DEVELOPMENT AND EARLY USE OF A NOVEL SPATIAL RELATIONS TASK IN EARLY CHILDHOOD

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Capturing the development of spatial reasoning is an important goal for researchers in early childhood mathematics, but few existing measures attend to young children's spatial intuition as they attempt to solve problems. This brief report describes the development of a novel measure of spatial reasoning. We describe design principles, the piloting process, and early findings from its use with 67 preschool children aged 37-56 months.

Keywords: Early Childhood Education, Geometry and Spatial Reasoning, Assessment, Problem-Solving

There is a growing need to capture the development of young children's spatial reasoning in ways that can inform policy and practice (Bruce et al., 2017; Davis & Spatial Reasoning Study Group, 2015; Pinilla, 2024). Children's spatial skills predict mathematics achievement (Mix & Cheng, 2012) and overlap with aspects of other STEM domains (Sarama & Clements, 2009). However, few existing measures attend to children's earliest intuitive practices as they attend to and manipulate shapes during problem solving. This brief report describes the development of a novel measure of spatial reasoning using a challenging shape puzzle task. We describe design principles, the piloting process, and early findings from the measure's use with preschoolers.

Perspectives and Theoretical Framework

This study adopts a view of learning that foregrounds people's evolving participation in socially and culturally organized activity (Lave, 2019; Nasir et al., 2020). This perspective focuses our attention on the situated nature of interactions between people, tasks, tools, and symbols that mediate doing mathematics. We also attend to the embodied, material nature of young children's puzzle activity (Radford, 2014) as they communicate their understandings through movement, gesture, facial expressions, and sounds of excitement or confusion.

Engaging children challenging tasks is a core tenant of current recommendations for mathematics education (National Council of Teachers of Mathematics, 2014), but some measures of early spatial skills (e.g., cards with images that depict rotations or reflections) narrow spatial ways of knowing and communicating by focusing on mental activity. These measures have provided valuable insights into young children's emerging spatial abilities but may overlook children's earliest intuitive practices as they attend to and manipulate within more playful problem-solving situations.

Methods

We sought to design a spatial relations measure for use in a longitudinal study of young children's participation in mathematical practices. Critically, the measure needed to invite young children to participate in rich and meaningful mathematics. We also needed the measure to be suitable for multiple uses over time for children aged three to six years. The following design principles guided our piloting and development process:

Design Principles

Playful. Our measure was to be used during classroom visits to various preschool sites in our study. Our goal was to administer assessments during indoor and/or outdoor play times, so as not to interrupt teachers' planned whole group or small group activities. We were also conscious that, for especially the initial round of preassessments, the researchers would be unfamiliar to children and that some children might be apprehensive about working with us. It was therefore important that the measure be engaging and inviting in a way that many of the materials in their classrooms were. Tasks that featured manipulable materials such as blocks or puzzles seemed promising as they were similar to the rich materials children had access to during indoor play.

Focused on children's mathematical thinking. Our study is concerned with the mathematics that emerges from children's engagement with and sense making about the world around them. This stance pushes us to focus not on the formalisms and conventions that dominate school mathematics, but on the approaches that children naturally generate as they draw on their intuition to solve problems (e.g., Carpenter et al., 2015). This called for a measure that offered multiple solutions or solution pathways. It also discouraged us from using measures relied on children's interpretations of extensive verbal directions (we hoped to minimize confusion about what the child was being asked to do) and steered us away from tasks that required the child to mimic an adult's use of material or comment on an adult's activity.

Challenging. Our goal was to be able to capture children's engagement with the measure at the beginning and end of each school year for three years. The task needed to be approachable enough for children to work with and experience success during their earliest experiences at school (as young as 36 months), but rich enough to still hold appeal and challenge for children at the end of their kindergarten school year (up to 79 months). This meant that many of the puzzles and block building tasks commonly used with preschoolers would be too simple for children later in our study. This also prompted us to look beyond common materials that children might be familiar with from home or school, such as wooden building blocks, Duplos, or Magna-Tiles.

