

# High-Resolution Characterization of Machining-Induced Residual stress In Single-Crystal Sapphire

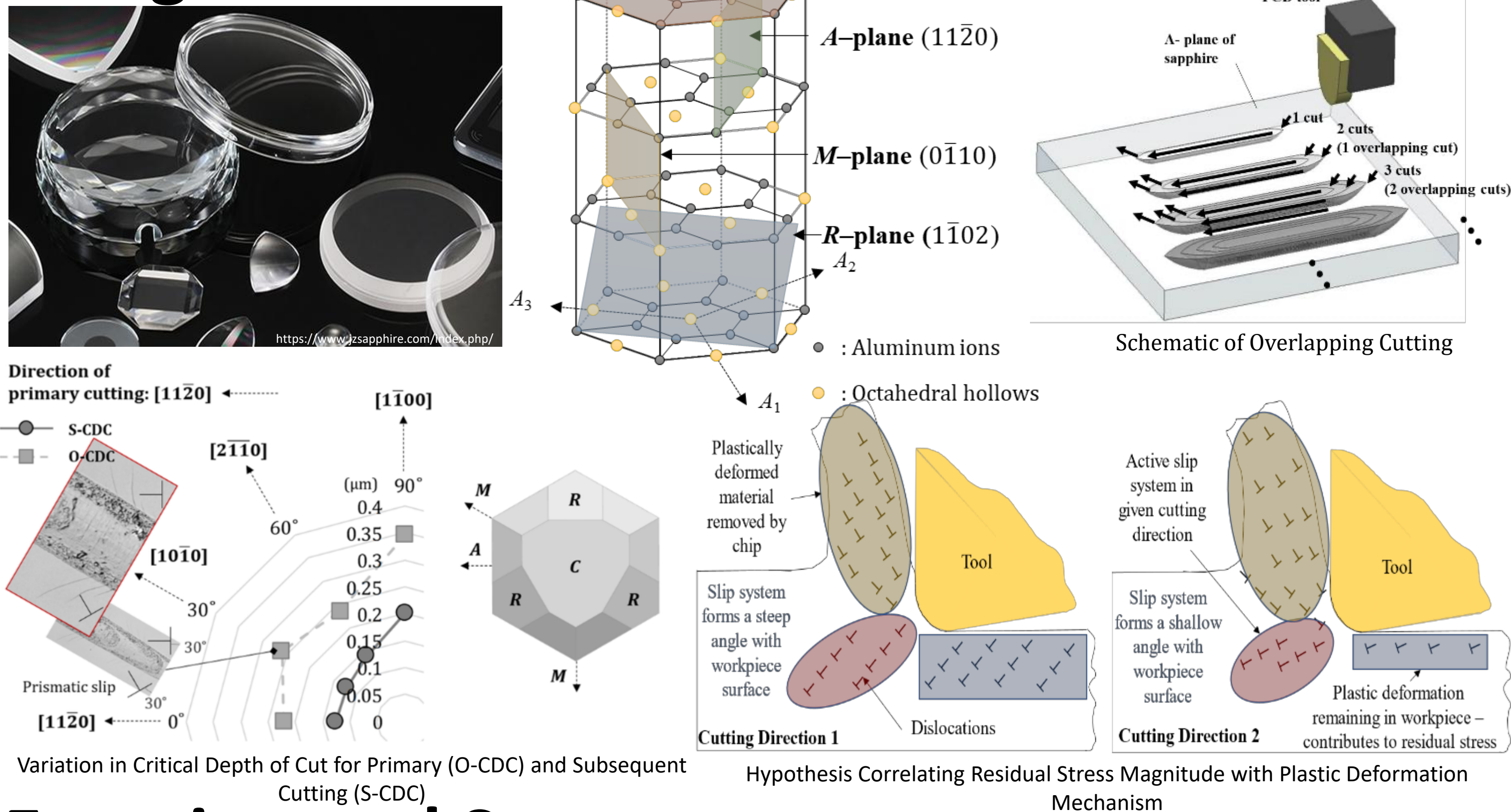
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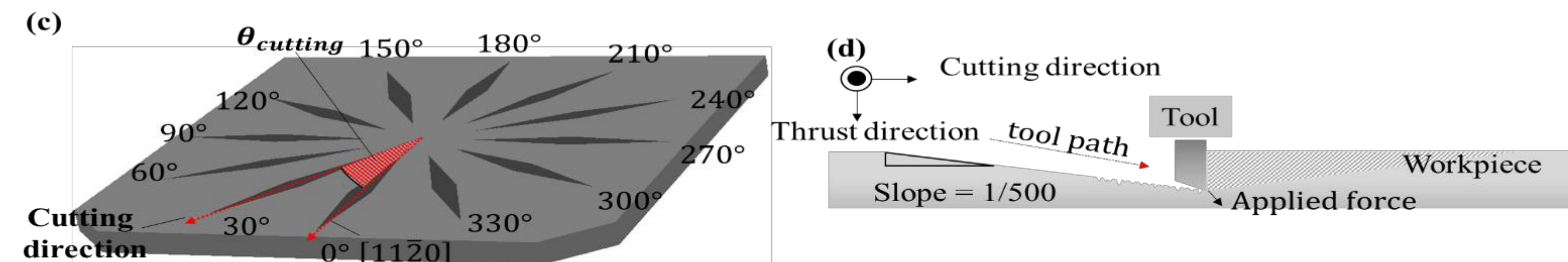
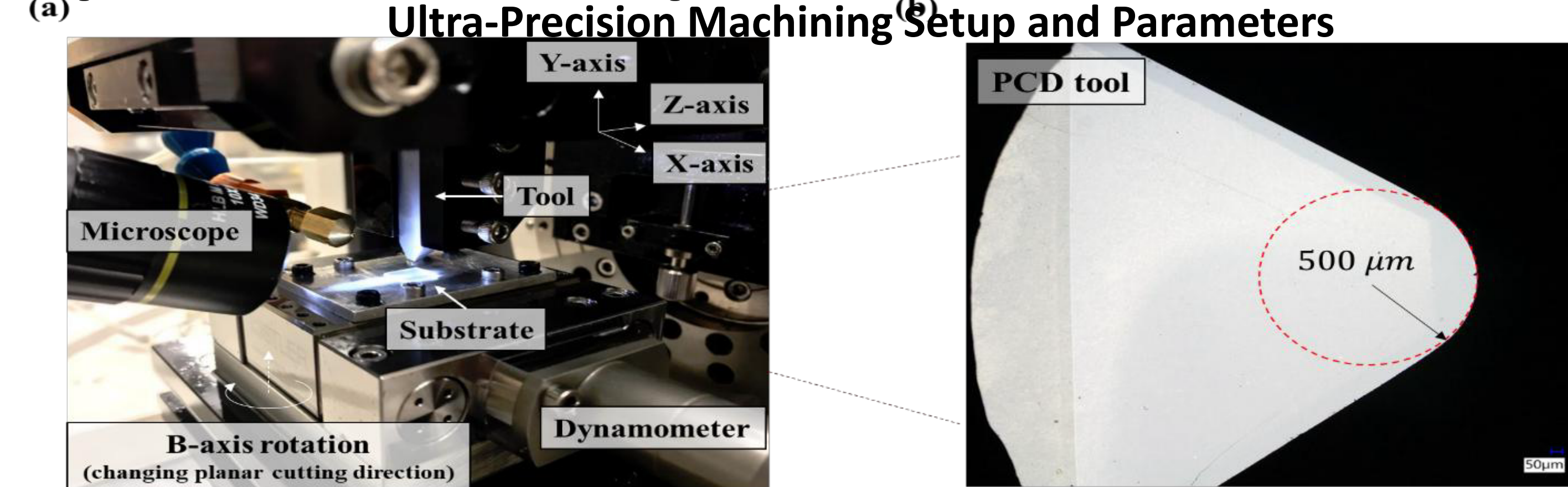
## Project Objectives and Goals

