MULTISENSOR DETECTION OF IMPROPER SIGNALS IN IMPROPER NOISE

Jitendra K. Tugnait

Department of Electrical & Computer Engineering
Auburn University, Auburn, AL 36849, USA
tugnajk@eng.auburn.edu

ABSTRACT

This paper addresses the problem of detecting the presence of a complex-valued, possibly improper, but unknown signal, common among two or more sensors (channels) in the presence of spatially independent, unknown, possibly improper and colored, noise. Past work on this problem is limited to signals observed in proper noise. A source of improper noise is IQ imbalance during down-conversion of bandpass noise to baseband. A binary hypothesis testing approach is formulated and a generalized likelihood ratio test (GLRT) is derived using the power spectral density estimator of an augmented sequence. An analytical solution for calculating the test threshold is provided. The results are illustrated via simulations.

***Index Terms*—** Multichannel signal detection; improper signals; generalized likelihood ratio test (GLRT)

1. INTRODUCTION

We consider the problem of detecting the presence of a complex-valued, possibly improper, but unknown signal, common among two or more sensors. The unknown common signal is observed at multiple sensors in the presence of unknown, possibly improper and colored, noise that is independent across sensors.

A zero-mean complex-valued random sequence is called proper if the cross-correlation function of the sequence with its complex conjugate (called complementary correlation) is vanishing [1]. Quite often, algorithms for complex signal processing in communications and statistical signal processing have been derived assuming that the complex signals are proper [1, 2]. However, this assumption of propriety is often not justified. For example, BPSK, offset QPSK, GMSK and ASK modulation based signals are improper [1]. Also, in-phase/quadrature-phase (IQ) imbalance during down-conversion of bandpass signals to baseband can result in impropriety in both signals and noise [3]. If the underlying signals are improper, much can be gained in performance if the information contained in the complementary correlation is also exploited [1, 4, 5].

A potential application of this problem is in spectrum sensing for cognitive radio (for other potential applications see [6]) to decide if the received signal, in addition to noise, contains signals from a single or multiple primary users (PUs). This is formulated as a binary hypothesis testing problem and is a well-investigated topic [7]. A wide variety of approaches exist based on differing signal and noise models [7]. A widely used model is that of temporally white but spatially correlated proper complex Gaussian PU signal in temporally and spatially uncorrelated proper complex Gaussian noise [8]. Temporally colored, proper signals in spatially uncorrelated but temporally correlated Gaussian noise have been considered in [9] assuming multiple independent realizations (snapshots) and Gaussian PU signals, whereas only one data realization is needed in [10]. [6] is an extension of [9]. The model of [11] is limited to temporally white but spatially correlated improper complex Gaussian PU signal in temporally and spatially uncorrelated proper complex Gaussian noise whereas [12] allows temporal correlation for both improper signal and proper noise. Both show improved performance compared to the case where improper signals are treated as proper.

Relation to Prior Work: The model of [12] is limited to improper signals in spatially independent proper noise. In this paper we allow noise to be improper also.

Contributions: A binary hypothesis testing approach is formulated and GLRT is derived using the power spectral density (PSD) estimator of an augmented sequence. An asymptotic analytical solution for calculating the test threshold is provided. The results are illustrated via computer simulations.

Notation: We use $\mathbf{S} \succeq 0$ and $\mathbf{S} \succ 0$ to denote that Hermitian $\mathbf{S}$ is positive semi-definite and positive definite, respectively. For a square matrix $\mathbf{A}$, $|\mathbf{A}|$ and $\mathrm{etr}(\mathbf{A})$ denote the determinant and the exponential of the trace of $\mathbf{A}$, respectively, i.e., $\mathrm{etr}(\mathbf{A}) = \exp(\mathrm{tr}(\mathbf{A}))$, $\mathbf{B}_{k;i:l,j:m}$ denotes the submatrix of the matrix $\mathbf{B}_k$ comprising its rows i through l and columns j through m, $\mathbf{B}_{k;ij}$ is its ijth element, and $\mathbf{I}$ is the identity matrix. The superscripts $*$, T and H denote the complex conjugate, transpose and the Hermitian (conjugate transpose) operations, respectively.

