Highly Accelerated Cardiac Cine parallel MRI using Low-Rank Matrix Completion and Partial Separability Model

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ABSTRACT

This paper presents a new approach to highly accelerated dynamic parallel MRI using low rank matrix completion, partial separability (PS) model. In data acquisition, *k*-space data is moderately randomly undersampled at the center *k*-space navigator locations, but highly undersampled at the outer *k*-space for each temporal frame. In reconstruction, the navigator data is reconstructed from undersampled data using structured low-rank matrix completion. After all the unacquired navigator data is estimated, the partial separable model is used to obtain partial *k-t* data. Then the parallel imaging method is used to acquire the entire dynamic image series from highly undersampled data. The proposed method has shown to achieve high quality reconstructions with reduction factors up to 31, and temporal resolution of 29ms, when the conventional PS method fails.

Keywords: cardiac cine MRI, low rank, partial separable model, parallel imaging, sparsity.

1. INTRODUCTION

The main goal of the proposed method is to recover the dynamic image sequence from highly under-sampled Fourier measurements with high spatiotemporal resolution. Joint use of partial separability (PS) [1-5] and spatial spectral sparsity constraints has shown success in improving the temporal resolution of dynamic magnetic resonance imaging (MRI) [6-11]. In the PS model, temporal basis is usually obtained from the dominant singular vectors through the singular value decomposition (SVD) of data from center *k*-space navigator locations. However, there are many circumstances that acquiring sufficient navigator data would become the bottleneck of high accelerations (shown in Fig. 1). For example, in dynamic MRI, the temporal resolution is expected to be increased. As a result, very limited phase encoding lines can be sampled within each temporal frame. In non-Cartesian imaging, such as spirals, acquiring sufficient data requires longer readouts, which can result in artifacts due to off-resonance.

In this work, we developed a new approach to highly accelerated cardiac cine MRI using low rank matrix completion, partial separability model [1-5], and the parallel imaging method GRAPPA [12]. It is a *k*-space based reconstruction method that does not require explicit navigator data. Different from conventional PS methods which continuously and fully acquire navigator data at central *k*-space to estimate the temporal basis, we moderately randomly undersample *k*-space data at the center *k*-space navigator locations but highly undersampled at the outer *k*-space for each temporal frame.

In the first step, the linear dependency in the *k-t* domain is exploited to reconstruct the missing navigator data. The difference, however, is that we estimate and impose the linear dependency simultaneously by organizing acquired navigator data into a structured matrix, which consists of vectorizd overlapping blocks in *k*-space. Under the assumption that the navigator data in the *k-t* domain is of locally low rank, estimating the unacquired navigator data becomes a low rank matrix completion problem. Then the temporal basis can be obtained from the few dominant right singular vectors through the SVD [5].

In the second step, we adopt the PS model which assumes the dynamic image series and the corresponding *k-t* data (Fourier domain) are spatial-temporal partially separable. Thus the *k-t* data can be represented as the product a temporal basis and spatial basis. We further assume the dynamic image series and the *k-t* data to be sparse in the temporal frequency domain to reconstruct partial Fourier data.

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In image reconstruction, *k*-space based parallel image reconstruction method GRAPPA was used to get the final image series. The proposed method has shown to achieve high quality, artifacts-free reconstructions with temporal resolution up to 29ms (reduction factor 31), when conventional PS method fails.

2. PROPOSED METHOD

2.1 Problem formulation

The main goal is to recover the dynamic image sequence $\gamma_\ell(\mathbf{x},t)$ from highly under-sampled Fourier measurements acquired from multiple channels of phased array coils, where $\mathbf{x}$ and t represent spatial location and time respectively, ℓ counts all channels. The sequence can be represented as the $M\times N$ Casorati matrix:

$$\mathbf{\Gamma}_\ell = \begin{pmatrix} \gamma_\ell(x_1,t_1) & \cdots & \gamma_\ell(x_1,t_N) \\ \gamma_\ell(x_2,t_1) & \cdots & \gamma_\ell(x_2,t_N) \\ . & \cdots & . \\ \gamma_\ell(x_M,t_1) & \cdots & \gamma_\ell(x_M,t_N) \end{pmatrix} \in \mathbb{C}^{M\times N} \tag{1}$$

