A Hammerstein Approach for Compact Model of Nonlinear Circuits with Arbitrary Terminations

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***Abstract*—We formulate a behavioral model order reduction approach for nonlinear circuits based on the canonical Hammerstein architecture. The work extends the ideas of [1] to arbitrary loads and improves the efficiency of the model inference by training the linear time invariant (LTI) block in the frequency domain. We demonstrate the approach with an LM741 operational amplifier and present simulation results showing its application over a range of input frequencies and loads.**

I. Introduction

Transistor-level SPICE simulations solve systems of differential algebraic equations (DAEs) assembled from compact device models via modified nodal analysis [2]. The size of these DAEs is proportional to the number of devices in the circuit, which dramatically increases simulation costs for large circuits. Macromodeling [3] attempts to reduce the computational burden of SPICE simulations by using simplification and build-up to replace the original circuit by a smaller circuit with approximately the same functionality. However, macromodeling is a manual effort that relies on intuition and requires significant subject matter expertise. More “automated” model order reduction (MOR) techniques such as PRIMA [4] and NORM [5] use moment matching, rational approximation, and Volterra series to obtain reduced order models (ROMs) of the circuit DAEs. These approaches are effective for linear and weakly nonlinear systems, but their application to general nonlinear circuits is challenging. In addition, macromodeling and the above MOR require access to full-order models (FOM) of the circuit, which may be unavailable for proprietary or new technologies that lack adequate modeling support. Trajectory piecewise linear and piecewise polynomial models offer the ability to capture nonlinear behavior by connecting linearized or higher order polynomial models at given trajectory points but also require access to the circuit’s FOM [6]. Harmonic balance [7] is another popular approach to speed up simulation of nonlinear analog circuits but its scope is limited to circuits operating in a periodic, or quasi-periodic, steady-state regimes. Finally, methods such as nonlinear autoregressive network with exogenous inputs (NARX) are typically trained in the time domain [8], whereas the effects of circuits such as amplifiers are often functions of frequency.

$$\mathbf{u}[n] \dashrightarrow \boxed{f(\mathbf{u})} \xrightarrow{\mathbf{w}[n]} \boxed{\mathbf{h}[n] * \mathbf{w}[n]} \dashrightarrow \mathbf{y}[n]$$

Figure 1. Hammerstein model nomenclature.

In this paper we develop a non-intrusive behavioral MOR that does not require access to a circuit FOM and learns compact circuit models solely from input-output data. Our approach targets general nonlinear circuits that operate primarily in memoryless capacity at low to moderate frequencies, such as operational and differential amplifiers, comparators, and voltage regulators. The behavior of these circuits can be modeled accurately at such frequencies by a static nonlinearity that regresses the input-output voltage characteristics of the circuit. Phase shift, degradation and distortion due to parasitic capacitance at high frequencies are then “corrected” by connecting a dynamic LTI block in series with the static nonlinear DC block. Thus, from a behavioral modeling perspective, the operation of these circuits can be mapped to the canonical Hammerstein architecture in Figure 1, which forms the basis of our approach.

Our contribution builds on the ideas of [1] and extends them in two important ways. First, we adopt the Hammerstein model sequential inference procedure in [1] but train the LTI block in the frequency domain. In so doing we reduce the amount of training data needed by approximately three orders of magnitude from the 5×10^5 data points required in [1] to just 95 frequency points. Second, we generalize the LTI models to operate across a range of loads by sampling the load range and interpolating the LTI blocks at the desired load, while simultaneously ensuring the interpolated LTI is stable.

II. Behavioral MOR approach

To demonstrate our approach we use an LM741 operational amplifier [9] in a feedback configuration. We first develop its Hammerstein model for a specific load and then extend it to arbitrary loads by interpolating the LTI blocks.

To analyze the behavior of LM741, it is common practice to first find the DC operating point and then linearize the MOSFET model at that point to find the effects of small AC signals. The gain of the amplifier is a function of frequency due to parasitic capacitances from the transistors. This analysis prompts the selection of the Hammerstein model architecture, which comprises a static nonlinear block in series with a dynamic linear time invariant block. The former is a nonlinear function f of the input $\mathbf{u}[n]$, whereas the latter is a transfer function $\mathbf{h}[n]$; see Figure 1.