2. SYSTEM MODEL

Let $p \times 1$ $\mathbf{n}(t)$ denote a zero-mean spatially independent, stationary, possibly improper, random sequence (noise) and $p \times 1$ $\mathbf{s}(t)$ denote a zero-mean stationary, possibly improper random sequence (signal) which is independent of $\{\mathbf{n}(t)\}$. Both noise and signal may be non-Gaussian. Let $\mathcal{H}_0$ denote the null hy-

This work was supported by NSF Grant CCF-1617610.

pothesis that the user is receiving just noise, and $\mathcal{H}_1$ is the alternative that signal common to all sensors is also present. We consider the following binary hypothesis testing problem for the measurement sequence $\mathbf{x}(t)$:

$$\begin{array}{ll} \mathcal{H}_0 : & \mathbf{x}(t) = \mathbf{n}(t), \text{ noise only} \\ \mathcal{H}_1 : & \mathbf{x}(t) = \mathbf{s}(t) + \mathbf{n}(t), \text{ signal and noise.} \end{array} \quad (1)$$

We assume that noise is independent across sensors.

A stationary complex zero-mean process $\{\mathbf{x}(t)\}$ of dimension p is said to be proper [1] if its matrix complementary correlation (covariance) function $\tilde{\mathbf{R}}_{xx}(\tau)$ vanishes, i.e.,

$$\tilde{\mathbf{R}}_{xx}(\tau) = E\{\mathbf{x}(t+\tau)\mathbf{x}^T(t)\} = 0, \ \tau = 0, \pm 1, \cdots, \quad (2)$$

where $\mathbf{x}(t) = \mathbf{x}_r(t) + j\mathbf{x}_i(t)$, with $\mathbf{x}_r(t)$ and $\mathbf{x}_i(t)$ denoting its real and imaginary components, respectively. Define $\mathbf{R}_{xx}(\tau) = E\{\mathbf{x}(t+\tau)\mathbf{x}^H(t)\}$, the conventional matrix correlation function. The PSD $\mathbf{S}_x(f)$ of $\{\mathbf{x}(t)\}$ is the Fourier transform of $\mathbf{R}_{xx}(\tau)$, $\mathbf{S}_x(f) = \sum_{\tau=-\infty}^{\infty} \mathbf{R}_{xx}(\tau) e^{-j2\pi f\tau}$, whereas the complementary PSD (C-PSD) $\tilde{\mathbf{S}}_x(f)$ of $\{\mathbf{x}(t)\}$ is $\tilde{\mathbf{S}}_x(f) = \sum_{\tau=-\infty}^{\infty} \tilde{\mathbf{R}}_{xx}(\tau) e^{-j2\pi f\tau}$. Clearly, for a proper process, the C-PSD vanishes.

We observe $\mathbf{x}(t)$ for $t = 0, 1, \cdots, N-1$ (N samples). Since $\mathbf{s}(t)$ is assumed to be improper, we will exploit both PSD and C-PSD. Define the augmented complex process $\{\mathbf{y}(t)\}$ and the real-valued process $\{\mathbf{z}(t)\}$ as

$$\mathbf{y}(t) = \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{x}^*(t) \end{bmatrix}, \quad \mathbf{z}(t) = \begin{bmatrix} \mathbf{x}_r(t) \\ \mathbf{x}_i(t) \end{bmatrix}. \quad (3)$$

We assume that $\{\mathbf{z}(t)\}$ satisfies Assumption 2.6.1 of [13] so that some asymptotic results from [13] regarding PSD estimators can be invoked; the time series need not be Gaussian but its moments of all orders should be finite.