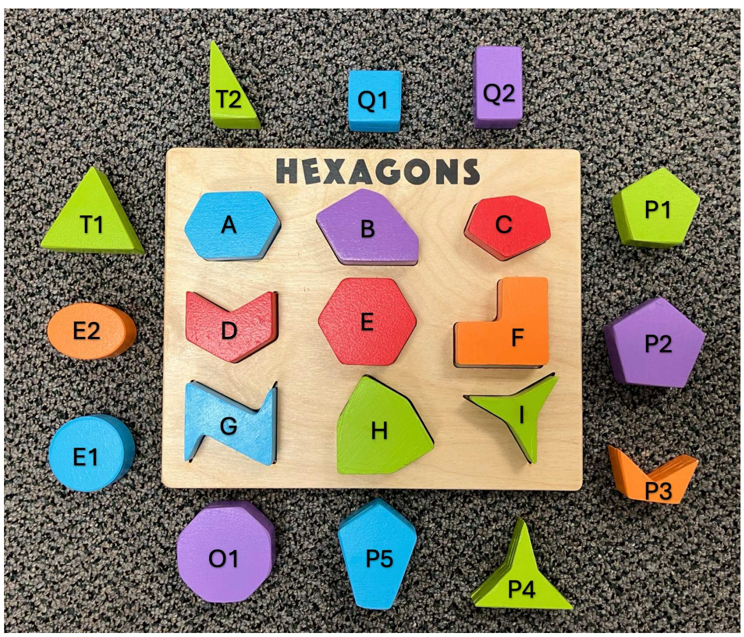
Piloting and Development

We were intrigued by the possibilities for spatial reasoning offered by 21st Century Pattern Blocks and the assortment of wooden shape puzzles used at Math On-A-Stick (n.d.) and piloted these materials during our design phase. During piloting several trends emerged. Children were particularly drawn to the wooden shape puzzles. They were heavy, fit well into small hands, and appeared satisfying to successfully fit into their recessed spots on the puzzle bases. Moreover, they required no introduction; simply dumping the pieces and puzzle bases onto the rug ensured a crowd of young people would gather and work collectively on completing the six different puzzles (triangles, rectangles, pentagons, hexagons, ellipses, and regular polygons). A particularly striking moment during piloting was when a young child who had only recently received cochlear implants struggled to participate in a counting activity with her teacher but, moments later, demonstrated facility in completing the shape puzzles, outpacing her peers. This convinced us of the importance of including a spatial task that could be administered nonverbally that would complement the *Describe, Draw, Describe* activity that teachers were using to invite children to create spatial representations and elicit their descriptions (Turrou et al., 2021).

Perhaps most importantly, the shape puzzled offered possibilities to observe a range of rich mathematics. Rather than the canonical representations of shapes offered by most puzzles, these puzzles featured many variations of shapes (including convex and concave polygons) with a range of angles, sides, and symmetric properties. The specific geometric properties of a given piece shaped the necessary slides, flips, or turns required to place that piece. For example, pieces with different kinds of symmetry offered multiple "solutions" or placements that would work (the rectangles could be rotated 180 degrees or flipped over, yielding four possible orientations that would work for a given piece, whereas only one possible orientation would allow for the correct placement of the scalene right triangle). However, during piloting two limitations of the shape puzzles emerged. First, the colored backings for each spot on the puzzle bases corresponded with the colors of the piece that would fit into that particular spot. This allowed for children's attempts to place a particular piece to be influenced by, not only its geometric features, but also by whether the color corresponded, limiting where it might be tried. Second, as the other puzzles were completed there were fewer and fewer possibilities for which pieces were available to fill the most challenging spaces; the task narrowed considerably as the puzzles neared completion. We addressed these by, with their creator's help, (1) removing the colored backings from the puzzle bases, and (2) inviting the child to complete only one of the puzzle bases and including additional pieces from the other puzzles.

We settled on a task that centered on completing the hexagon puzzle. The final set of shapes included both the 9 hexagonal pieces (A-E) needed to successfully complete the puzzle, along with 12 "distractor" puzzle pieces: two ellipses (E1-E2), two triangles (T1-T2), two quadrilaterals (Q1-Q2), five pentagons (P1-P5), and one octagon (O1; see Figure 1).

Figure 1: Hexagon puzzle with distractor pieces



Task Administration

Previous work with shape puzzles found that the "messy" condition yielded greater participation in puzzle activity, both in terms of time spent working on puzzles and the number of children who chose to work with them (Danielson, n.d.). This aligned with our previous work in counting which showed children were sometimes reluctant to manipulate objects when the researcher had carefully organized and placed them prior to administering the task. For these reasons we randomized the initial state of the puzzle pieces by dumping them into an unorganized, overlapping pile on the ground next to the empty puzzle base.

Participants

Our sample drew from six preschool classrooms. We video recorded 67 preschoolers as they engaged with and attempted to complete the hexagon puzzle. At the time of the interview children's age ranged from 37-56 months (mean age 46 months, SD=5).

Initial Findings

Children's engagement with the shape puzzle varied, both in terms of how many pieces they successfully placed and in the ways they strategically attempted placing certain pieces. The number of pieces successfully placed ranged from 0-9, with a mean of 5.8 (SD=2.6). We had hypothesized that the relative difficulty of pieces might correspond with their possible "solutions" according to their rotational and reflexive symmetry. For example, we expected the regular hexagon E, with its reflective symmetry and six possible rotational placements, would be among the easiest shapes for children to place successfully. Similarly, we expected that the hexagon without any rotational or reflexive symmetry (labeled B) would be the most difficult for children to place correctly. However, both hypotheses were incorrect. Children were most successful (88%) in correctly placing the "L shaped" hexagon (labeled F), with hexagon A (39%) the most challenging to place correctly. The hexagon without any symmetry (B) was third most difficult (42%), while surprisingly regular hexagon E was the fourth most difficult (64%).

Video analyses of the details of children's attempt processes suggested children were attending to shape features in unexpected ways during their trials. For example, an analysis showed that children frequently tried regular shapes (P1, P2, & O1) in the spot for hexagon E, while rarely selecting other non-regular pieces. Many children also seemed to attend to concavity, and they attempted to place P3 in spots D and G, but not in other possible spots. Finally, some children appeared strategic in the pieces they *did not* attempt to place in the puzzle. For example, Edwin picked up the circle and ellipse (E1, E2), looked at the puzzle base, and then discarded the ellipses without attempting to place them. As a goal of our assessments is to unearth children's intuitive mathematical understandings, we are interested to more thoroughly explore these tendencies in our current data and are curious if they will surface again in the post assessments completed at the end of the school year.

Discussion and Implications

Our early findings suggest the potential of the shape puzzles task to uncover children's early processes related to transformational geometry and shape properties. Our findings also offer a different interpretation of children's early conceptions of shapes. Views of development that prioritize naming and classifying shapes show that children first form loose, holistic visual schemes for shapes before attending to relationships between parts of shapes (National Research Council, 2009). However, when a child tries a given piece because it "looks like" it might fit into the puzzle, they may be attending to other relevant attributes such as regularity or concavity. Such an interpretation invites a restructuring of school mathematics to center children's—rather than mathematicians'—ways of knowing geometry and space.

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