- Address machining challenges by investigating the effects of sapphire's high hardness, brittleness, and crystallographic anisotropy on material removal
- Quantify residual stress and that contribute to crack initiation and degradation of machining performance
- Utilize Raman spectroscopy to evaluate residual stress providing insights for process modeling and optimization
- Employ X-ray nanobeam diffraction microscopy, enabling high-resolution correlation between deformation mechanisms and stress evolution

## Background



## Experimental Setup



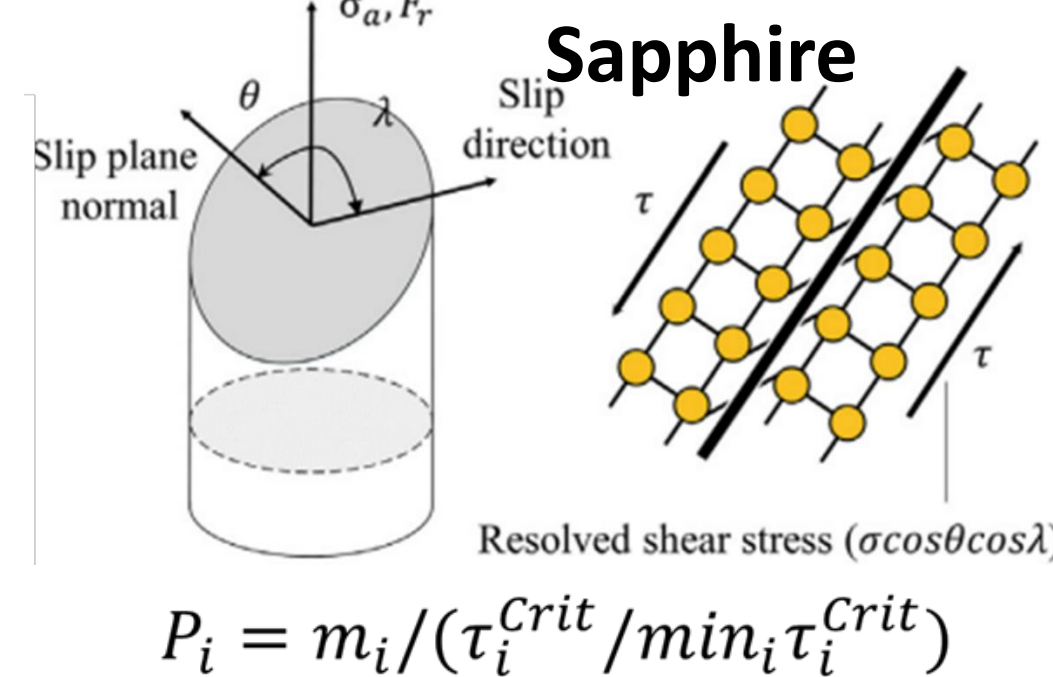
Parameter	Description
Machine Tool	ROBONANO α-0/B, FANUC Corp., Japan
Cutting Tool	Binderless Nano-PCD Tool, 500 μm nose radius, 0° rake angle (A.L.M.T. Corp., Japan)
Substrate	Sapphire, Czochralski growth; 3 mm x 5 mm x 0.5 mm (MTI Corp., U.S.A)
Dynamometer	Type 9119AA1 (Kistler Instrument Corp., Switzerland)
Cutting oil	Mineral oil based (JX Nippon Oil & Energy, Japan)
Feed rate	5 mm/min