Consider the finite Fourier transform (FFT) $\mathbf{d}_y(f_n)$ of $\mathbf{y}(t)$, $t = 1, 2, \cdots, N-1$, given by

$$\mathbf{d}_y(f_n) := \sum_{t=0}^{N-1} \mathbf{y}(t) e^{-j2\pi f_n t} \quad (4)$$

where $f_n = n/N$, $n = 0, 1, \cdots, N-1$. Then the estimator of the PSD of $\mathbf{y}(t)$ at frequency f_n, based on the Daniell method, is given by

$$\hat{\mathbf{S}}_y(f_n) = \frac{1}{K} \sum_{l=-m_t}^{m_t} \left(N^{-1} \mathbf{d}_y(f_{n+l}) \mathbf{d}_y^H(f_{n+l}) \right) \quad (5)$$

where $K = 2m_t + 1$ is the smoothing window size. Based on [13, Theorem 7.3.3], it is shown in [12] that $\hat{\mathbf{S}}_y(f_n)$ is asymptotically ("large" N) distributed as $W_C\left(2p, K, K^{-1}\mathbf{S}_y(f_n)\right)$ (denoted as $\overset{a}{\sim}$) where $W_C\left(2p, K, K^{-1}\mathbf{S}_y(f_n)\right)$ denotes the complex Wishart distribution of dimension $2p$, degrees of freedom K, and mean value $\mathbf{S}_y(f_n)$, and we exclude $n = 0, N/2$ on the right-side of (5). If a random matrix $\mathbf{X} \sim W_C\left(p, K, \mathbf{S}(f)\right)$, then by [13, Sec. 4.2], $E\{\mathbf{X}\} = K\mathbf{S}(f)$, $\text{cov}\{\mathbf{X}_{jk}, \mathbf{X}_{lm}\} = K\mathbf{S}_{jl}(f)\mathbf{S}^*_{km}(f)$, and for $K \geq p$, the probability density function (pdf) of $\mathbf{X}$ is given by

$$f_{\mathbf{X}}(\mathbf{X}) = \frac{1}{\Gamma_p(K)} \frac{1}{|\mathbf{S}(f)|^K} |\mathbf{X}|^{K-p} \operatorname{etr}\{-\mathbf{S}^{-1}(f)\mathbf{X}\} \quad (6)$$

where the pdf (6) is defined for $\mathbf{S}(f) \succ 0$ and $\mathbf{X} \succeq 0$, and is otherwise zero, and $\Gamma_p(K) := \pi^{p(p-1)/2} \prod_{j=1}^{p} \Gamma(K-j+1)$ where $\Gamma(n)$ denotes the (complete) Gamma function $\Gamma(z) := \int_0^\infty t^{z-1} e^{-t}\, dt$.

We will confine our attention to the frequency points over which the spectral estimators are approximately mutually independent, which for the Daniell method are given by

$$\tilde{f}_k = \frac{(k-1)K + m_t + 1}{N}, \ 1 \leq k \leq M = \left\lfloor \frac{\frac{N}{2} - m_t - 1}{K} \right\rfloor. \quad (7)$$

Let $\mathcal{M} := \{\tilde{f}_k : 1 \leq k \leq M\}$ denote the set of M frequency bins as in (7) of interest.

Under $\mathcal{H}_0$, the ℓth component $\mathbf{x}_\ell(t)$ of $\mathbf{x}(t)$ is independent of $\mathbf{x}_m(t)$ for $\ell \neq m$. Let ($\ell = 1, 2, \cdots, p$)

$$\mathbf{S}_y^{(\ell)}(f) := \begin{bmatrix} \mathbf{S}_{y;\ell\ell}(f) & \mathbf{S}_{y;\ell(\ell+p)}(f) \\ \mathbf{S}_{y;(\ell+p)\ell}(f) & \mathbf{S}_{y;(\ell+p)(\ell+p)}(f) \end{bmatrix}. \quad (8)$$

