Bayesian Complex Amplitude Estimation and Adaptive Matched Filter Detection in Low-Rank Interference†

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Abstract

We propose a Bayesian method for complex amplitude estimation in low-rank interference. We assume that the received signal follows the generalized multivariate analysis of variance (GMANOVA) patterned-mean structure and is corrupted by low-rank spatially correlated interference and white noise. An iterated conditional modes (ICM) algorithm is developed for estimating the unknown complex signal amplitudes and interference and noise parameters. We also discuss initialization of the ICM algorithm and propose a (non-Bayesian) adaptive-matched-filter (AMF) signal detector that utilizes the ICM estimation results. Numerical simulations demonstrate the performance of the proposed methods.

I. Introduction

The topic of signal detection and estimation in low-rank interference has recently attracted considerable attention (see e.g. [1]–[6] and references therein) as it has great potential in many signal-processing applications facing the curse-of-dimensionality problem and data (or ”snapshot”) constraints [4], [5]. In space-time adaptive processing (STAP) for radar, low-rank interference is due to clutter and jamming, see e.g. [6]–[8]. In [3] and [9], maximum likelihood (ML) and least-squares estimators of low-rank covariance matrices are derived assuming that secondary (interference-plus-noise only) data is available. Structured ML covariance estimation from interference-only measurements is discussed in [10]. Intrinsic Cramér-Rao bounds (CRBs) for low-rank subspace estimation are developed in [1] and low-rank subspace tracking is discussed in [11]. Bayesian and non-Bayesian approaches for complex amplitude estimation have been developed and analyzed in [12]–[13] and [14]–[17] (see also references therein) assuming unstructured covariance matrix of interference and noise. A Bayesian approach for estimating interference-plus-noise covariance matrices in knowledge-aided radar is outlined in [18], where numerous examples of available *a priori* information are given for the radar problem. The methods in [12]–[18] *ignore* the low-rank structure of the interference. In this paper, we develop an iterated conditional modes (ICM) algorithm for Bayesian estimation of complex signal amplitudes in low-rank interference and propose a (non-Bayesian) adaptive-matched-filter (AMF) detector that utilizes the ICM estimates of the unknown parameters. Computational complexity of our iterative scheme *does not* depend on the array size; rather, it is determined by the ranks of signal and interference.

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Measurement model and prior specifications are introduced in Section II, where we also justify use of the Bayesian methodology. In Section III, we develop the ICM algorithm for Bayesian complex amplitude estimation, discuss its initialization and interference rank estimation (Section III-A), and derive an AMF signal detector (Section III-B). In Section IV, we evaluate the performance of the proposed methods via numerical simulations. Concluding remarks are given in Section V.

II. Measurement Model and Prior Specifications

Denote by $\boldsymbol{y}(t)$ an $m \times 1$ complex data vector (snapshot) received at time t and assume that we have collected N snapshots. Consider the following model for the received snapshots:

$$\boldsymbol{y}(t) = AX\boldsymbol{\phi}(t) + B\boldsymbol{h}(t) + \boldsymbol{e}(t), \quad t = 1, \ldots, N \tag{2.1}$$

where the first term describes the generalized multivariate analysis of variance (GMANOVA) patterned mean-signal structure with a known $m \times q$ signal array-response matrix A ($m \geq q$), known $d \times 1$ signal temporal response vectors $\boldsymbol{\phi}(t)$, $t = 1, 2, \ldots, N$ ($d \leq N$), and an $q \times d$ matrix X of *unknown* complex signal amplitude coefficients. (See [14] and references therein for a detailed exposition on the GMANOVA model and its applications.) The second term in (2.1) corresponds to low-rank interference described by

- an unknown $m \times r$ interference array-response matrix B ($m \geq r$) and
- random zero-mean independent, identically distributed (i.i.d.) $r \times 1$ vectors $\boldsymbol{h}(t)$, $t = 1, 2, \ldots, N$ of interference signals following a circularly symmetric complex Gaussian distribution with unknown covariance $\sigma^2 \Sigma$. (Here, Σ is a *normalized covariance matrix* of the interference signals.)

Finally, $\boldsymbol{e}(t)$ is zero-mean circularly symmetric i.i.d. complex Gaussian noise with covariance $\sigma^2 I_m$, where σ^2 is unknown noise variance and I_m denotes the identity matrix of size m. Denote by $\boldsymbol{\xi}$ the set of all unknown parameters:

$$\boldsymbol{\xi} = \{X, B, \Sigma, \sigma^2\} \tag{2.2}$$

where Σ is a positive-semidefinite Hermitian matrix. We allow the interference rank r to be *unknown* as well, estimated in the initialization stage (i.e. separately from the estimation of $\boldsymbol{\xi}$), see Section III-A.

We define the data, temporal signal response, and interference-signal matrices:

$$Y = [\boldsymbol{y}(1) \cdots \boldsymbol{y}(N)], \quad \Phi = [\boldsymbol{\phi}(1), \boldsymbol{\phi}(2) \cdots \boldsymbol{\phi}(N)], \quad H = [\boldsymbol{h}(1), \boldsymbol{h}(2) \cdots \boldsymbol{h}(N)] \tag{2.3}$$

and assume that Φ has full rank equal to d. Before we proceed, let us introduce the following definition: a matrix-variate circularly symmetric complex Gaussian probability density function (pdf) of an $p \times q$ random matrix Z with mean M (of size $p \times q$) and positive-definite covariance matrices $\mathcal{S}$ and Φ (of dimensions $p \times p$ and $q \times q$, respectively) is

$$\mathcal{N}_{p \times q}(Z; M, \mathcal{S}, \Phi) = \frac{1}{\pi^{pq} \, |\mathcal{S}|^q \cdot |\Phi|^p} \cdot \exp\big\{ - \mathrm{tr}[\Phi^{-1}(Z - M)^H \mathcal{S}^{-1}(Z - M)]\big\} \tag{2.4}$$

where $|\cdot|$, $\mathrm{tr}(\cdot)$, and "^{H}" denote the determinant, trace, and Hermitian (conjugate) transpose. This definition is an extension of [19, eq. (2.7.4)] and [20, Definition 1.1] to the complex-data case.