## Data and Results—Raman Spectroscopy

### Raman Spectroscopy Parameter

Parameter	Description
Raman Instrument and Model	Horiba LabRAM HR Evolution
Laser Wavelength	633 nm
Grating Density	1800 grooves/mm
Spectral Acquisition Time	10 s/position, 2 scans per spot
Raman map spacing	~10 μm intervals

### Modelling Plastic Deformation In Sapphire



$$m_i = \cos \theta_i \cos \lambda_i$$

$P_i$ : Plastic deformation parameter

$m_i$ : Schmid factor

$\tau_i^{crit}$ : Critical resolved shear stress

$\theta_i$ : The angle between the direction of applied stress and normal to the slip plane

$\lambda_i$ : The angle between direction of applied stress and slip direction

### Residual Stress Tensor Calculation

Based on the relationship between peak shift ( $\Delta \nu$ ) and stress tensor ( $\sigma_{ij}$ )

$$\Delta \nu = \Pi_{ij} \sigma_{ij} = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} \\ \Pi_{21} & \Pi_{22} & \Pi_{23} \\ \Pi_{31} & \Pi_{32} & \Pi_{33} \end{bmatrix} \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \quad (1)$$

$\Pi_{ij}$  is the piezo spectroscopic coefficient

Accounting for the crystal orientation in the lab frame

$$\begin{bmatrix} \sigma'_{x'x'} & \sigma'_{x'y'} & \sigma'_{x'z'} \\ \sigma'_{y'x'} & \sigma'_{y'y'} & \sigma'_{y'z'} \\ \sigma'_{z'x'} & \sigma'_{z'y'} & \sigma'_{z'z'} \end{bmatrix} = \Phi_{xyz} \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \tilde{\Phi}_{xyz} \quad (2)$$