Here, M is the number of voxels in the image and N is the number of image frames in the dataset. The columns of $\mathbf{\Gamma}_\ell$ correspond to the voxels of each time frame of the ℓ-th channel. Assuming that D samples are acquired in *k*-space from each channel at each time, the imaging equation can be written as:

$$\mathbf{d}_\ell = \mathbf{\Omega F \Gamma}_\ell + \boldsymbol{e}_\ell \tag{2}$$

where $\mathbf{d}_\ell \in \mathbb{C}^{D\times N}$ and $\boldsymbol{e}_\ell \in \mathbb{C}^{D\times N}$ are the measured data matrix in ($\boldsymbol{k}$, t)-space and noise matrix at the ℓ-th channel respectively; $\mathbf{F}$ is the 2D Fourier operator, and $\mathbf{\Omega}$ is the under sampling operator. The measured data from all channels can be combined together to form the complete imaging equation that represents the entire encoding process in space, and time. In highly accelerated dynamic imaging, Eq. (2) is highly underdetermined and thus solving Eq. (2) for $\mathbf{\Gamma}_\ell$ is highly ill-posed. To simplify Eq. (2), we define $\mathbf{\Gamma}_{\mathbf{k},\ell}=\mathbf{F\Gamma}_\ell$, and exploit the following model and constraints.

PS model on dynamic data

The PS model assumes $\gamma_\ell(\mathbf{x},t)$ to be spatial-temporal partially separable [3, 4, 5]. Then $\mathbf{\Gamma}_\ell$ can be represented as the product of a spatial coefficient matrix $\mathbf{U}_s$ and a temporal basis $\mathbf{V}_t$:

$$\mathbf{\Gamma}_\ell^{M\times N} = \mathbf{U}_{\mathbf{s},\ell}^{M\times R}\mathbf{V}_t^{R\times N}, \tag{3}$$

where R is the order of the PS model, or rank of $\mathbf{\Gamma}_\ell$. **The PS model is able to capture spatial-temporal correlation often observed in dynamic image sequences. The higher the order R is, the less the spatial-temporal correlation is present in the image sequence. The PS model can also be applied in the (k, t) domain, since the 2D Fourier operator is linear:**

$$\mathbf{\Gamma}_{\mathbf{k},\ell}^{M\times N} = \mathbf{U}_{\mathbf{k},\ell}^{M\times R}\mathbf{V}_t^{R\times N},$$
$$\mathbf{d}_\ell = \mathbf{\Omega}(\mathbf{U}_{\boldsymbol{k},\ell}\mathbf{V}_t) + \boldsymbol{e}_\ell,$$

where $\mathbf{U}_{\boldsymbol{k},\ell}$ is the spatial coefficient matrix of the ℓ-th channel, and $\mathbf{U}_{\boldsymbol{k},\ell} = \mathbf{F}\mathbf{U}_{\boldsymbol{s},\ell}$.

Sparse constraint on temporal frequency

We further assume the dynamic image sequences present slow variations in time [5, 11]. Therefore the signal in the spatial and temporal frequency domain is sparse. That is, $\mathbf{\Gamma}_\ell\mathbf{F}_t$ has very few significant elements, where $\mathbf{F}_t$ represents the temporal Fourier transform.

Shift-invariant property and Locally Low rank

Locally low rank in either image or *k*-space has been used in parallel imaging reconstruction [13, 14], dynamic imaging [15, 16], and parameter mapping [17]. In *k*-space, the data points are of locally low rank [13]. Such property has been used in several parallel MR image reconstruction methods [13, 18, 19]. Here, we assume the navigator data in each time frame is of locally low rank (as shown in Fig. 1). A small sliding window is applied on the navigator lines in nth time frame. The data points inside the window are vectored to a row and a low rank matrices X_n are generated as the sliding window shifts to generate different rows. After we stack all matrices $\mathrm{X}_1, \mathrm{X}_2, \ldots, \mathrm{X}_N$ from different time frames, a big rank deficient matrix $\mathbf{X} = \begin{bmatrix} \mathrm{X}_1 \\ \vdots \\ \mathrm{X}_N \end{bmatrix}$ is formed.

