling* or *similarity laws*, we use a scaling factor $\delta(h) = N(h)/N_0$, where N is the air number density at altitude h and $N_0 = 2.688 \times 10^{25}$ m^3 is neutral density corresponding to standard atmospheric conditions at sea level on Earth. Note that z_0 and h represent the same location in space but carry different meaning. Although h is simply the altitude, z_0 is the vertical coordinate of the source, where the ground is located at $z = 0$ and might be at an altitude greater than zero. This distinction is important because h is used only in the calculation of δ, while the streamer current distribution is represented along the z-axis with the origin located at the ground. The streamer develops in a region with locally applied reduced electric field E_a/δ. It has been demonstrated in a number of works that as a streamer propagates, its head radius and velocity exponentially increase [17]. The length of a streamer is linearly proportional to its velocity and, thus, exponentially grows as well [17]. The time scale or time constant of this exponential growth τ_s is defined as a function of E_a/δ

$$\tau_s \delta = \frac{E_{cr}^-/\delta}{E_a/\delta} \frac{\tau_{s0}}{1 - \frac{E_{cr}^-/\delta}{E_a/\delta}} \tag{1}$$

where $E_{cr}^-/\delta = 12.5$ kV/cm is the minimum electric field required for stable propagation of negative streamers and $\tau_{s0} = 6.1524$ ns is a parameter whose value was determined based on values of τ_s reported in previous works [17–20]. For exponential growth of streamers, $E_a > E_{cr}^-$. There are some variations around the value for E_{cr}^- in the existing literature. For consistency with previous publications, we use the value listed in [17].

Because streamer current $i_s \propto r_s^2 v_s$, where r_s and v_s are the streamer head radius and velocity, respectively, the current exponentially grows with time constant $\tau_s/3$. The streamer grows until time t_{peak}, after which the streamer current exponentially decays. This decay is primarily caused by electron losses due to two- and three-body attachment processes. The time constant of this exponential decay is approximated as $\tau_{23b} = 1/[\nu_{2b}(E_{cr}^-/\delta) + \nu_{3b}(E_{cr}^-/\delta)]$, where ν_{2b} and ν_{3b} are the two- and three-body attachment frequencies, respectively. After t_{peak}, the velocity and radius remain constant until the streamer current vanishes. The velocity is so defined as

$$v_s(t) = \begin{cases} v_{s0} e^{t/\tau_s}, & t \le t_{peak} \\ v_{s0} e^{t_{peak}/\tau_s}, & t > t_{peak} \end{cases} \tag{2}$$

where $v_{s0} = 3 \times 10^5$ m/s is equivalent to electron drift velocity at $3E_k$ in which $E_k/\delta = 31.845$ kV/cm is the conventional breakdown field defined by the equality of the ionization and two-body attachment coefficients in air [21], because the electric field in the streamer head zone when the streamer initiates is about $3E_k$. Note that although E_k (and, more generally, all electric fields) scale with altitude as $\propto \delta$, velocity does not scale. Similarly, the streamer head radius is defined as

$$r_s(t) = \begin{cases} r_{s0} e^{t/\tau_s}, & t \le t_{peak} \\ r_{s0} e^{t_{peak}/\tau_s}, & t > t_{peak} \end{cases} \tag{3}$$

where $r_{s0}\delta = 0.2$ mm, which broadly agrees with the starting radii of positive and negative streamers in [22]. The streamer velocity is limited to a maximum possible value of $c/10$, where c is the speed of light, because this is about the highest velocity recorded in naturally occurring streamers [23]. The time t_{peak} separating the growth and decay stages is defined as $t_{peak} = \min[4.6\tau_s, \tau_{23b}]$ and $v(t = 4.6\tau_s) = c/10$. The streamer current is defined as