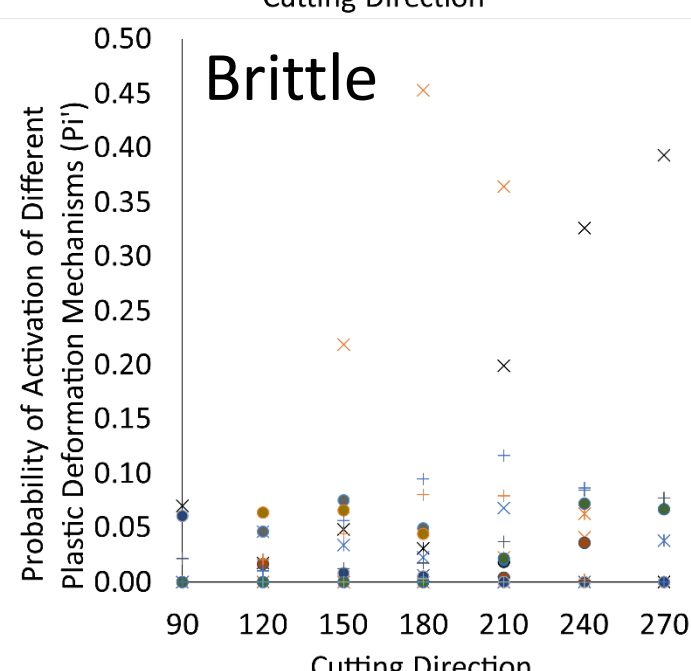
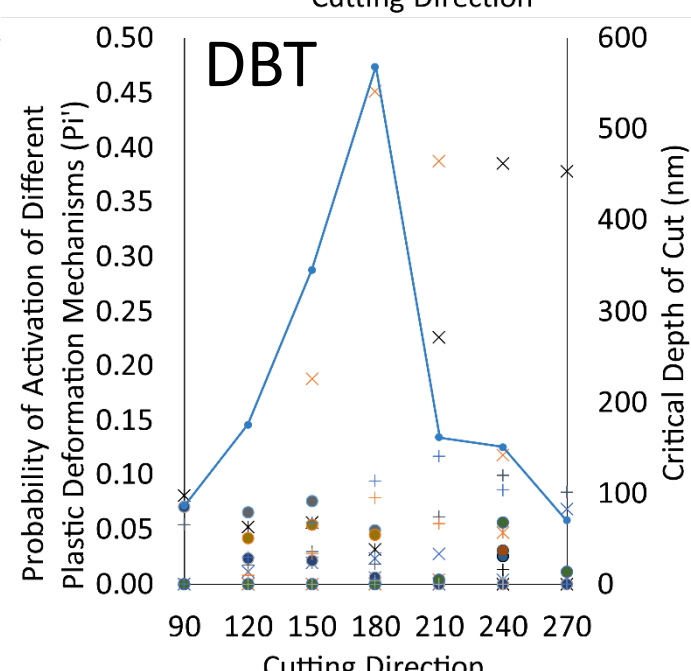
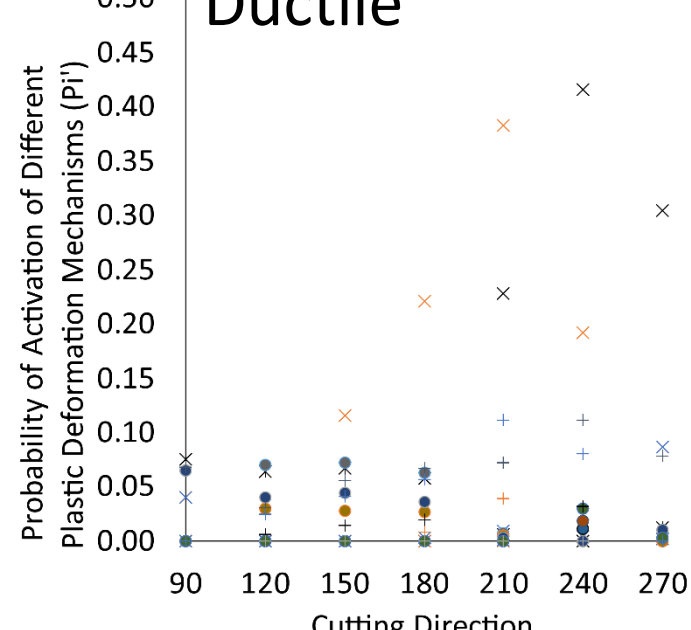
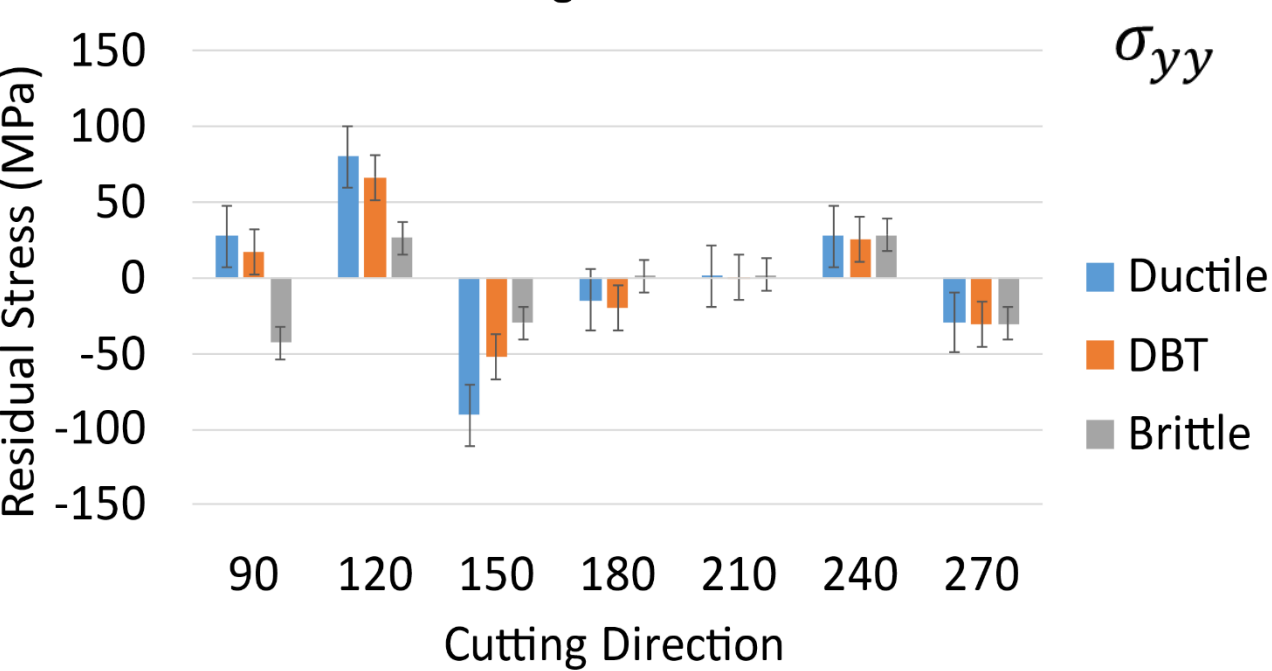