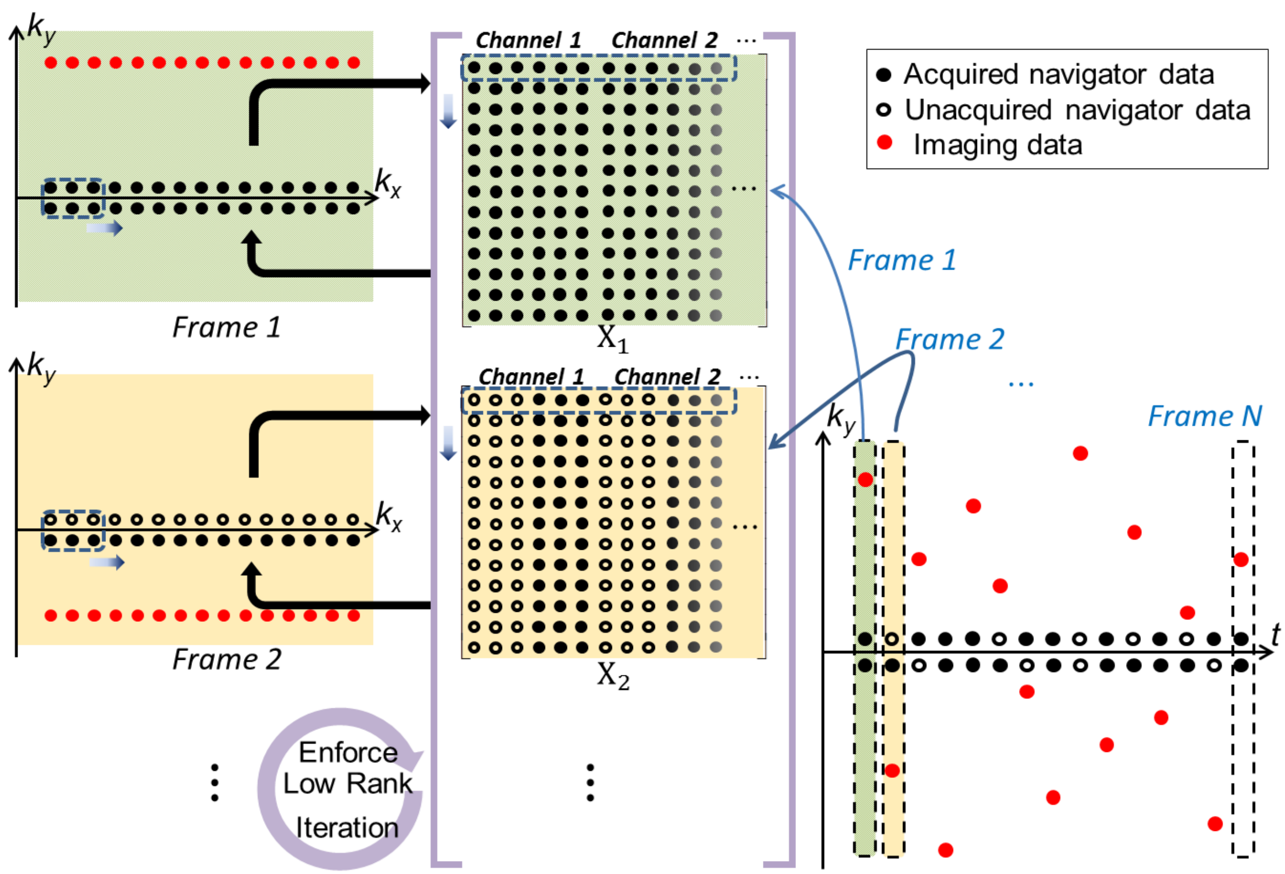

Fig.1 Illustration of how navigator data and imaging data are sampled in the (k-t) space for a dynamic object.