The static nonlinear DC block is charged with finding the ideal DC gain of the LM741 and the linear block will capture the degradation of the gain and the phase shift at high frequencies shown in Figure 2. Both blocks will be inferred solely from input-output data without assuming access to the circuit's FOM. This is a key advantage of our approach compared to the methods in, e.g., [4], [5] and [6] that explicitly exploit the FOM structure and require access to its equations.

To obtain our models we will follow the sequential identification procedure in [1]. Such sequential approach greatly increase model's flexibility and modularity, and simplifies the identification process. However, in contrast to [1] here we shall use a residual neural network (ResNet) for the DC block and train the LTI block in the frequency domain.

In addition to the effects in Figure 2, the LM741 exhibits another subtle frequency-dependent behavior comprising a slight shift in the DC offset at high frequencies [10].

To model this shift, we write the output of the DC block as $\mathbf{w}[n] = \mathbf{w}_{AC}[n] + \mathbf{w}_{DC}[n]$, where $\mathbf{w}_{AC}[n] = \mathbf{w}[n] - \overline{\mathbf{w}[n]}$ and $\mathbf{w}_{DC}[n] = \overline{\mathbf{w}[n]}$, where $\overline{\mathbf{w}[n]}$ is the time-average of $\mathbf{w}[n]$. We similarly write $\mathbf{y}[n] = \mathbf{y}_{AC}[n] + \mathbf{y}_{DC}[n]$. Thus, $\mathbf{y}_{AC}[n]$ captures the gain degradation and phase shift that results from AC portion of the input, $\mathbf{w}_{AC}[n]$, while $\mathbf{y}_{DC}[n]$ captures the DC offset effect from the DC portion of the input, $\mathbf{w}_{DC}[n]$. Thus, the output $\mathbf{y}[n]$ of the linear block in our model is the superposition of the outputs $\mathbf{y}_{AC}[n]$ and $\mathbf{y}_{DC}[n]$ of two independently trained transfer functions.

III. LTI BLOCK

Training data for the model was obtained by simulating LM741 in Xyce [11] with inputs given by sine waves with frequencies sampling the interval $[1, 1e7]$ Hz. The output was sampled at a time-step of 10 ns, and every input was simulated for either one complete cycle or 200 time steps, whichever was greater.

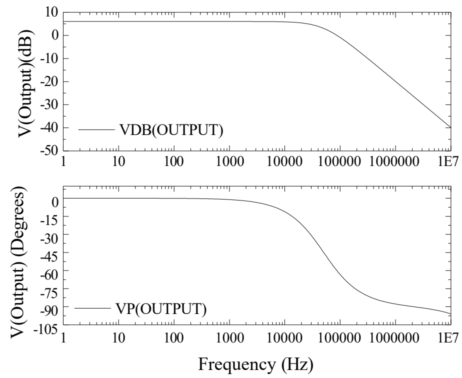

Figure 2. Magnitude response (top) and phase response (bottom) of the LM741 with a 5 pF capacitor in parallel with a 100Ω at the load.

Training data for the LTI block was obtained by combining the output $w[n]$ of the nonlinear block with the Xyce output $y[n]$. Both signals were further split into "AC" and "DC" components and converted to frequency domain using the Fast Fourier Transform (FFT), resulting in a total of 95 data points. In contrast, inference of the LTI block of the model in [1] in the time domain required time domain data with 5×10^5 points.

We denote the frequency domain data using capital letters, e.g., $Y_{AC}(f_i)$ is the complex-valued frequency response from an input at frequency f_i obtained by taking the FFT of $y_{AC}[n]$. We arrange this data into a vector of training frequencies

$$\mathbf{F} = \begin{bmatrix} f_1 f_2 \ldots f_n \end{bmatrix}^T \quad (1)$$

and vectors

$$\mathbf{Y}_\kappa = \begin{bmatrix} Y_\kappa(f_1) \\ Y_\kappa(f_2) \\ \vdots \\ Y_\kappa(f_n) \end{bmatrix}; \quad \mathbf{W}_\kappa = \begin{bmatrix} W_\kappa(f_1) \\ W_\kappa(f_2) \\ \vdots \\ W_\kappa(f_n) \end{bmatrix}.$$

where $\kappa = \{AC, DC\}$ We the use MATLAB's function TFEST with each pair of vectors above to estimate two transfer functionshaving the form

$$\frac{Y_\kappa(z)}{W_\kappa(z)} = \frac{a_\kappa}{1 + b_{1,\kappa} z^{-1} + b_{2,\kappa} z^{-2}}; \; \kappa = \{AC, DC\}. \quad (2)$$

Figure 3 summarizes the training process for (2).