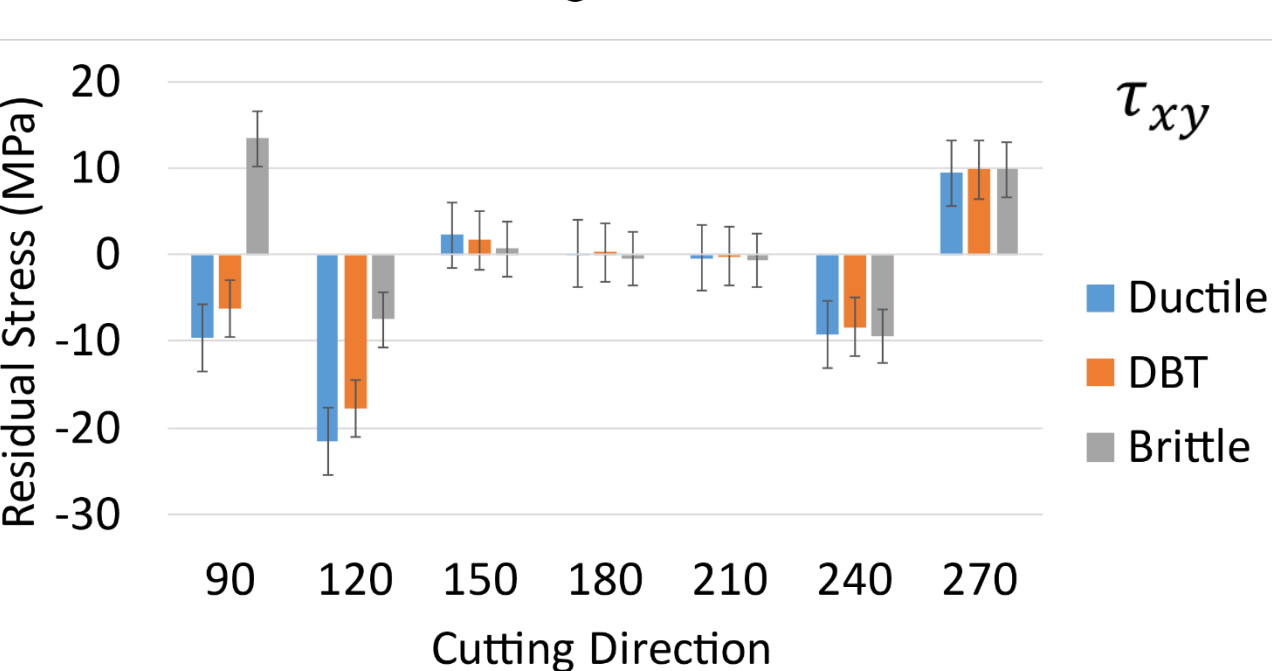
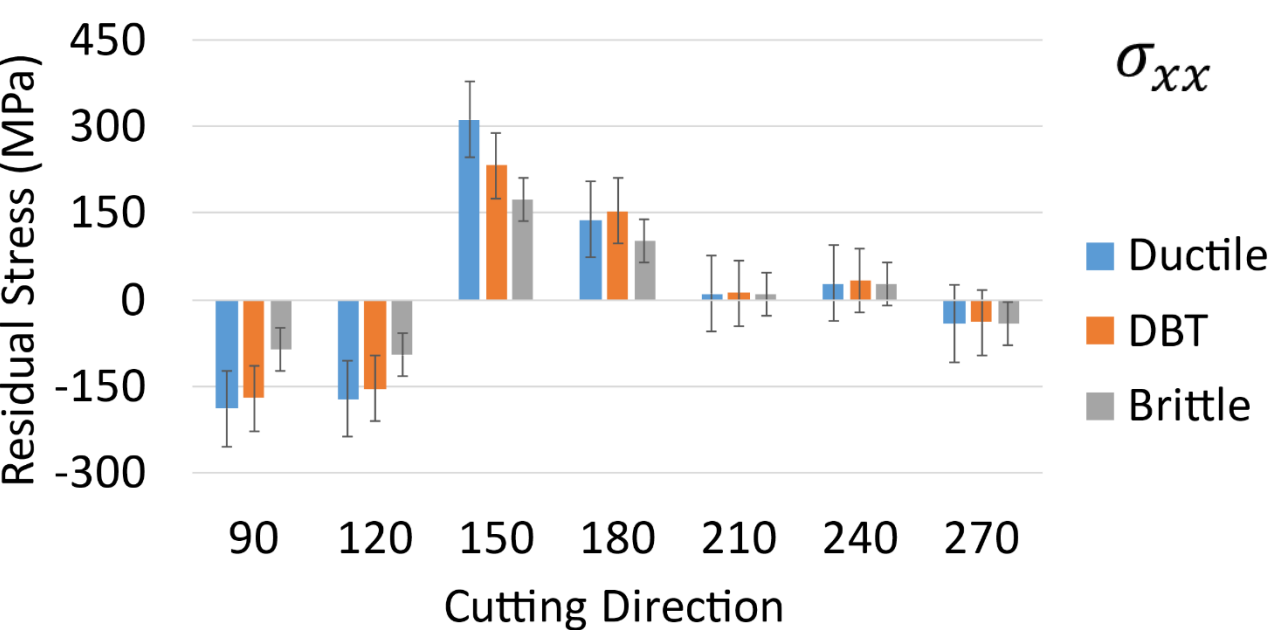
$\Phi_{xyz}$  is rotation matrix and  $\tilde{\Phi}_{xyz}$  is the transpose of the rotation matrix.