2.2 Image reconstruction algorithm

Estimation of $\mathbf{V}_t$

Since the Casorati matrix $\mathbf{\Gamma}_{\mathbf{k},\ell}$ (with each column corresponding to one time frame) of the k-t data is of low rank, the block-wise Hankel matrix of the navigator data X also presents low rankness (Each block X_n corresponds to some linear transform of the nth image frame). We enforce the low rankness of matrix X iteratively as we update the unknowns in X. As a result, estimating the unacquired navigator data becomes a low rank matrix completion problem.

After all missing navigator data being calculated, the temporal basis V_t can be obtained from the R dominant right singular vectors through the singular value decomposition (SVD) of the navigator data [3]. Selection of R often needs to balance the representation capability of the model and the numerical condition of the resulting model fitting problem. When R is too low, the model may fail to capture some temporal features, although the corresponding model fitting problem is often well-conditioned. When R is too high, the model fitting problem becomes under-determined, which can amplify modeling errors and increase computation complexity.

Calculation of U_k

We further assume the dynamic image series to be sparse in the spatial and temporal frequency domain [5, 13]. $\mathbf{U}_{k,l}$ is calculated by the following optimization function: $\widehat{\mathbf{U}}_{k,l} = \operatorname{argmin}\left\|\mathbf{d}_l - \boldsymbol{\Omega}\mathbf{U}_{k,l}\mathbf{V}_t\right\|_2^2 + \lambda\left\|vec(\mathbf{U}_{k,l}\mathbf{V}_t\mathbf{F}_t)\right\|_1$, where $\mathbf{d}_l$ represents the undersampled *k*-space data, Ω represents an operator which samples the data with a specified undersampling trajectory in (k, *t*)-space, F_t is the temporal Fourier transform, and the L_1 norm term enforces the sparsity constraint in spatial and temporal frequency domain.

Parallel Imaging

To further accelerate acquisition in the *k-t* space, GRAPPA is exploited on top of the above-mentioned method. Specifically, the sampling pattern in k-t space is actually further random undersampling of the uniformly undersampled GRAPPA data, similar to CS-SENSE [20]. After the navigator data is recovered (Step 1 in Fig. 2), the PS model is used to reconstruct the k-t space data for reduced field of view only (i.e., the *k*-space is uniformly undersampled), as shown in Fig. 2, Step 2. Then in Step 3, GRAPPA is applied to reconstruct the rest missing data for the full field of view. We use RP-GRAPPA [21] in this step, which saves more computation time than GRAPPA.