The model description (2.1)–(2.3) leads to the likelihood function of the unknown parameters:

$$p(Y \,|\, \boldsymbol{\xi}) \;=\; \mathcal{N}_{m\times N}\Big(Y; AX\varPhi, \mathcal{R}(B,\varSigma,\sigma^2), I_N\Big) \tag{2.5a}$$

where $p(Y \,|\, \boldsymbol{\xi})$ denotes the conditional pdf of Y given $\boldsymbol{\xi}$,

$$\mathcal{R}(B,\varSigma,\sigma^2) = \mathrm{E}\left\{[B\boldsymbol{h}(t)+\boldsymbol{e}(t)]\,[B\boldsymbol{h}(t)+\boldsymbol{e}(t)]^H\right\} = \sigma^2\, R(B,\varSigma) \tag{2.5b}$$

is the spatial covariance matrix of interference and noise, and

$$R(B,\varSigma) = I_m + B\varSigma B^H. \tag{2.5c}$$

This measurement model is equivalently represented using a two-stage hierarchical formulation:

$$p(Y \,|\, X,B,H,\sigma^2) \;=\; \mathcal{N}_{m\times N}(Y; AX\varPhi + BH, \sigma^2 I_m, I_N) \tag{2.6a}$$

$$p(H \,|\, \varSigma,\sigma^2) \;=\; \mathcal{N}_{r\times N}(H; 0_{r\times N}, \sigma^2\varSigma, I_N) \tag{2.6b}$$

where $0_{r\times N}$ denotes the $r \times N$ matrix of zeroes.

A. Prior Specifications

We assume that the unknown parameters are independent *a priori,* i.e.

$$\pi(\boldsymbol{\xi}) = \pi(X,B,\varSigma,\sigma^2) = \pi(X)\cdot\pi(B)\cdot\pi(\varSigma)\cdot\pi(\sigma^2). \tag{2.7a}$$

Here, $\pi(\boldsymbol{\xi})$ denotes the prior pdf of $\boldsymbol{\xi}$ and analogous notation is used for the prior pdfs of the components of $\boldsymbol{\xi}$. Let us adopt an (improper) "flat" Jeffreys' noninformative prior pdf for the complex signal amplitudes[1]

$$\pi(X) \;\propto\; 1 \tag{2.7b}$$

and select *conjugate prior pdfs* for the noise and interference parameters:[2]

$$\pi(\sigma^2) \;=\; \text{Inv-}\chi^2(\sigma^2\,;\,2\,\nu_{\sigma^2},\sigma_0^2) \propto (\sigma^2)^{-(1+\nu_{\sigma^2})}\cdot\exp(-\nu_{\sigma^2}\cdot\sigma_0^2/\sigma^2) \tag{2.7c}$$

$$\pi(B) \;\propto\; \mathcal{N}_{m\times r}(B; M_B, I_m, \varGamma_B) \tag{2.7d}$$

$$\pi(\varSigma) \;=\; \text{Inv-Wishart}_r\Big(\varSigma;\nu_\varSigma,[(\nu_\varSigma - r)\,\varLambda_0]^{-1}\Big) \propto |\varSigma|^{-(r+\nu_\varSigma)}\cdot\exp[-(\nu_\varSigma - r)\cdot\mathrm{tr}(\varSigma^{-1}\varLambda_0)] \tag{2.7e}$$

where

[1]See [21, Ch. 2.9] for an introduction to noninformative prior distributions.

[2]Utilizing conjugate priors simplifies Bayesian computations.

- Inv-$\chi^2(\sigma^2;\, 2\,\nu_{\sigma^2}, \sigma_0^2)$ denotes the pdf of a scaled inverse chi-square distribution with $2\,\nu_{\sigma^2}$ degrees of freedom and a scale parameter σ_0^2, see [21, p. 50 and App. A];[3]
- Inv-Wishart$_r(\Sigma; \nu_\Sigma, \Sigma_0^{-1})$ denotes the pdf of an $r\times r$ random matrix Σ having complex inverse Wishart distribution with ν_Σ degrees of freedom and positive definite Hermitian $r \times r$ scale matrix Σ_0;[4]
- prior mean and precision matrices M_B and Γ_B^{-1} (of dimensions $m \times r$ and $r \times r$, respectively) quantify our prior knowledge about the interference array response B.

The degrees of freedom ν_Σ and ν_{σ^2} can be interpreted as numbers of virtual "observations" describing our prior knowledge about Σ and σ^2, where each "observation" is equal to Λ_0 or σ_0^2, respectively. Prior information about the noise level σ^2 is available in many applications (e.g. in systems operating at the microwave frequencies, see [3]), thus justifying use of informative priors for $\pi(\sigma^2)$ and the Bayesian approach in general. Complex inverse-Wishart priors have been utilized in [18] to improve estimation of interference-plus-noise covariance matrices in knowledge-aided radar. The interference array response prior (2.7d) can be constructed from previous estimates of B. The Bayesian framework allows us to utilize sequential-Bayesian ideas and *track* the interference. In practice, we may select the precision Γ_B^{-1} to be a diagonal matrix whose diagonal elements describe prior precisions of the interference array-response vectors (i.e. the columns of B). Lack of prior information on B can be expressed by choosing a "flat" noninformative prior with $\Gamma_B^{-1} = 0_{r\times r}$.