Considering the crystal symmetries of sapphire, Eq. 1 reduces to

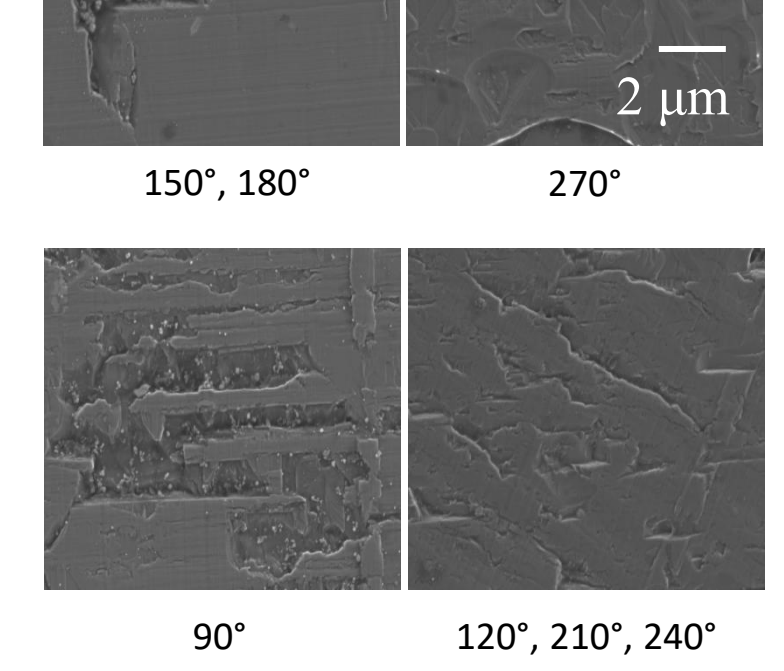
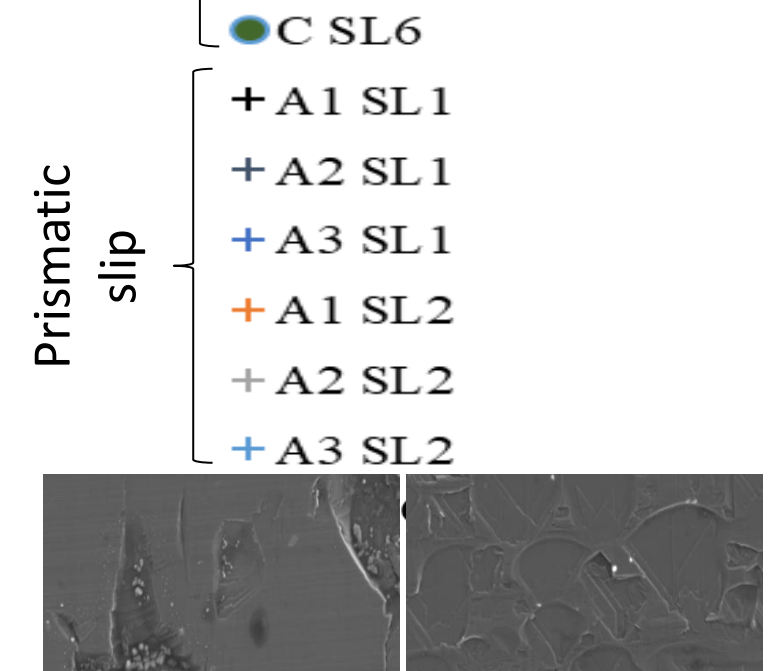
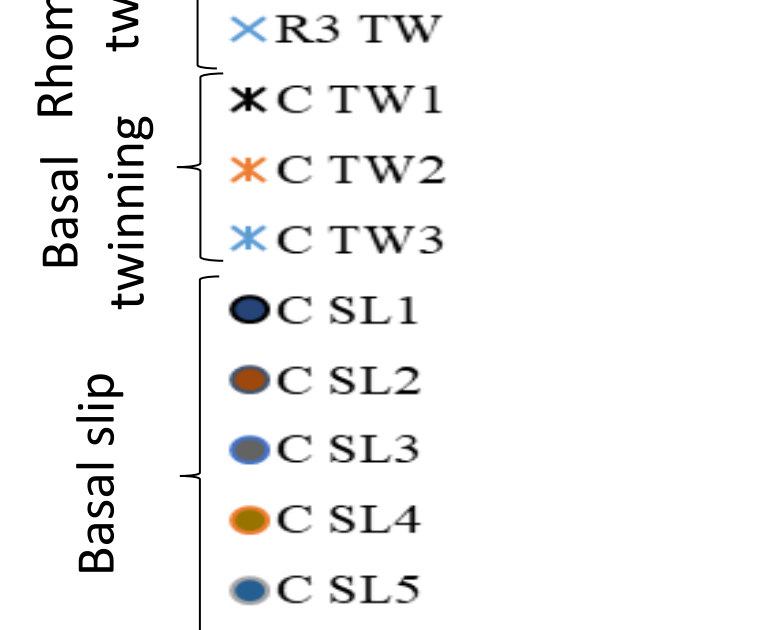
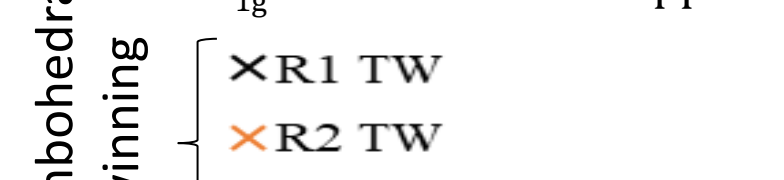
$$\Delta \nu = \begin{bmatrix} \Pi_a^{(n)} & 0 & 0 \\ 0 & \Pi_b^{(n)} & 0 \\ 0 & 0 & \Pi_c^{(n)} \end{bmatrix} \Phi_{xyz} \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \tilde{\Phi}_{xyz} \quad (3)$$