$$I_s(z,t) = i_s(t) \begin{cases} \frac{1}{2}\left\{1 + \tanh\left[\frac{(z - z_0) + \overline{v}_s(t)t}{r_s(t)}\right]\right\}, & z < z_0 \\ 1, & z = z_0 \\ \frac{1}{2}\left\{1 - \tanh\left[\frac{(z - z_0) - \overline{v}_s(t)t}{r_s(t)}\right]\right\}, & z > z_0 \end{cases} \tag{4}$$

and

$$i_s(t) = \begin{cases} i_0 e^{3t/\tau_s}, & t \le t_{peak} \\ i_0 e^{3t_{peak}/\tau_s} e^{-(t - t_{peak})/\tau_{23b}}, & t > t_{peak} \end{cases} \tag{5}$$

where $i_0 = \pi r_{s0}^2 q_e n_e v_{s0} \approx 0.6$ A, q_e is the charge of electron, $n_e \approx 10^{20}$ m^{-3} is the electron number density, and $\overline{v}_s$ is the average of v_s from 0 to t, given by

$$\overline{v}_s(t) = \begin{cases} \frac{v_{s0}\tau_s}{t}\left(e^{t/\tau_s} - 1\right), & t \le t_{peak} \\ \frac{v_{s0}}{t}\left[\tau_s\left(e^{t_{peak}/\tau_s} - 1\right) + e^{t_{peak}/\tau_s}(t - t_{peak})\right], & t > t_{peak} \end{cases} \tag{6}$$

We use current moment $[IL](t) = \int_z I(z,t)dz$ and dipole moment $[QL] = \int_0^\infty [IL](t)dt$ for representation of streamer currents and NBE source current. We also define the effective time duration of a streamer as $\tau_Q = [QL]/[IL]_{peak}$, where $[IL]_{peak}$ is the maximum current moment, and effective length of a streamer as $l_Q = [QL]/Q_s$, where $Q_s = \int_0^\infty i_s(t)dt$.

Now, we assume that the entire source current of an NBE is composed of N_s identical streamers colocated in

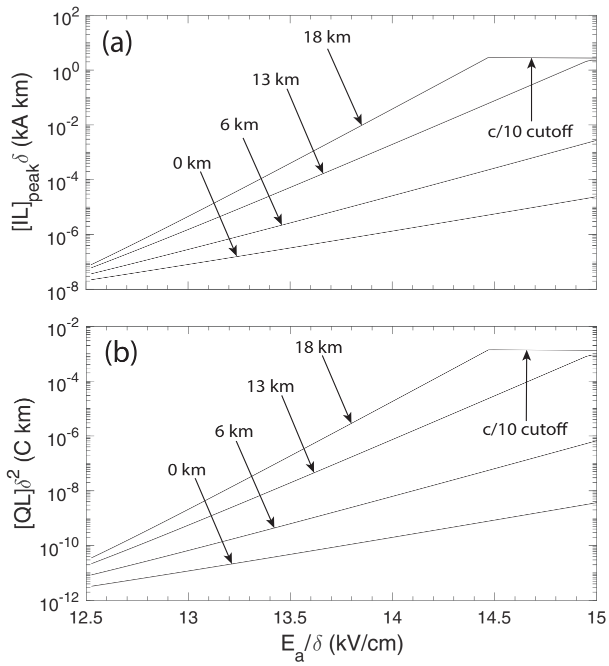

Figure 1. (a) Peak current moment $[IL]_{\text{peak}}\delta$ and (b) dipole moment $[QL]\delta^2$ versus reduced applied electric field E_{a}/δ for the streamer model at different altitudes.