III. Bayesian Estimation of Complex Signal Amplitudes and Application to Adaptive Detection

We first develop a Bayesian approach for estimating the unknown parameters $\boldsymbol{\xi}$ under the measurement and prior models in Section II and then discuss its application to adaptive signal detection. The joint posterior pdf of the parameters $\boldsymbol{\xi}$ and random interference signals H is given by

$$\begin{aligned} p(\boldsymbol{\xi}, H \,|\, Y) &\propto p(Y \,|\, X, B, H, \sigma^2) \cdot p(H|\Sigma, \sigma^2) \cdot \pi(X) \cdot \pi(B) \cdot \pi(\Sigma) \cdot \pi(\sigma^2) \\ &\propto |\sigma^2 I_m|^{-N} \exp\big\{ -\operatorname{tr}[(Y - AX\Phi - BH)\,(Y - AX\Phi - BH)^H]/\sigma^2 \big\} \\ &\quad \cdot |\sigma^2 \Sigma|^{-N} \exp\big[-\operatorname{tr}(\Sigma^{-1} H H^H)/\sigma^2 \big] \cdot \exp\big\{ -\operatorname{tr}[(B - M_B)\,\Gamma_B^{-1}\,(B - M_B)^H] \big\} \\ &\quad \cdot |\Sigma|^{-(r+\nu_\Sigma)} \cdot \exp[-(\nu_\Sigma - r) \cdot \operatorname{tr}(\Sigma^{-1}\Lambda_0)] \cdot (\sigma^2)^{-(1+\nu_{\sigma^2})} \cdot \exp(-\nu_{\sigma^2} \cdot \sigma_0^2/\sigma^2) \end{aligned} \tag{3.1a}$$

[3]Provided that $\nu_{\sigma^2} > 1$, the mean of the Inv-$\chi^2(\sigma^2\,;\, 2\,\nu_{\sigma^2}, \sigma_0^2)$ pdf is $\nu_{\sigma^2}/(\nu_{\sigma^2} - 1) \cdot \sigma_0^2$.

[4]Provided that $\nu_\Sigma > r$, the mean of the Inv-Wishart$_r(\Sigma; \nu_\Sigma, \Sigma_0^{-1})$ pdf is $(\nu_\Sigma - r)^{-1} \cdot \Sigma_0$ (see [22, eq. (39)]); consequently, the mean of the prior pdf in (2.7e) is Λ_0. See [22] for a detailed discussion on complex inverse-Wishart distribution.

implying that

$$p(H \mid \boldsymbol{\xi}, Y) \;=\; \mathcal{N}_{r\times N}\Big(H;\;\; (\Sigma B^H B + I_r)^{-1} \Sigma B^H \,(Y - AX\,\Phi), \sigma^2 \cdot (\Sigma B^H B + I_r)^{-1} \Sigma, I_N\Big) \tag{3.1b}$$

$$\begin{aligned} p(B \mid X, H, \Sigma, \sigma^2, Y) \;=\; & \mathcal{N}_{m\times r}\Big(B;\;\; [(Y - AX\,\Phi)H^H + \sigma^2\, M_B \Gamma_B^{-1}]\,(HH^H + \sigma^2\,\Gamma_B^{-1})^{-1}, I_m, \\ & \sigma^2 \cdot (HH^H + \sigma^2\,\Gamma_B^{-1})^{-1}\Big) \end{aligned} \tag{3.1c}$$

which follow by keeping the terms in (3.1a) that depend on H and B (respectively) and completing the squares in the exponents of the resulting expressions. The result (3.1b) is consistent with the well-known result in [23, eqs. (15.64)–(15.67)]. Integrating H out from (3.1a) yields the posterior pdf of $\boldsymbol{\xi}$:

$$\begin{aligned} p(\boldsymbol{\xi} \mid Y) \;\propto\; & p(Y \mid \boldsymbol{\xi}) \cdot \pi(X) \cdot \pi(B) \cdot \pi(\Sigma) \cdot \pi(\sigma^2) \\ \propto\; & \frac{1}{|\sigma^2 R(B, \Sigma)|^N} \cdot \exp\{-\operatorname{tr}[R(B, \Sigma)^{-1}(Y - AX\,\Phi)(Y - AX\,\Phi)^H]/\sigma^2\} \\ & \cdot \exp\big\{-\operatorname{tr}[(B - M_B)\,\Gamma_B^{-1}\,(B - M_B)^H]\big\} \\ & \cdot |\Sigma|^{-(r+\nu_\Sigma)} \cdot \exp[-(\nu_\Sigma - r) \cdot \operatorname{tr}(\Sigma^{-1} \Lambda_0)] \cdot (\sigma^2)^{-(1+\nu_{\sigma^2})} \cdot \exp(-\nu_{\sigma^2} \cdot \sigma_0^2/\sigma^2) \end{aligned} \tag{3.2}$$

and the conditional pdf $p(X \mid B, \Sigma, \sigma^2, Y)$ follows by keeping the terms in (3.2) that depend on X and completing the squares in the exponent:

$$p(X \mid \sigma^2, B, \Sigma, Y) \;=\; \mathcal{N}_{q\times d}\Big(X; \widehat{X}(B, \Sigma), [A^H \mathcal{R}(B, \Sigma, \sigma^2)^{-1} A]^{-1}, (\Phi\Phi^H)^{-1}\Big) \tag{3.3a}$$

where

$$\widehat{X}(B, \Sigma) \;=\; [A^H R(B, \Sigma)^{-1} A]^{-1} A^H R(B, \Sigma)^{-1} Y\,\Phi^H (\Phi\Phi^H)^{-1} \tag{3.3b}$$

which is consistent with the real-data result in [20, Theorem 6.7]. If we “vectorize” X, then (3.3a) implies that the posterior covariance matrix of $\operatorname{vec}(X)$ given σ^2, B, Σ, and Y is equal to the (non-Bayesian) CRB of $\operatorname{vec}(X)$ in [17, eq. (66)] and [15, App. 4.D].[5] Let us integrate X out from the joint pdf $p(X, \sigma^2 \mid B, \Sigma, Y)$ [see also (3.2) and (3.3a)]:

$$\begin{aligned} p(\sigma^2 \mid B, \Sigma, Y) = & \frac{p(X, \sigma^2 \mid B, \Sigma, Y)}{p(X \mid \sigma^2, B, \Sigma, Y)} \\ \propto\; & \frac{|\sigma^2 R(B, \Sigma)|^{-N} \cdot \exp\Big(-\operatorname{tr}[R(B, \Sigma)^{-1}\,(Y - AX\,\Phi)\,(Y - AX\,\Phi)^H]/\sigma^2\Big) \cdot (\sigma^2)^{-(1+\nu_{\sigma^2})} \cdot \exp(-\nu_{\sigma^2} \cdot \sigma_0^2/\sigma^2)}{(\sigma^2)^{-q\,d} \cdot \exp\Big(-\operatorname{tr}\{[X - \widehat{X}(B, \Sigma)]^H\, A^H R(B, \Sigma)^{-1} A\,[X - \widehat{X}(B, \Sigma)]\;\Phi\,\Phi^H\}/\sigma^2\Big)} \end{aligned} \tag{3.4a}$$

$$\begin{aligned} \propto\; & (\sigma^2)^{-(mN+1+\nu_{\sigma^2}-q\,d)} \cdot \exp\Big(-\operatorname{tr}\{R(B, \Sigma)^{-1}\,[Y - A\widehat{X}(B, \Sigma)\,\Phi] \cdot [Y - A\widehat{X}(B, \Sigma)\,\Phi]^H\}/\sigma^2 - \nu_{\sigma^2} \cdot \sigma_0^2/\sigma^2\Big) \\ =\; & \text{Inv-}\chi^2\Big(\sigma^2\,;\, 2\,(mN - q\,d + \nu_{\sigma^2}), \widehat{\sigma}^2(B, \Sigma) \cdot (mN + 1 - q\,d + \nu_{\sigma^2})/(mN - q\,d + \nu_{\sigma^2})\Big) \end{aligned} \tag{3.4b}$$

where $\widehat{\sigma}^2(B, \Sigma)$ denotes the mode of the pdf in (3.4b) (see [21, App. A]):

$$\widehat{\sigma}^2(B, \Sigma) = \frac{\operatorname{tr}\big\{R(B, \Sigma)^{-1}\,[Y - A\widehat{X}(B, \Sigma)\,\Phi]\,[Y - A\widehat{X}(B, \Sigma)\,\Phi]^H\big\} + \nu_{\sigma^2} \cdot \sigma_0^2}{mN + 1 - q\,d + \nu_{\sigma^2}}. \tag{3.4c}$$

[5] Here, the vec operator stacks the columns of a matrix one below another into a single column vector.

Here, (3.4b) follows by noting that $p(\sigma^2 \,|\, B, \Sigma, Y)$ *does not* depend on X and setting $X = \widehat{X}(B, \Sigma)$ in (3.4a).

We now estimate the unknown parameters $\boldsymbol{\xi}$ using an ICM approach [24, Sect. 4], [25, Ch. 10.2.1], [26, Ch. 6.2.2]. Our ICM algorithm consists of cycling between the following steps:

1) Fix $B = \widehat{B}$, $\Sigma = \widehat{\Sigma}$ and estimate X by maximizing the conditional posterior pdf $p(X \,|\, B, \Sigma, \sigma^2, Y)$ [see also (3.3b)]:
$$\widehat{X} = \widehat{X}(B, \Sigma). \tag{3.5a}$$

2) Fix $B = \widehat{B}$, $\Sigma = \widehat{\Sigma}$ and estimate σ^2 by maximizing the conditional posterior pdf $p(\sigma^2 \,|\, B, \Sigma, Y)$ in (3.4):
$$\widehat{\sigma}^2 = \widehat{\sigma}^2(B, \Sigma). \tag{3.5b}$$

3) Fix $B = \widehat{B}$, $X = \widehat{X}$, $\sigma^2 = \widehat{\sigma}^2$ and estimate Σ by maximizing the conditional posterior pdf $p(\Sigma \,|\, X, B, \sigma^2, Y)$. For this purpose, we utilize an expectation-maximization (EM) step where we treat the random interference signals H as the *unobserved* (or missing) data:
$$\widehat{\Sigma} = \arg\max_{\Sigma} \mathrm{E}_{\,H|\Sigma=\Sigma_{\mathrm{p}},X,B,\sigma^2,Y}\big\{\ln p(\boldsymbol{\xi}, H \,|\, Y)\big\} \tag{3.5c}$$
where Σ_{p} denotes the estimate of Σ from the previous iteration and $\mathrm{E}_{\,H|\Sigma,X,B,\sigma^2,Y}$ the expectation with respect to $p(H \,|\, \Sigma, X, B, \sigma^2, Y) = p(H \,|\, \boldsymbol{\xi}, Y)$ in (3.1b). (See [21, Ch. 12.3] for a detailed exposition on EM-type algorithms in the Bayesian context.) Consequently,
$$\widehat{\Sigma} = \frac{1}{N + r + \nu_\Sigma} \cdot [\widehat{H}\widehat{H}^H/\sigma^2 + N \cdot (\Sigma_{\mathrm{p}} B^H B + I_r)^{-1} \Sigma_{\mathrm{p}} + (\nu_\Sigma - r) \cdot \Lambda_0] \tag{3.5d}$$
where $\widehat{H} = (\Sigma_{\mathrm{p}} B^H B + I_r)^{-1} \Sigma_{\mathrm{p}} B^H (Y - AX\Phi)$.