Then in terms of $\mathbf{S}_x$ and $\tilde{\mathbf{S}}_x$,

$$\mathbf{S}_y^{(\ell)}(f) = \begin{bmatrix} \tilde{\mathbf{S}}_{x;\ell\ell}(f) & \tilde{\mathbf{S}}_{x;\ell\ell}(f) \\ \tilde{\mathbf{S}}^*_{x;\ell\ell}(-f) & \mathbf{S}^*_{x;\ell\ell}(-f) \end{bmatrix}. \quad (9)$$

Under $\mathcal{H}_0$, all entries in $\mathbf{S}_y(f)$ are zeros except for those in $\mathbf{S}_y^{(\ell)}(f)$, $\ell = 1, 2, \cdots, p$. Under $\mathcal{H}_1$, $\mathbf{x}(t)$ is improper with $\mathbf{S}_y(f) \succeq 0$ with no specific structure. Testing for the presence of an improper common signal in spatially independent improper noise is then reformulated as the problem:

$$\begin{array}{ll} \mathcal{H}_0 : & \mathbf{S}_{y;\ell m}(\tilde{f}_k) = 0 \text{ except for } \mathbf{S}_y^{(\ell)}(\tilde{f}_k) \\ & \quad \ell = 1, 2, \cdots, p \ \forall \tilde{f}_k \in \mathcal{M} \\ \mathcal{H}_1 : & \mathbf{S}_y(\tilde{f}_k) \succ 0 \text{ with no specific structure } \forall \tilde{f}_k \in \mathcal{M}. \end{array} \quad (10)$$

We assume that $\mathbf{S}_y(f) \succ 0$ for any f. Otherwise, one can add artificial proper white Gaussian noise to $\mathbf{x}(t)$ to achieve $\mathbf{S}_y(f) \succ 0$.

3. PSD-BASED GLRT

In this section we derive the GLRT. We will denote the spectral estimator at the k-th frequency bin $\tilde{f}_k$ (see (7)), acquired from $\{\mathbf{y}(t)\}_{t=0}^{N-1}$, via (5), as $\mathbf{Y}_k$. We have

$$\mathbf{Y}_k \overset{a}{\sim} W_C\left(2p, K, K^{-1}\mathbf{S}_y(\tilde{f}_k)\right) \quad (11)$$

and $\mathbf{Y}_k$s are mutually independent for $k \in [1, M]$. The joint pdf of $\mathbf{Y}_k$ for $\tilde{f}_k \in \mathcal{M}$ under $\mathcal{H}_0$ is maximized w.r.t. $\mathbf{S}_y^{(\ell)}(\tilde{f}_k)$ for $\hat{\mathbf{S}}_y^{(\ell)}(\tilde{f}_k) = \mathbf{Y}_k^{(\ell)}$ where

$$\mathbf{Y}_k^{(\ell)} := \begin{bmatrix} \mathbf{Y}_{k;\ell\ell} & \mathbf{Y}_{k;\ell(\ell+p)} \\ \mathbf{Y}_{k;(\ell+p)\ell} & \mathbf{Y}_{k;(\ell+p)(\ell+p)} \end{bmatrix} \in \mathbb{C}^{2\times 2}. \quad (12)$$

Under $\mathcal{H}_1$, the joint pdf of $\mathbf{Y}_k$ for $k \in [1, M]$ is maximized w.r.t. the Hermitian matrix $\mathbf{S}_y(\tilde{f}_k)$ for $\hat{\mathbf{S}}_y(\tilde{f}_k) = \mathbf{Y}_k$. Define $\mathcal{Y} = \{\mathbf{Y}_k, k \in \mathcal{M}\}$. Then one gets the GLRT

$$\mathcal{L} := \frac{f_{\mathcal{Y}}(\mathbf{Y}_k, k \in \mathcal{M} | \mathcal{H}_1, \hat{\mathbf{S}}_y(\tilde{f}_k))}{f_{\mathcal{Y}}(\mathbf{Y}_k, k \in \mathcal{M} | \mathcal{H}_0, \hat{\mathbf{S}}_y^{(\ell)}(\tilde{f}_k), \ell \in [1,p])} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \tau_1 \quad (13)$$