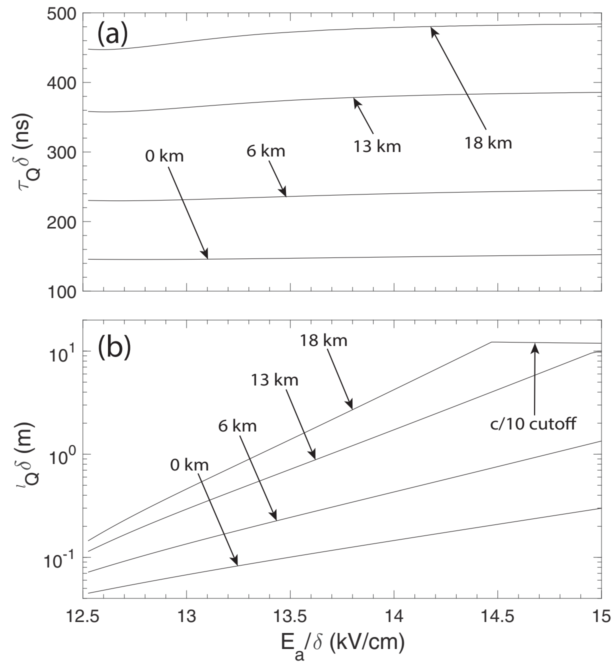

Figure 2. (a) Effective time duration $\tau_{\text{Q}}\delta$ and (b) effective length $l_{\text{Q}}\delta$ versus reduced applied electric field E_{a}/δ for the streamer model at different altitudes.

space at $z = z_0$. Each streamer will result in a dipole with dipole moment $[QL]_{\text{s}}$, and the collective dipole moment of all N_{s} identical streamers will create a dipole with dipole moment $N_{\text{s}}[QL]_{\text{s}}$. The dipole moment of the NBE is $[QL]_{\text{NBE}} = N_{\text{s}}[QL]_{\text{s}}$, so we have $N_{\text{s}} = [QL]_{\text{NBE}}/[QL]_{\text{s}}$. Because all the streamers are vertical, this superposition of dipole moments is scalar. If the streamers were oriented in different directions, the vector addition of dipole moments would be less than or equal to the scalar addition of the magnitudes. Thus, $[QL]_{\text{NBE}}$ represents an upper bound for the total dipole moment. Because the fast breakdown discharge associated with NBEs largely develops in a vertical direction [13], this is a good estimate.

To formulate the streamer-composed current of the NBE, we use an approach similar to the one in [14], where the NBE current is given by $I_{\text{NBE}}(z,t) = \sum_{i=1}^{N_{\text{s}}} I_{\text{s}}(z, t - t_i)$, and $I_{\text{s}}(z, t - t_i)$ is the current of the i^{th} streamer initiated at $t = t_i$. Because N_{s} can be a fairly large quantity, computing current for each streamer using (4) for every point in z can be computationally expensive. To circumvent this problem, we translate I_{s} to an equivalent current I_{d} on the basis of the methodology to model a vertical current source in [11], which we call the *half-wave dipole model* here. Hence, we have $I_{\text{d}}(z,t) = \frac{\pi}{2l_{\text{Q}}} g(z)[IL]_{\text{s}}(t)$, where $g(z) = \cos(\pi(z-h)/l)$ and $z \in [z_0 - l_{\text{Q}}/2, z_0 + l_{\text{Q}}/2]$. Note that I_{d} has the same streamer current moment $[IL]_{\text{s}}$ as I_{s}. This method requires calculation of I_{s} using (4) and I_{d} for a single streamer, and the current for each streamer can then be obtained by shifting in time because I_{d} independently varies with respect to z and t such that $I_{\text{NBE}}(z,t) = \sum_{i=1}^{N_{\text{s}}} I_{\text{d}}(z, t - t_i) = \frac{\pi}{2l_{\text{Q}}} g(z) \sum_{i=1}^{N_{\text{s}}} [IL]_{\text{s}}(t - t_i)$.

The starting times t_i of each streamer are randomly sampled from a probability density function $f_{\text{NBE}}(t)$. If the current moment $[IL]_{\text{NBE0}}$ of the NBE source generating the smooth VLF-LF sferic is known, for example, by using the method described in [11], then f_{NBE} can be obtained by normalizing this current moment as $f_{\text{NBE}}(t) = [IL]_{\text{NBE0}}(t)/\int_{t'} [IL]_{\text{NBE0}}(t')dt'$. Finally, once I_{NBE} is obtained, the radiated electric field observed on the ground at a horizontal distance d away from the source can be calculated using either equation (2) or (3) in [11].