4) Fix $X = \widehat{X}$, $\Sigma = \widehat{\Sigma}$, $\sigma^2 = \widehat{\sigma}^2$ and estimate B by maximizing the conditional posterior pdf $p(B \,|\, X, \Sigma, \sigma^2, Y)$. As before, we treat H as the missing data and utilize the following EM step:
$$\widehat{B} \;=\; \arg\max_{B} \mathrm{E}_{\,H|B=B_{\mathrm{p}},X,\Sigma,\sigma^2,Y}\big\{\ln p(\boldsymbol{\xi}, H \,|\, Y)\big\} \tag{3.5e}$$
where B_{p} denotes the estimate of B from the previous iteration. Consequently,
$$\widehat{B} = \{(Y - AX\Phi)\,\widetilde{H}^H + \sigma^2 M_B \Gamma_B^{-1}\} \cdot \big[\widetilde{H}\widetilde{H}^H + N \cdot \sigma^2 \cdot (\Sigma B_{\mathrm{p}}^H B_{\mathrm{p}} + I_r)^{-1} \Sigma + \sigma^2\, \Gamma_B^{-1}\big]^{-1} \tag{3.5f}$$
where $\widetilde{H} = (\Sigma B_{\mathrm{p}}^H B_{\mathrm{p}} + I_r)^{-1} \Sigma B_{\mathrm{p}}^H (Y - AX\Phi)$.

This iterative scheme can also be viewed as a parameter-expanded expectation-maximization (PX-EM) algorithm [21, Ch. 12.3]. Here, *parameter expansion* refers to the fact that B and Σ overparametrize the interference. The computation of $R(B, \Sigma)^{-1}$ in Steps 1) and 2) is performed efficiently using the matrix inversion lemma (e.g. [27, Theorem 18.2.8]):
$$R(B, \Sigma)^{-1} = (I_m + B\Sigma B^H)^{-1} = I_m - B\,(\Sigma B^H B + I_r)^{-1}\,\Sigma\, B^H \tag{3.6}$$

which requires inversion of an $r \times r$ matrix and thus leads to significant computational savings compared with directly inverting $R(B, \Sigma)$. We utilize (3.6) to simplify Steps 1) and 2) and summarize the resulting ICM algorithm: compute

$$P^{(i)} = [\Sigma^{(i)}(B^{(i)})^H B^{(i)} + I_r]^{-1} \Sigma^{(i)} \tag{3.7a}$$

$$X^{(i)} = \left[A^H A - A^H B^{(i)} P^{(i)} (B^{(i)})^H A\right]^{-1} \left[A^H Y - A^H B^{(i)} P^{(i)} (B^{(i)})^H Y\right] \Phi^H (\Phi \Phi^H)^{-1} \tag{3.7b}$$

$$H^{(i)} = P^{(i)} \, (B^{(i)})^H \, (Y - A X^{(i)} \Phi) \tag{3.7c}$$

$$(\sigma^2)^{(i)} = \left\{ \mathrm{tr}\left[((Y - A X^{(i)} \Phi - B^{(i)} H^{(i)}) \, (Y - A X^{(i)} \Phi)^H\right] + \nu_{\sigma^2} \cdot \sigma_0^2 \right\} / \, (mN + 1 - q\, d + \nu_{\sigma^2}) \tag{3.7d}$$

$$T^{(i)} = H^{(i)} \, (H^{(i)})^H + N \cdot (\sigma^2)^{(i)} \cdot P^{(i)} \tag{3.7e}$$

and update the interference parameters as follows:

$$\Sigma^{(i+1)} = \frac{1}{N + r + \nu_\Sigma} \cdot \left\{ \frac{T^{(i)}}{(\sigma^2)^{(i)}} + (\nu_\Sigma - r) \cdot \Lambda_0 \right\} \tag{3.8a}$$

$$B^{(i+1)} = \left[(Y - A X^{(i)} \Phi) \, (H^{(i)})^H + (\sigma^2)^{(i)} \, M_B \, \Gamma_B^{-1}\right] \left[T^{(i)} + (\sigma^2)^{(i)} \, \Gamma_B^{-1}\right]^{-1}. \tag{3.8b}$$

The above iteration is performed until the unknown parameters $\boldsymbol{\xi}^{(i)}$ do not change significantly between two consecutive cycles. Each cycle requires one $q \times q$ and two $r \times r$ matrix inversions. Hence, this method is computationally efficient in applications with the signal and interference ranks q and r much smaller than m, such as STAP [6]–[8], [18] where m is the product of the numbers of sensors and pulse returns, see also [4] and [5]. Our ICM algorithm also provides estimates of the interference signals H, see (3.7c). Observe the intuitively appealing weighted-average forms of the expressions (3.7d), (3.8a), and (3.8b), where the weights of the prior terms are dictated by the prior degrees of freedom ν_{σ^2}, ν_Σ and precision Γ_B^{-1}, respectively.

In the following, we describe initialization of the ICM iteration and AMF signal detection.

A. *Initialization and Interference Rank Estimation*

Our initialization scheme is based on the matrix $Y \, [I_N - \Phi^H (\Phi \Phi^H)^{-1} \Phi] \, Y^H$ which is complex Wishart with $N - d$ degrees of freedom and scale $\mathcal{R}(B, \Sigma, \sigma^2)$. (This is a standard result in multivariate analysis of variance (MANOVA) whose real-Wishart version is given in e.g. [28, Ch. 2.2].) Then

$$S = \frac{1}{N - d} \cdot Y \, [I_N - \Phi^H (\Phi \Phi^H)^{-1} \Phi] \, Y^H \tag{3.9a}$$

is an unbiased estimator of $\mathcal{R}(B, \Sigma, \sigma^2)$ with eigenvalue decomposition

$$S = [\boldsymbol{u}(1), \boldsymbol{u}(2) \cdots \boldsymbol{u}(m)] \; \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_m\} \; [\boldsymbol{u}(1), \boldsymbol{u}(2) \cdots \boldsymbol{u}(m)]^H \tag{3.9b}$$

where $\lambda_1 \geq \lambda_2 \geq \cdots \lambda_m \geq 0$ are the eigenvalues and $\boldsymbol{u}(1), \boldsymbol{u}(2) \cdots \boldsymbol{u}(m)$ are the corresponding eigenvectors of S. Then, define

$$\widetilde{\sigma}^2 = \frac{\sum_{i=r+1}^{\min\{m, N-d\}} \lambda_i}{\min\{m, N - d\} - r} \tag{3.9c}$$

and initialize the iteration (3.7)–(3.8) using

$$B^{(0)} = [\boldsymbol{u}(1) \cdots \boldsymbol{u}(r)], \qquad \Sigma^{(0)} = \frac{\operatorname{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_r\}}{\widetilde{\sigma}^2} - I_r. \tag{3.9d}$$

The above estimates can be computed efficiently using the algorithms in [11] and should be utilized when diffuse priors are employed for the unknown parameters; otherwise, prior information should be incorporated into the initial values as well.