Reducing to plane strain condition and simplifying Eq. 3,

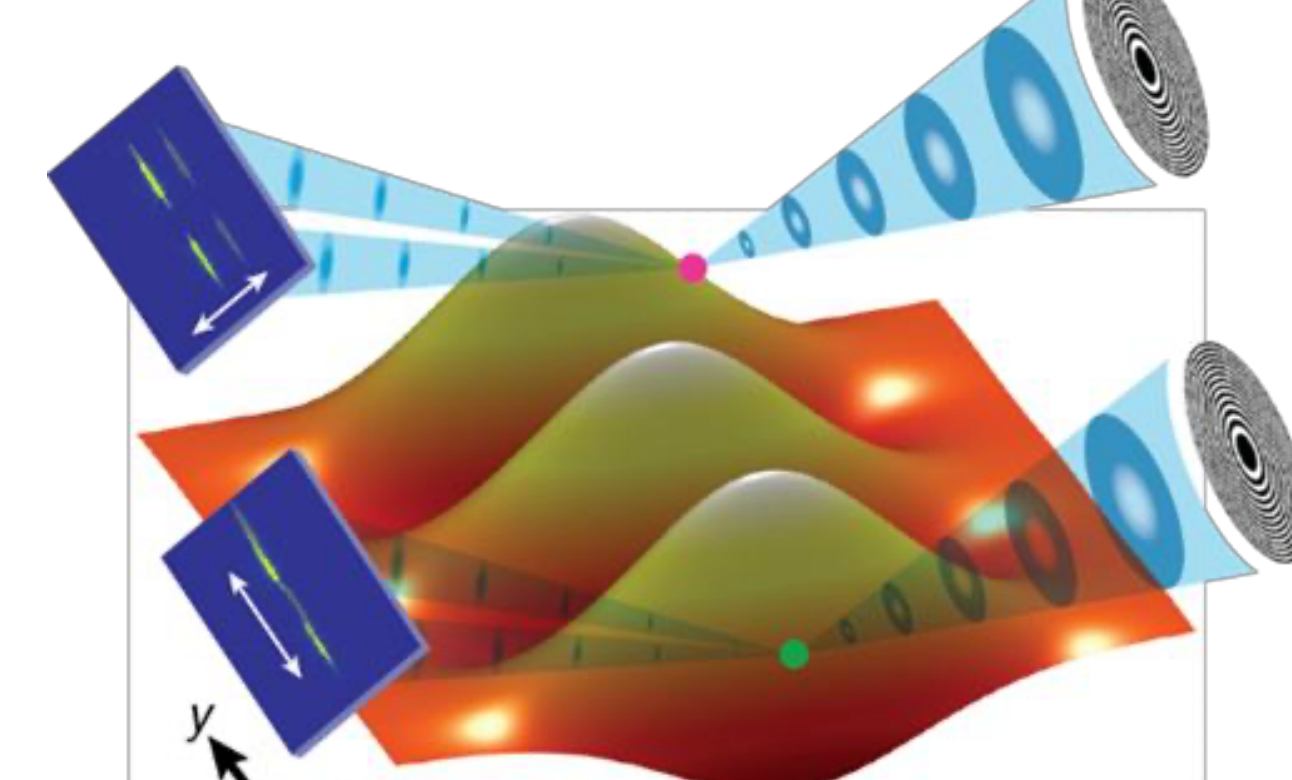
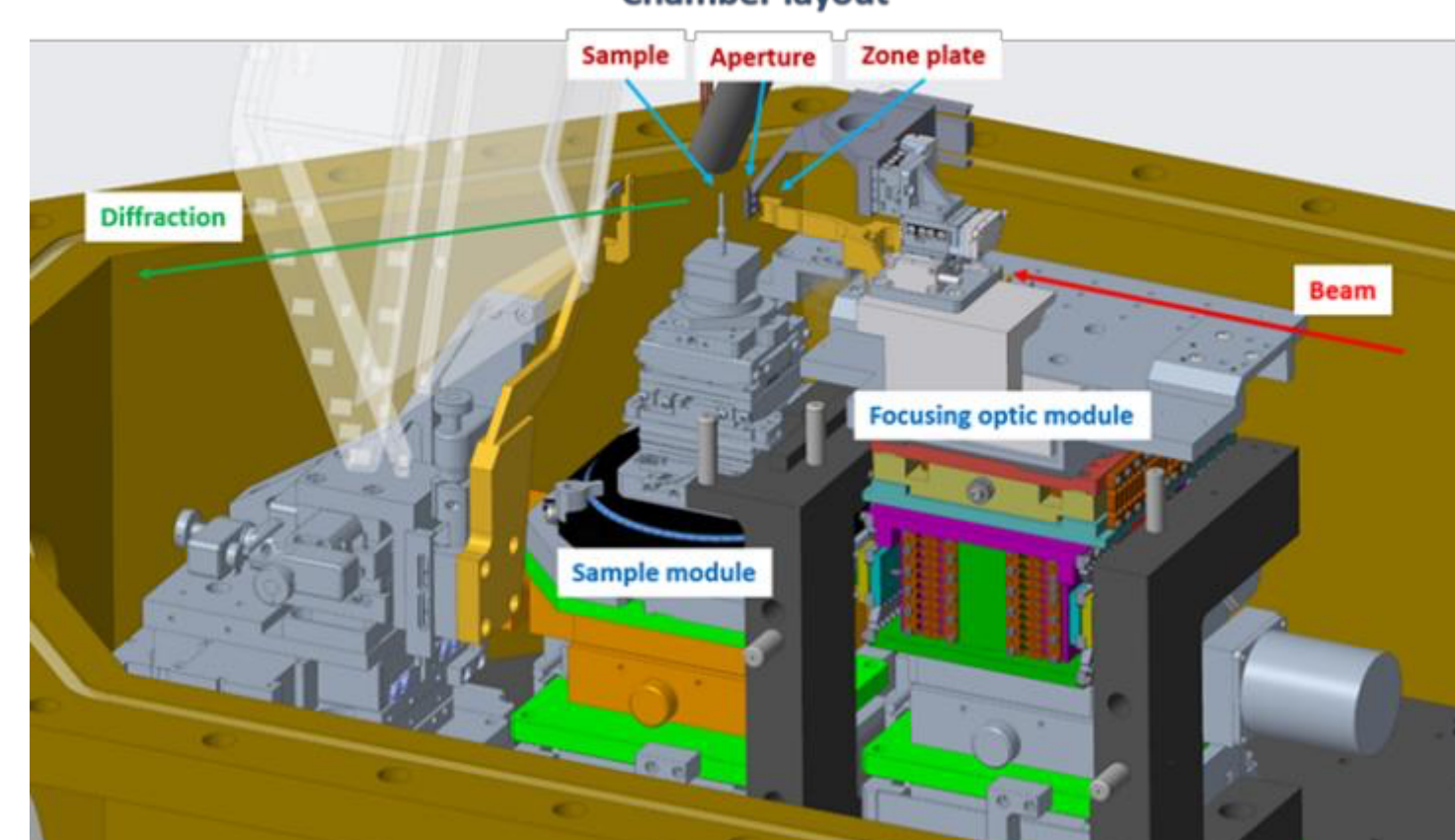
$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{xy} \\ \sigma_{yy} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}^{-1} \begin{pmatrix} \Delta \nu_1 \\ \Delta \nu_2 \\ \Delta \nu_3 \end{pmatrix} \quad (4)$$