3. Results and Discussion

Figures 1a and 1b show the altitude scaled peak current moment $[IL]_{\text{peak}}\delta$ and dipole moment $[QL]\delta^2$, respectively, for reduced applied fields E_{a}/δ ranging from 12.5 kV/cm to 15.0 kV/cm at altitudes of 0 km ($\delta = 1$), 6 km ($\delta \approx 0.5$), 13 km ($\delta \approx 0.2$), and 18 km ($\delta \approx 0.1$) for a streamer current from the model in Section 2. Figures 2a and 2b show the scaled effective streamer time duration $\tau_{\text{Q}}\delta$ and length $l_{\text{Q}}\delta$, respectively, for the same fields and altitudes. All physical parameters used in the model follow similarity laws and scale with altitude. Time, length, and current moment scale as $\propto 1/\delta$, and dipole moment scales as $\propto 1/\delta^2$. However, because the three-body attachment frequency scales as $\propto \delta^2$, it violates scalability for other quantities. Thus, all quantities shown in Figures 1 and 2 are different at

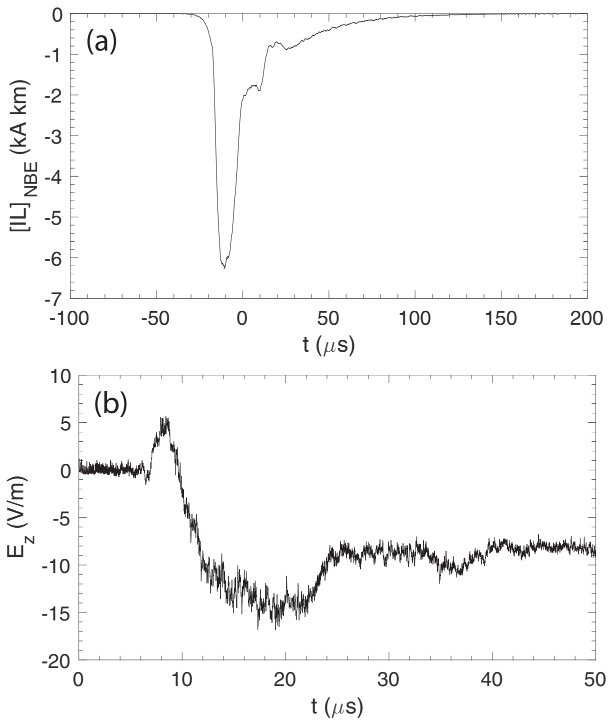

Figure 3. (a) Current moment for the streamer ensemble and (b) the resulting electric field at the point of observation for NBE geometry.

different altitudes. If three-body attachment was ignored, the scaled results for different altitudes would coincide. There is significant growth in all four quantities at higher altitudes, because of the diminishing effect of three-body attachment with altitude, which is the primary process responsible for the decay of streamers. With reducing air pressure, streamers become free and powerful, as seen in blue and gigantic jets that propagate several kilometers above cloud tops [24]. Also, $[IL]_{\text{peak}}\delta$, $[QL]\delta^2$, and $l_{\text{Q}}\delta$ values level out to nearly constant values at high altitude and high applied fields due to streamer velocity being limited to a maximum value of $c/10$. Growing $\tau_{\text{Q}}\delta$ with altitude indicates that the spectral content of streamers shifts to lower frequencies with the reduction of air pressure. Although $\tau_{\text{Q}}\delta$ grows with altitude, it remains fairly constant with changing E_{a}/δ.