where the threshold τ_1 is picked to achieve a pre-specified probability of false alarm $P_{fa} = P\{\mathcal{L} \geq \tau_1 \,|\, \mathcal{H}_0\}$. This requires pdf of $\mathcal{L}$ under $\mathcal{H}_0$ which is discussed in Sec. 4. Simplifying, one obtains

$$\mathcal{L} = \prod_{k=1}^{M} \mathcal{L}_k, \quad \mathcal{L}_k := \frac{\prod_{\ell=1}^{p} |\mathbf{Y}_k^{(\ell)}|^K}{|\mathbf{Y}_k|^K} \quad (14)$$

Invariance of GLRT: Note that $\mathcal{L}_k$ is invariant to transformation $\mathbf{Y}_k^{(\ell)} \rightarrow \mathbf{A}_k^{(\ell)} \mathbf{Y}_k^{(\ell)} \mathbf{A}_k^{(\ell)H}$ for any non-singular Hermitian $\mathbf{A}_k^{(\ell)} \in \mathbb{C}^{2\times 2}$, leaving the other entries of $\mathbf{Y}_k$ unchanged. In particular, by choosing $\mathbf{A}_k^{(\ell)} = \sqrt{K}(\mathbf{S}_y^{(\ell)}(\tilde{f}_k))^{-1/2}$ for $\ell \in [1,p]$ we can transform any $\mathbf{Y}_k$ to $\tilde{\mathbf{Y}}_k$ such that $\tilde{\mathbf{Y}}_k^{(\ell)} \sim W_C(2, K, \mathbf{I})$ and $\tilde{\mathbf{Y}}_k \sim W_C(2p, K, \mathbf{I})$ under $\mathcal{H}_0$. Then $\mathcal{L}$ is invariant.

4. THRESHOLD SELECTION

We now turn to determination of an asymptotic expansion of the distribution of $\mathcal{L}$ under $\mathcal{H}_0$ following [14, 15, 16]. First we need the following result:
Lemma 1 : Under $\mathcal{H}_0$, for any $h = 0, 1, 2, \cdots$, $E\{\frac{1}{\mathcal{L}^h} \,|\, \mathcal{H}_0\}$

$$= \frac{\prod_{k=1}^{2} \Gamma^{Mp}(K-k+1)}{\prod_{j=1}^{2p} \Gamma^{M}(K-j+1)} \frac{\prod_{k=1}^{2p} \Gamma^{M}(K(1+h)-k+1)}{\prod_{j=1}^{2} \Gamma^{Mp}(K(1+h)-j+1)} \quad (15)$$

Proof : Using the transformation specified in Sec. 3 to obtain $\tilde{\mathbf{Y}}_k \sim W_C(2p, K, \mathbf{I})$ under $\mathcal{H}_0$, we have

$$E\{1/\mathcal{L}_k^h \,|\, \mathcal{H}_0\} = \int \frac{|\tilde{\mathbf{Y}}_k|^{Kh+K-2p}}{\prod_{\ell=1}^{p} |\tilde{\mathbf{Y}}_k^{(\ell)}|^{Kh}} \frac{\mathrm{etr}\{-\tilde{\mathbf{Y}}_k\}}{\Gamma_{2p}(K)} d\tilde{\mathbf{Y}}_k$$
$$= \frac{\Gamma_{2p}(K+Kh)}{\Gamma_{2p}(K)} E\left\{ |\tilde{\mathbf{Y}}_k'^{(\ell)}|^{-Kh} \right\}, \quad (16)$$

where $\tilde{\mathbf{Y}}_k' \sim W_C(2p, K(1+h), \mathbf{I})$. Hence $\tilde{\mathbf{Y}}_k'^{(\ell)}$s (defined similar to (12)) are independent for $\ell \in [1,p]$ and $\tilde{\mathbf{Y}}_k'^{(\ell)} \sim W_C(2, K(1+h), \mathbf{I})$. Since (see [18, Theorem 3.8, p. 51])

$$E\{|\tilde{\mathbf{Y}}_k'^{(\ell)}|^{-Kh}\} = E\{(\prod_{m=1}^{2} (1/2)V_m)^{-Kh}\}, \quad (17)$$