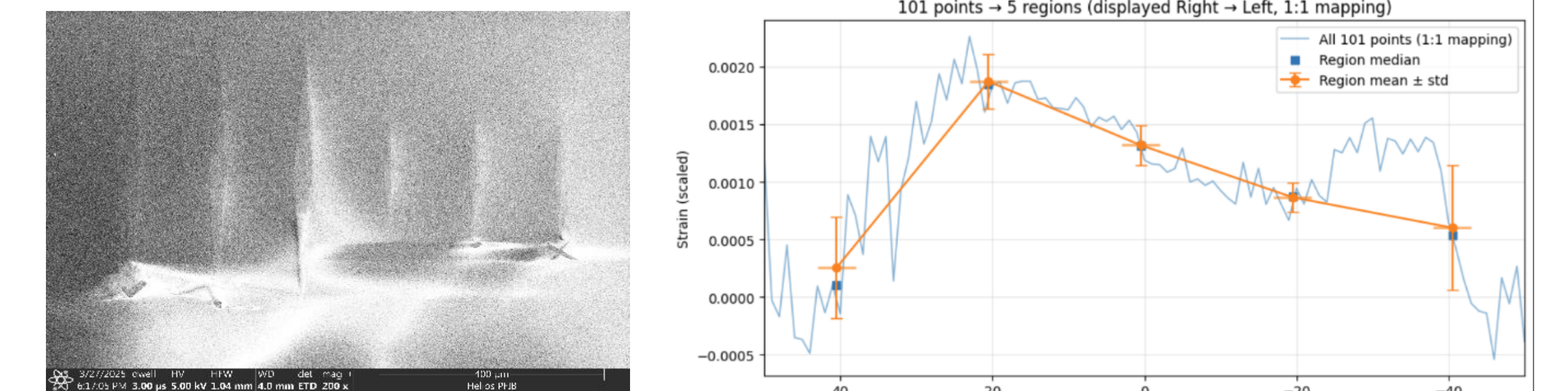
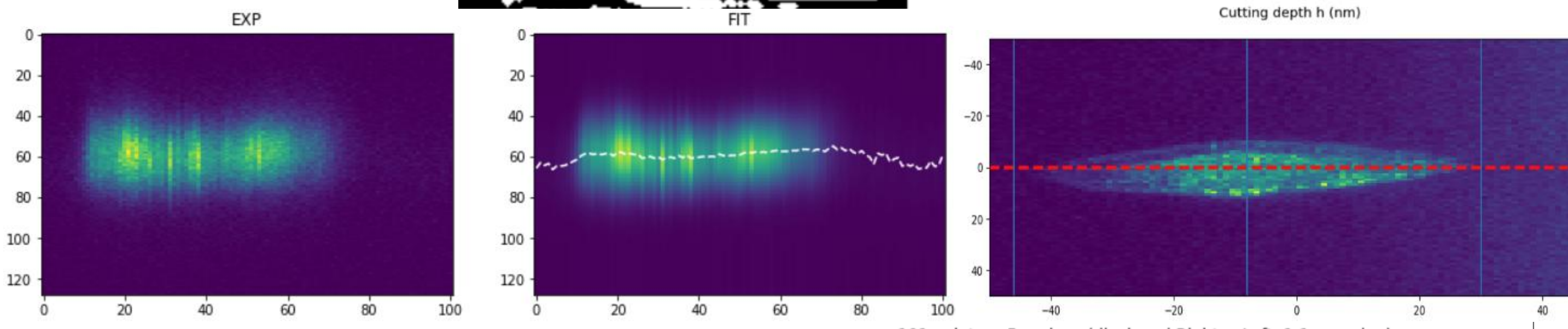
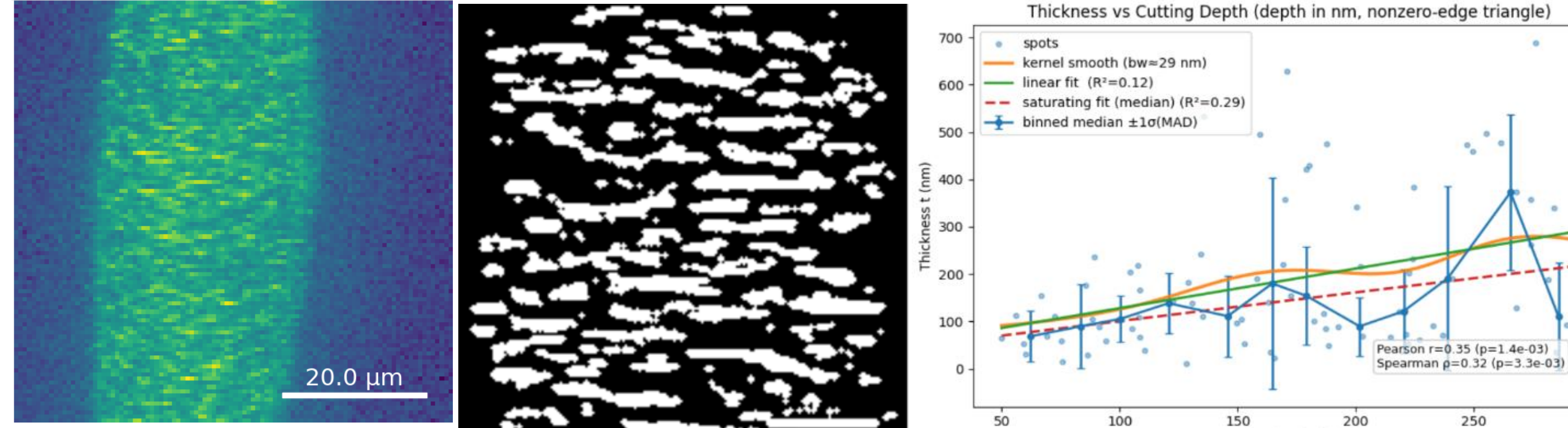
### A<sub>1g</sub> Phonon Peak in Sapphire



## Data and Results—X-ray Nanobeam



### Domains of ROIs



Within the ductile zone around the scratch interior 270° curves are larger, the strains decrease after the ductile region and DBT region.

## Conclusion and future work:

- Expand measurements to more crystallographic orientations to study stress–deformation coupling
- Develop predictive models linking machining parameters, crystallography, and stress evolution
- Optimize machining strategies for improved surface integrity and reduced subsurface damage

## References and Publications:

- [1] Zhu, W., & Pezzotti, G. (2011). Raman analysis of three-dimensionally graded stress tensor components in sapphire. Journal of Applied Physics, 109(7).
- [2] S. J. Whiteley, F.J. Heremans, et.al, (2019). Imaging dynamically-driven strain at the nanometer-scale using stroboscopic scanning X-ray diffraction microscopy. Nature Physics, 10:3386.
- [3] Nagaraj, A., & Min, S. (2022). Effect of Crystallography on residual stresses during Ultra-precision machining of sapphire. CIRP Annals, 71(1).

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