Interference Rank Estimation: If the interference rank r is *not known*, we estimate it in the initialization stage using the minimum-description-length (MDL) criterion (derived along the lines of [29]):

$$r_{\text{MDL}} = \arg \max_{r \in \{1,2,\ldots,n_{\text{MIN}}-1\}} \Big\{ n_{\text{MAX}} \cdot \Big(\sum_{i=r+1}^{n_{\text{MIN}}} \ln \lambda_i \Big) - (n_{\text{MIN}} - r)\, n_{\text{MAX}} \cdot \ln \Big(\frac{\sum_{l=r+1}^{n_{\text{MIN}}} \lambda_l}{n_{\text{MIN}} - r} \Big) - \tfrac{1}{2}\, r\, (2 n_{\text{MIN}} - r)\, \ln n_{\text{MAX}} \Big\} \tag{3.10}$$

where $n_{\text{MIN}} = \min\{m, N-d\}$ and $n_{\text{MAX}} = \max\{m, N-d\}$. If n_{MIN} is extremely small, the MDL approach performs poorly and better estimation of r can be achieved using statistical eigen-inference in [30], [31].

Here, we adopt a "plug-in" approach: once we estimate $r = r_{\text{MDL}}$, we treat it as if it were known and proceed with the initialization (3.9c)–(3.9d) and ICM iteration.

B. *Adaptive Matched Filter Signal Detection*

We propose the following AMF detector for testing $\mathrm{H}_0 : X = 0_{q \times d}$ (signal absent) versus the alternative $\mathrm{H}_1 : X \neq 0_{q \times d}$ (signal present):

Compare the test statistic $\mathrm{AMF}(\boldsymbol{\xi}^{(\infty)})$ with a threshold τ and declare the presence of signal if $\mathrm{AMF}(\boldsymbol{\xi}^{(\infty)}) > \tau$

where $\mathrm{AMF}(\boldsymbol{\xi}) = \operatorname{tr}\left[X^H A^H \mathcal{R}(B, \Sigma, \sigma^2)^{-1} A X \Phi\, \Phi^H\right]$ is the matched-filter test statistic for the above testing problem assuming that the parameters $\boldsymbol{\xi}$ are *known.* (The AMF approach to signal detection was introduced in [32] and [33].) Here, $\boldsymbol{\xi}^{(\infty)}$ denotes the estimate of $\boldsymbol{\xi}$ obtained upon convergence of the ICM algorithm; consequently, we refer to the proposed detector as the *AMF ICM detector*. Note that $\mathrm{AMF}(\boldsymbol{\xi}^{(\infty)})$ can be further simplified using (3.6) [see also (3.7a)]:

$$\mathrm{AMF}(\boldsymbol{\xi}^{(\infty)}) = \frac{1}{(\sigma^2)^{(\infty)}} \cdot \operatorname{tr}\Big\{ (X^{(\infty)})^H \left[A^H A - A^H B^{(\infty)} P^{(\infty)} (B^{(\infty)})^H A\right] X^{(\infty)} \Phi\, \Phi^H \Big\}. \tag{3.11}$$

IV. Numerical Examples

Consider an $m = 50$-sensor uniform linear antenna array with array response

$$\boldsymbol{a}(\varphi) = \left[1, \exp(-j\varphi), \exp(-2j\varphi), \ldots, \exp(-(m-1)j\varphi)\right]^T \tag{4.1}$$

where $\varphi = 2\pi\Delta \sin\theta/\lambda$. Here, "^{T}" denotes a transpose, Δ the interelement spacing, θ the angle of arrival, and λ the wavelength of the center frequency. We focus on estimating and detecting the complex amplitude $X = x$ of a planewave signal arriving from direction $\varphi_{\rm s}$, implying that $q = d = 1$ and signal array and temporal responses are vectors: $A = \boldsymbol{a}(\varphi_{\rm s})$ and $\Phi = [\phi(1), \phi(2) \cdots \phi(N)] = [1, 1, \ldots, 1]$. We have simulated $r = 6$ independent Gaussian interference sources at angles $\varphi \in \{-0.8\pi, -0.4\pi, 0.2\pi, 0.5\pi, 0.7\pi, 0.9\pi\}$ with 0, 40, 30, 20, 40, and 10 dB interference-to-white-noise ratios (respectively), similar to the simulated interference model in [34]. We have chosen a 0 dB signal level and direction $\varphi_{\rm s} = -0.81\,\pi$ satisfying $\boldsymbol{a}(\varphi_{\rm s})^H\,[I_m - B(B^H B)^{-1} B^H]\,\boldsymbol{a}(\varphi_{\rm s}) / \boldsymbol{a}(\varphi_{\rm s})^H\,\boldsymbol{a}(\varphi_{\rm s}) = 0.19$ implying that, in this case, interference "nulling" leads to a significant signal-power reduction [34]. Here, the interference-to-white-noise ratios are the diagonal elements of Σ and the signal level is $\mathrm{abs}(x/\sigma)$, where $\mathrm{abs}(\cdot)$ denotes absolute value.