$V_m \sim \chi^2_{2(K(1+h)-m+1)}$ and are independent, and (see [14, p. 101])

$$E\{W^r\} = \frac{2^r \Gamma((n/2)+r)}{\Gamma((n/2))} \quad \text{for} \quad W \sim \chi_n^2, \quad (18)$$

we obtain $\forall \ell \in [1,p]$

$$E\left\{ |\tilde{\mathbf{Y}}_k'^{(\ell)}|^{-Kh} \right\} = \frac{\prod_{m=1}^{2} \Gamma(K+1-m)}{\prod_{m=1}^{2} \Gamma(K(1+h)+1-m)}. \quad (19)$$

Now using $\Gamma_p(K) := \pi^{p(p-1)/2} \prod_{j=1}^{p} \Gamma(K-j+1)$, (14), (16) and (19), we get the desired result. $\square$

In order to exploit Lemma 2 (stated next), we need to establish that $0 \leq \mathcal{L}^{-1} \leq 1$. Since $\mathbf{Y}_k \succ 0$ (hence $\mathbf{Y}_k^{(\ell)} \succ 0$ $\forall \ell$), $\mathcal{L}^{-1} \geq 0$ follows immediately. By Fischer's inequality [17, p. 477], we have $|\mathbf{Y}_k| \leq \prod_{\ell=1}^{p} |\mathbf{Y}_k^{(\ell)}|$ which implies $\mathcal{L}^{-1} \leq 1$. The following result follows from [14, Sec. 8.2.4], [15, Sec. 8.5.1]:
Lemma 2 : Consider a random variable W ($0 \leq W \leq 1$) with the hth moment ($h = 0, 1, 2, \cdots$)

$$E\{W^h\} = C \left(\frac{\prod_{j=1}^{b} y_j^{y_j}}{\prod_{k=1}^{a} x_k^{x_k}} \right)^h \frac{\prod_{k=1}^{a} \Gamma(x_k(1+h)+\xi_k)}{\prod_{j=1}^{b} \Gamma(y_j(1+h)+\eta_j)}, \quad (20)$$

where a and b are integers, C is a constant such that $E\{W^0\} = 1$ and $\sum_{k=1}^{a} x_k = \sum_{j=1}^{b} y_j$. Let $B_r(h)$ denote the Bernoulli polynomial of degree r and order unity. Define
$\nu = -2\left[\sum_{k=1}^{a} \xi_k - \sum_{j=1}^{b} \eta_j - \frac{1}{2}(a-b)\right]$, $\rho = 1 - \frac{1}{\nu}\left[\sum_{k=1}^{a} x_k^{-1}\left(\xi_k^2 - \xi_k + \frac{1}{6}\right) - \sum_{j=1}^{b} y_j^{-1}\left(\eta_j^2 - \eta_j + \frac{1}{6}\right)\right]$,
$\beta_k = (1-\rho)x_k$, $\quad \epsilon_j = (1-\rho)y_j$ and
$\omega_r = \frac{(-1)^{r+1}}{r(r+1)}\left\{\sum_{k=1}^{a} \frac{B_{r+1}(\beta_k+\xi_k)}{(\rho x_k)^r} - \sum_{j=1}^{b} \frac{B_{r+1}(\epsilon_j+\eta_j)}{(\rho y_j)^r}\right\}$.
Then with χ_n^2 denoting a random variable with central chi-square distribution with n degrees of freedom (as well as the distribution itself),