Conversion of the AC and DC offset models back to the time domain is given by

$$y_\kappa[n] = a_\kappa w[n] - b_{1,\kappa} y[n-1] - b_{2,\kappa} y[n-2], \quad (3)$$

where the output from the LTI block is computed as $y[n] = y_{AC}[n] + y_{DC}[n]$. Each signal $y_\kappa[n]$ can be built recursively from the previous 2 time steps.

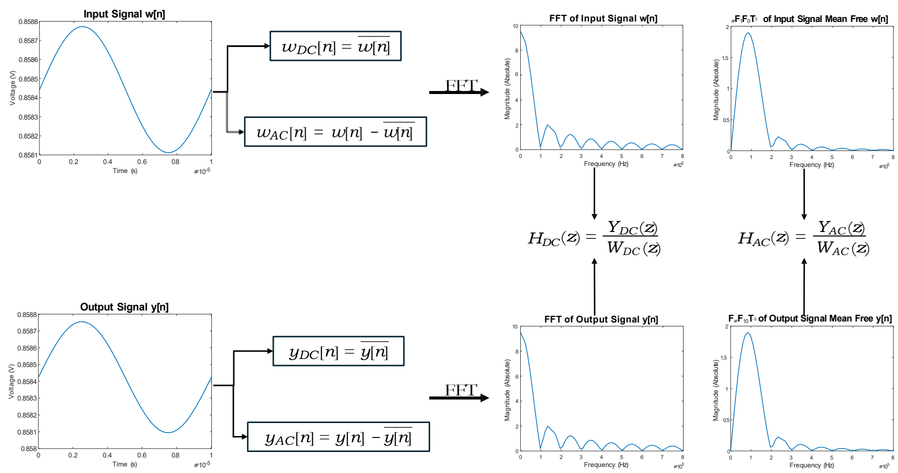

Figure 3. Flow chart of the training process for AC and DC offset transfer functions.

IV. LM741 Hammerstein Model with Variable Loads

We consider loads defined by a resistor R in parallel with a capacitor C. Our goal is to define Hammerstein models that can accurately predict the response of LM741 for any arbitrary load $(R, C) \in \mathsf{D} = [R_{\min}, R_{\max}] \times [C_{\min}, C_{\max}]$.

Let $\mathsf{S}(N_R, N_C) = (R_i, C_j) \in \mathsf{D}$, $i = 1, \ldots, N_R$, $j = 1, \ldots, N_C$ denote a sampling of D. For every pair (R_i, C_j) we estimate a transfer function

$$H^{ij}(z) = \frac{a^{ij}}{1 + b_1^{ij} z^{-1} + b_2^{ij} z^{-2}} \tag{4}$$

as in Section III. Given a load $(R^*, C^*) \in \mathsf{D}$ that is not included in $\mathsf{S}(N_R, N_C)$ we define a transfer function $H(R^*, C^*)$ using that all H^{ij} have the same functional form (4) and interpolating their coefficients. Assume for simplicity that D is sampled on a Cartesian grid and let (R_k, C_l), $k, l = 1, 2$ denote the four vertices of the mesh cell containing (R^*, C^*). Let a^{kl}, b_1^{kl} and b_2^{kl} denote the coefficients of (4) at these vertices. To obtain the values of these coefficients at the target load we use the bilinear interpolant of the vertex coefficients given by

$$\alpha(R^*, C^*) = \sum_{k,l=1}^{2} \psi_{kl}(R^*, C^*) \alpha^{kl} \tag{5}$$

where ψ_{kl} is the bilinear basis function for vertex kl and $\alpha = a, b_1, b_2$. The coefficients in (5) provide the initial guesses for the desired transfer function. To ensure pole stability $b_1(R^*, C^*)$ and $b_2(R^*, C^*)$ have conditions on their interpolation that are coupled together. We can uncouple the conditions by rewriting the denominator as