Diffuse Prior Specifications: We have selected diffuse prior pdfs for the unknown parameters with

- $\Gamma_B^{-1} = 0_{r\times r}$ (corresponding to the "flat" Jeffreys' noninformative prior for B);
- $\nu_\Sigma = r + 1 = 7$ and $\Lambda_0 = \Sigma^{(0)}$ [see (3.9d) and footnote 4];
- $\nu_{\sigma^2} = 2$ and $\sigma_0^2 = (\nu_{\sigma^2} - 1)/\nu_{\sigma^2} \cdot \widetilde{\sigma}^2 = \widetilde{\sigma}^2/2$ [yielding the mean of $\pi(\sigma^2)$ equal to $\widetilde{\sigma}^2$, see (3.9c) and footnote 3].

Under this prior model, the ICM estimates $\boldsymbol{\xi}^{(\infty)}$ of the unknown parameters are approximately equal to their (non-Bayesian) ML estimates. Consequently, the mean-square error (MSE) performance of $X^{(\infty)}$ is bounded from below by the CRB derived in [15, App. 4.D] and [17, App. A]. Using [27, eq. (1.25) at p. 338], we simplify the trace of this CRB to

$$\mathrm{CRB}(x) = \sigma^2 \Big/ \Big\{ \big[\boldsymbol{a}(\varphi_{\rm s})^H \boldsymbol{a}(\varphi_{\rm s}) - \boldsymbol{a}(\varphi_{\rm s})^H\, B\, (\Sigma\, B^H B + I_r)^{-1}\, \Sigma\, B^H\, \boldsymbol{a}(\varphi_{\rm s})\big] \cdot \sum_{t=1}^{N} |\phi(t)|^2 \Big\}. \tag{4.2}$$

Interference Rank Estimation: In all the examples, we employ the "plug-in" estimate $r_{\rm MDL}$ in (3.10), computed in the initialization stage.

Complex-Amplitude Estimation: In the first simulation example, we study the performance of the proposed estimator of $X = x$. Our performance metric is the MSE of an estimator, calculated using 30 000 independent trials. (In each trial, we generated independent interference and noise realizations.) We compare the ICM estimator of x with the GMANOVA method for unstructured covariance matrix of interference and noise (see [14]–[16]):

$$\widehat{x}_{\rm UC} = \frac{\boldsymbol{a}(\varphi_{\rm s})^H\, S^{-1}\, \sum_{t=1}^N \boldsymbol{y}(t)\phi(t)^*}{\boldsymbol{a}(\varphi_{\rm s})^H S^{-1} \boldsymbol{a}(\varphi_{\rm s}) \cdot \sum_{t=1}^N |\phi(t)|^2} \tag{4.3a}$$

where "$*$" denotes complex conjugation and S has been defined in (3.9a). We need $N \geq \mathrm{rank}(\Phi) + m = 51$ for S to be invertible with probability one, see [14, eq. (4)]. When the number of snapshots N is smaller than this bound, we apply diagonal loading as follows (similar to [35], see also [34] and references therein):

$$\widehat{x}_{\rm UC,DL} = \frac{\boldsymbol{a}(\varphi_{\rm s})^H (S + \delta^2 I_m)^{-1}\, \sum_{t=1}^N \boldsymbol{y}(t)\phi(t)^*}{\boldsymbol{a}(\varphi_{\rm s})^H (S + \delta^2 I_m)^{-1} \boldsymbol{a}(\varphi_{\rm s}) \cdot \sum_{t=1}^N |\phi(t)|^2} \tag{4.3b}$$

where the loading factors have been selected as $\delta^2 \in \big\{\widetilde{\sigma}^2, 3\,\widetilde{\sigma}^2\big\}$, corresponding to 0 and 5 dB above the estimated noise level (3.9c) [8]. In Fig. 1, we show normalized MSEs and corresponding CRBs for the following estimates of the

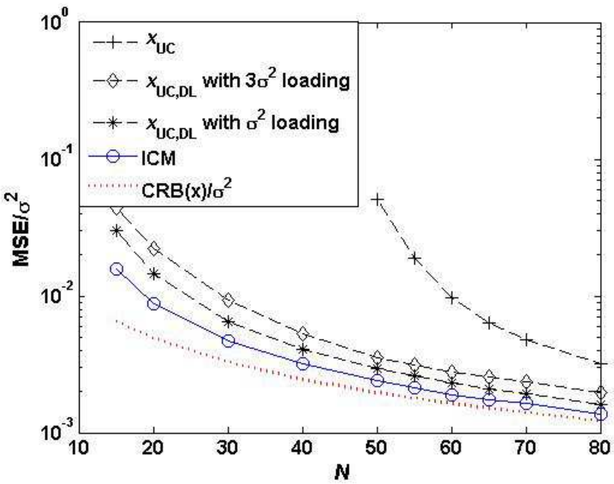

Fig. 1. Normalized MSEs and CRBs for the ICM, unstructured GMANOVA, and loaded unstructured GMANOVA estimators of the complex amplitude x as functions of the number of snapshots N, for an array with $m = 50$ sensors.

complex signal amplitude: (i) $x^{(\infty)}$ obtained upon convergence of the ICM algorithm and (ii) $\widehat{x}_{\rm UC}$ in (4.3a) (requiring $N \geq 51$) and $\widehat{x}_{\rm UC,DL}$ in (4.3b), as functions of the number of snapshots N. The ICM algorithm converged within ten iteration steps. For small N, $x^{(\infty)}$ outperforms $\widehat{x}_{\rm UC,DL}$ approximately by a factor of two (in terms of MSE). Note that the (unstructured) GMANOVA estimator $\widehat{x}_{\rm UC}$ performs poorly.