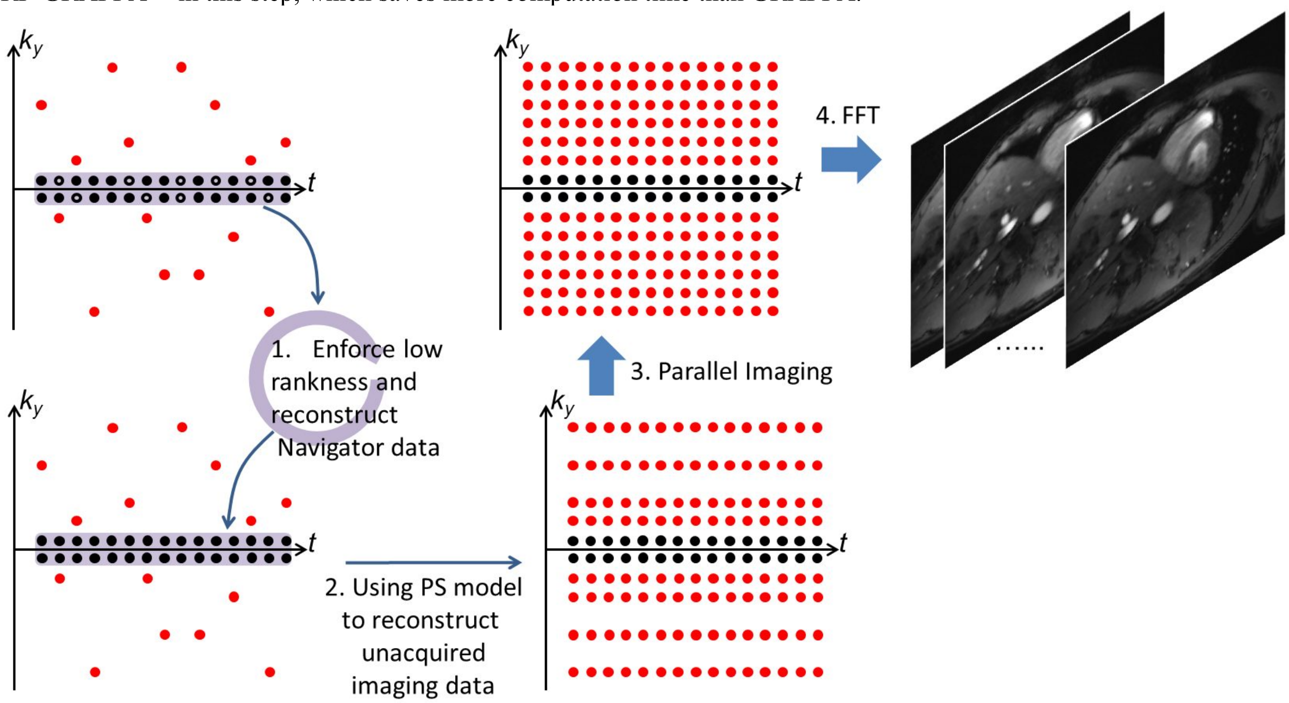

Fig. 2 Flowchart of the proposed method

3. RESULTS

In this section, we show a set of representative results from in vivo cardiac cine MRI experiments to evaluate the performance of the proposed method. The dataset was acquired with a 12-channel cardiac coil using a special designed sequence (TR=5.8 ms; matrix size = 156x192; field of view [FOV] = 284mm x 350mm). The data was continuously sampled with a variable density, where 156 phase encoding lines full sample the k_x-k_y plane.

The central 5 phase encoding lines in the (*k*, *t*)-space were considered as navigation locations. The *k*-space data in the navigation location has a random undersampling factor of 2. The *k*-space data outside the navigation location is randomly undersampled with a much higher reduction factor (only 2 and 3 lines at each frame, corresponding to net reduction factors of 31), where the composite *k*-space data from all frames are undersampled uniformly with a reduction factor of 2.