$$1 + b_1 z^{-1} + b_2 z^{-2} = (1 - p_1 z^{-1})(1 - p_2 z^{-1}) \tag{6}$$

where

$$\begin{cases} p_1 < p_2 & \text{if } p_1, p_2 \in \mathbb{R} \\ \angle p_1 < \angle p_2 & \text{if } p_1 = p_2^* \end{cases} \tag{7}$$

Since b_1 and b_2 are real, the poles must be complex conjugates if they are not purely real. Now, when we perform the interpolation, the constraint on the interpolation of the poles' magnitudes is that their range is $[0, 1]$, while the phases of the poles are unconstrained. Although there are now 4 interpolations to be done for the denominator since the poles are complex, the interpolations are uncoupled, as opposed to leaving the denominator in the form $1 + b_1 z^{-1} + b_2 z^{-2}$, which requires two interpolations that are coupled together. Using the methodology described in Section III, we build four complete Hammerstein models with loads $(100\Omega, 5pF)$, $(100\Omega, 7pF)$, $(200\Omega, 5pF)$, and $(200\Omega, 7pF)$. We then test the interpolation of coefficients on a load of $(150\Omega, 6pF)$. We also compare the prediction to the DC model's prediction to highlight the critical corrections that the LTI model makes with regards to gain degradation and phase shifting at high frequencies. We report model fit as a percentage of the R^2 metric, i.e., fit $= 100\,(1 - \|y - \hat{y}\| / \|y - \bar{y}\|)$ [%], where y is the Xyce output, $\bar{y}$ is the mean of y, and $\hat{y}$ is the model's prediction. The results of the interpolation are shown in Figure 4.

V. Conclusion and Future Work

We demonstrated an accurate and effective reduced-order model for LM741 circuits based on Hammerstein architecture that can be developed using only input-output data collected at the external ports, without *explicit* and/or *implicit* knowledge of the circuit's FOM. Compared to the time-domain inference of the LTI block in [1], switching to a frequency domain identification of this block reduced

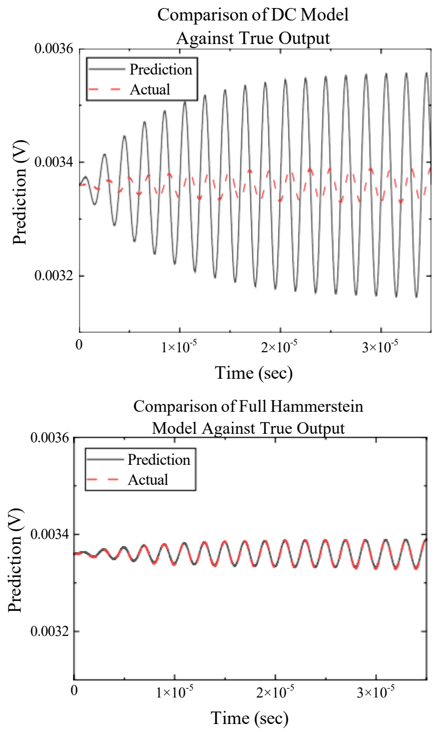

Figure 4. Comparison of the Xyce output with the DC model's prediction (top). The fit of the DC model alone is −544%. Comparison of the Xyce output with a Hammerstein model generated via interpolation (bottom). The fit of the model is 91.23%.

the training data burden by approximately three orders of magnitude, while maintaining the ability of the model to accurately predict the output signals in the time domain. Our novel parametric approach successfully extends the model to multiple loads, providing the flexibility to continue to generalize as the number of parameters increases. While our current model is SISO, the underlying Hammerstein architecture naturally extends to MIMO configurations that can be used to model multi-port behaviors, as long as the underlying circuit operates in a primarily memoryless capacity. Future extensions of this work will consider such configurations, as well as incorporating current into the model to enhance its flexibility and generalizability to arbitrary loads, allowing for effective simulation of a wider range of circuit configurations.

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