Signal Detection: In the second set of simulations, we compare the adaptive detector in Section III-B with several existing methods. Our performance metric is the *average probability of detection*, where averaging is performed over the random interference and noise realizations *as well as* Gaussian complex-amplitude realizations following the $\mathcal{N}_{1\times 1}(x; 0, \sigma_x^2, 1)$ distribution. Define the output signal-to-noise ratio (SNR) as ${\rm SNR_o} = \sigma_x^2/{\rm CRB}(x)$. We compare the AMF ICM detector with

- the *clairvoyant* detector, which assumes perfect knowledge of the interference and noise properties and compares ${\rm tr}[\widehat{X}(B,\Sigma)^H A^H \mathcal{R}(B,\Sigma,\sigma^2)^{-1} A\, \widehat{X}(B,\Sigma)\, \Phi\, \Phi^H]$ with a threshold, see also (3.3b) and (2.5b);
- generalized likelihood ratio (GLR) and loaded GLR signal detectors for unstructured covariance matrix of interference and noise, which compare

$$\frac{\boldsymbol{a}(\varphi_{\rm s})^H S^{-1} \boldsymbol{a}(\varphi_{\rm s})}{\boldsymbol{a}(\varphi_{\rm s})^H [(1/N)\cdot YY^H]^{-1} \boldsymbol{a}(\varphi_{\rm s})}$$

  and

$$\frac{\boldsymbol{a}(\varphi_{\rm s})^H (S + \delta^2 I_m)^{-1} \boldsymbol{a}(\varphi_{\rm s})}{\boldsymbol{a}(\varphi_{\rm s})^H [(1/N)\cdot YY^H + \delta^2 I_m]^{-1} \boldsymbol{a}(\varphi_{\rm s})}$$

  (respectively) with appropriate thresholds, along the lines of [14, eq. (8)] and [34]–[35].

Fig. 2 shows the performances of the above detectors as functions of ${\rm SNR_o}$ for fixed false-alarm probability $P_{\rm FA} = 10^{-2}$. The average detection performance of the clairvoyant detector can be computed analytically and is given by

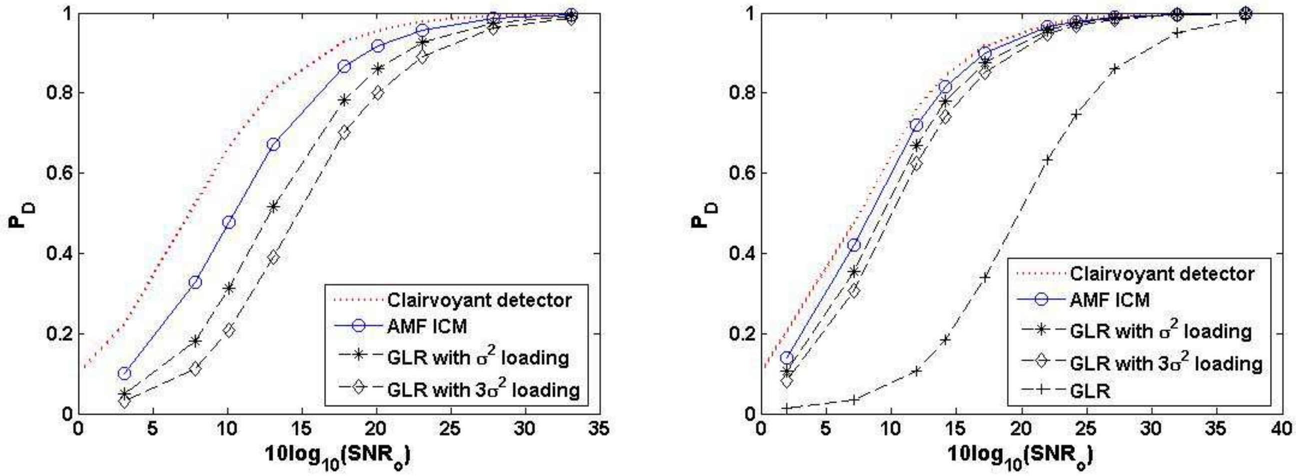

Fig. 2. Detection probabilities of the clairvoyant, AMF, GLR and loaded GLR detectors as functions of the output SNR in decibels for (left) $N = 20$ and (right) $N = 55$ snapshots, assuming $P_{\mathrm{FA}} = 10^{-2}$ and an array with $m = 50$ sensors.

$P_{\mathrm{d,av}} = P_{\mathrm{FA}}^{(1+\mathrm{SNR_o})^{-1}}$. The AMF ICM detector outperforms other adaptive methods, with significant performance improvement for $N = 20$. As expected, the unstructured GLR detector performs poorly. Note that $N = 20$ corresponds to the increasingly important "snapshot-constrained" scenario [4], [5].

Clearly, the proposed estimators and detectors will achieve better performance (compared with that shown in this section) if we utilize informative priors for the unknown interference and noise parameters.

V. Concluding Remarks

We proposed an ICM algorithm for Bayesian estimation of complex signal amplitudes in low-rank interference and an adaptive signal detector based on the ICM estimates of the signal amplitudes and interference and noise parameters.

Interestingly, the initialization method in Section III-A is the main source of computational complexity since it requires eigenvalue decomposition of a (potentially large) $m \times m$ matrix. [The diagonal-loading approaches also require this decomposition to obtain the loading factor, see (3.9c).] If the array size m is large, then the proposed ICM algorithm and AMF detector will be computationally efficient compared with the existing methods. Our approach also provides a framework for avoiding the complex initialization step, in particular, utilizing sequential-Bayesian concepts will allow *interference tracking.*

Further research will include: developing alternative initialization approaches and sequential-Bayesian methods for interference tracking (possibly using the "blending" ideas in [18]) and extensions to the STAP scenarios and comparison with related methods, such as the parametric adaptive matched filter (PAMF) in [36] and [37]. It is also of interest to compute analytical false-alarm and detection probability expressions for the proposed adaptive detector and to study Bayesian performance measures [38].

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