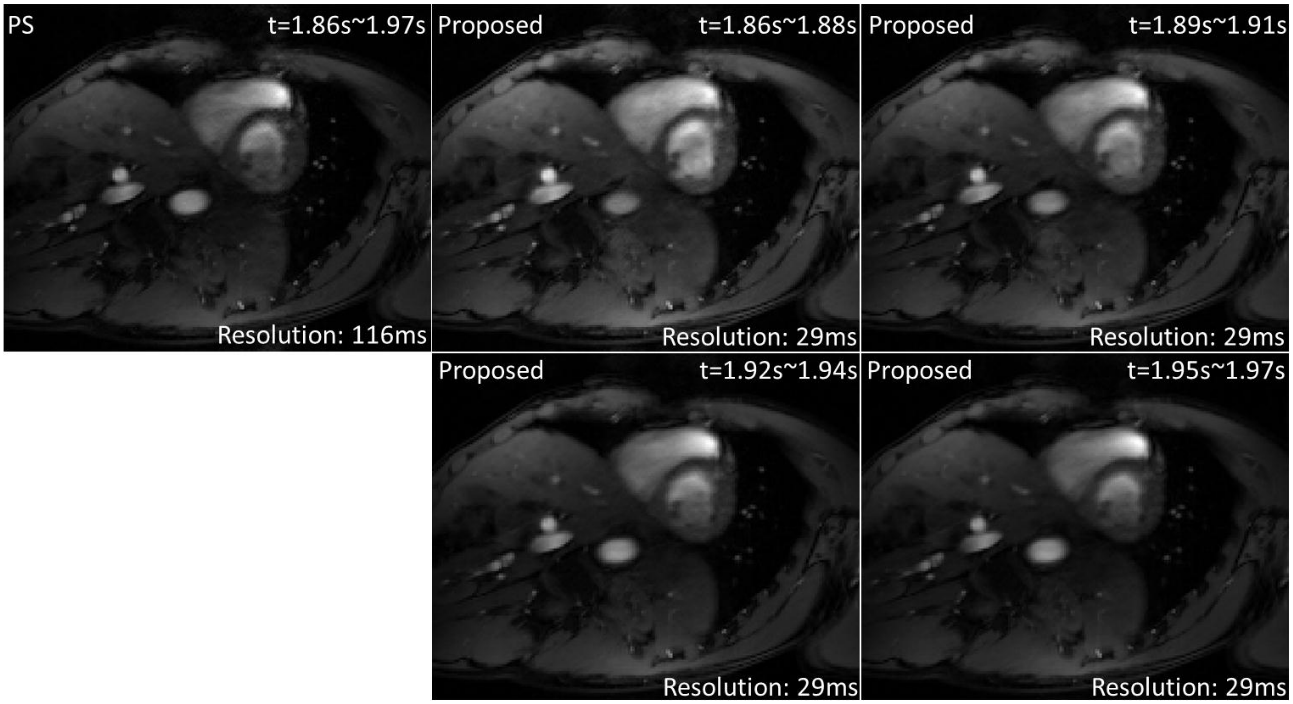

Fig.3a Reconstruction results from PS and the proposed method. 20 PE lines were used to for each frame so as to so as to include enough navigator data for PS model, while only 5 PE lines were used for the proposed method.

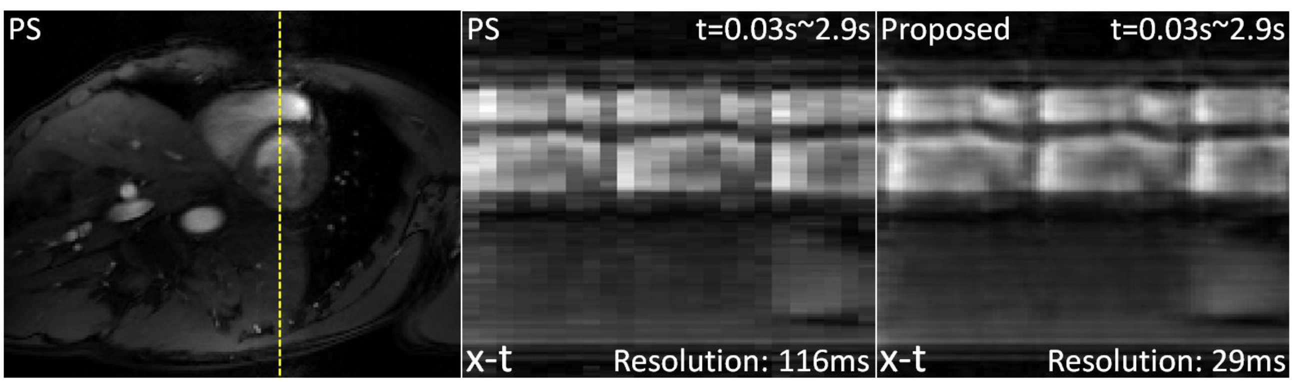

Fig. 3b Comparison of reconstruction results from PS and the proposed method in x-t domain

As can be seen in Fig. 3a, the reconstruction results from the proposed method are clean without any aliasing artifacts. For the 116 ms period, movements in the cardiac region can be clearly seen from the proposed reconstruction method, where the PS model fails due to the lower temporal resolution. In Fig. 3b, the ROI temporal profiles from both methods are shown. It is seen that the results from proposed method has a higher temporal resolution while containing much more morphological details at the same time.

4. CONCLUSION

We have proposed an approach to reconstruct highly undersampled dynamic MRI with high spatial and temporal resolution. The approach effectively exploits locally low rankness within the navigator region to recover the full from undersampled navigator data, and combines PS with the sparsity constraint and GRAPPA in the same framework. The proposed method has shown to achieve high quality, artifacts-free reconstructions with reduction factors up to 31 and temporal resolution up to 29 ms, where the conventional PS method fails.

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