To entirely compose an NBE current of streamers, as described in Section 2, NBE3 from [3] was used, the current for which was estimated in [11], and shown in Figure 3c. The NBE source current moment $[IL]_{\text{NBE0}}$ (also given in [11]) and, consequently, f_{NBE} can be obtained from this current. The NBE source was located at $h = 9.6$ km ($\delta \approx 0.33$) and vertical distance $z_0 = 6.6$ km above the ground. For a representative applied field $E_{\text{a}}/\delta = 14.04$ kV/cm corresponding to $\tau_{\text{s}}\delta \approx 50$ ns, the obtained streamer has $[IL]_{\text{peak}}\delta = 2.41 \times 10^{-4}$ kAkm, $[QL]\delta^2 = 7.35\times10^{-8}$ Ckm, $\tau_{\text{Q}}\delta = 0.305$ μs, and $l_{\text{Q}}\delta = 0.88$ m. The number of streamers required to compose this current moment $[IL]_{\text{NBE}}$ is calculated to be $N_{\text{s}} = 216,588$. Figure 3a

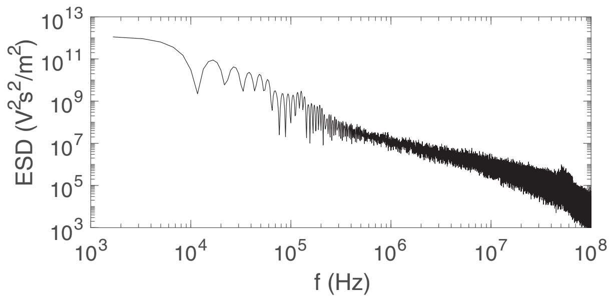

Figure 4. ESD of the NBE under consideration, averaged over 10 streamer ensembles.

shows $[IL]_{\text{NBE}}$ constructed from the streamer ensemble that is similar to the original current moment $[IL]_{\text{NBE0}}$ taken from [11]. It is, however, not as smooth and contains higher frequency components. Figure 3b shows the electric field E_z observed on the ground at a horizontal distance $d = 3.3$ km away from the source. This E_z closely follows the recorded NBE but is not smooth because it is dominated by the radiation term that is proportional to $\partial[IL]_{\text{NBE}}/\partial t$.

In studying the spectral content of the NBE, the energy spectral density (ESD) is considered, which is defined as $\text{ESD} = 2\,|\widetilde{E}_z(f)\,|^2$, where $\widetilde{E}_z(f)$ is the Fourier transform of $E_z(t)$ [14]. Figure 4 shows ESD for the NBE under consideration averaged over 10 instances of randomized streamer ensembles. Figure 3 is an example of one such instance. The ESD falls with frequency $\propto f^{-1.6}$, is relatively smoother at lower frequencies, and significantly fluctuates above 1 MHz. It falls about one to two orders of magnitude between 10 MHz and 100 MHz, similar to recorded VHF spectra for NBEs shown in [14]. Unlike the modeling results in [14], from a few megahertz to tens of megahertz, the ESD is not fairly constant.

4. Conclusions

This work develops a new analytical model for effective representation of currents responsible for electromagnetic radiation from exponentially growing streamers at different applied electric fields, air densities, and altitudes in the Earth's atmosphere. The model results indicate significant growth of streamers at higher altitudes compared with ground pressure ones in terms of physical dimensions, durations, and current and dipole moments due to the diminishing effects of three-body attachment losses of electrons with the reduction of air pressure. For a system of streamers known as fast breakdown present around an NBE source, believed to be the cause of the strong VHF radiation observed for NBEs, the model is used to study the source and VHF spectrum of an NBE reported in [3]. The results indicate that VHF emissions, observed in association with smooth current sources with spatial extents of hundreds of meters, leading to VLF-LF NBE sferics are likely produced due to fast-growing streamers seeded by an abundance of low-energy electrons generated by the relativistic photoelectric feedback discharges in the same physical space. Future work will involve a more quantitative analysis of the streamer ensemble spectral features to gain deeper insight into the VHF spectrum of NBEs.

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