$$\begin{aligned} P\{-2\rho \ln(W) \leq z\} &= P\{\chi_\nu^2 \leq z\} + \omega_2 \left[P\{\chi_{\nu+4}^2 \leq z\} \right. \\ &\left. -P\{\chi_\nu^2 \leq z\}\right] + \omega_3 \left[P\{\chi_{\nu+6}^2 \leq z\} - P\{\chi_\nu^2 \leq z\}\right] \\ &+ \Big\{ \omega_4 \left[P\{\chi_{\nu+8}^2 \leq z\} - P\{\chi_\nu^2 \leq z\}\right] + \\ &\frac{1}{2}\omega_2^2 \left[P\{\chi_{\nu+8}^2 \leq z\} - 2P\{\chi_{\nu+4}^2 \leq z\} + P\{\chi_\nu^2 \leq z\}\right] \Big\} \\ &+ \sum_{k=1}^{a} \mathcal{O}(x_k^{-5}) + \sum_{j=1}^{b} \mathcal{O}(y_j^{-5}) \quad \bullet \end{aligned} \quad (21)$$

Comparing (20) with (15), we find the correspondence

$$\begin{aligned} &a = 2Mp, \ b = 2Mp, \ x_k = K, \\ &\xi_k = -[(k-1) \bmod (2p)] \text{ for } k = 1, 2, \cdots, a, \\ &y_j = K, \ \eta_j = -[(j-1) \bmod (2)] \text{ for } j = 1, 2, \cdots, b. \end{aligned} \quad (22)$$

Comparing Lemmas 1 and 2, we further have

$$\beta_k = (1-\rho)K \ \forall k, \quad \epsilon_j = (1-\rho)K \ \forall j. \quad (23)$$

Furthermore, one has $E\{1/\mathcal{L}^0 \,|\, \mathcal{H}_0\} = 1$. Thus, Lemma 2 is applicable with $W = 1/\mathcal{L}$ and parameters specified in (22). Using these values in Lemma 2 and simplifying, one gets

$$\nu = 4Mp(2p-1), \quad \rho = 1 - \frac{2(p+1)}{3K}, \quad (24)$$

$$\sum_{k=1}^{a} \frac{B_{r+1}(\beta_k+\xi_k)}{(\rho x_k)^r} = M \sum_{l=1}^{2p} \frac{B_{r+1}((1-\rho)K+1-l)}{(\rho K)^r}, \tag{25}$$

$$\sum_{j=1}^{b} \frac{B_{r+1}(\epsilon_j+\eta_j)}{(\rho y_j)^r} = Mp \sum_{l=1}^{2} \frac{B_{r+1}((1-\rho)K+1-l)}{(\rho K)^r}. \tag{26}$$

Therefore, we have

$$\omega_r = \frac{(-1)^{r+1}M}{r(r+1)(\rho K)^r}\Bigg\{ \left(\sum_{l=1}^{2p} B_{r+1}((1-\rho)K+1-l) \right) - p \left(\sum_{l=1}^{2} B_{r+1}((1-\rho)K+1-l) \right) \Bigg\}. \tag{27}$$

It then follows from Lemma 2 that

$$\begin{aligned} P\{2\rho \, \ln(\mathcal{L}) \le z \,|\, \mathcal{H}_0\} &= P\{\chi^2_\nu \le z\} + \omega_2 \left[P\{\chi^2_{\nu+4} \le z\} - P\{\chi^2_\nu \le z\} \right] + \omega_3 \left[P\{\chi^2_{\nu+6} \le z\} - P\{\chi^2_\nu \le z\} \right] \\ &+ \Big\{ \omega_4 \left[P\{\chi^2_{\nu+8} \le z\} - P\{\chi^2_\nu \le z\} \right] + \frac{1}{2}\omega_2^2 \left[P\{\chi^2_{\nu+8} \le z\} - 2P\{\chi^2_{\nu+4} \le z\} + P\{\chi^2_\nu \le z\} \right] \Big\} + \mathcal{O}(K^{-5}) \end{aligned} \tag{28}$$

where ω_r's are given by (25)-(27), and

$$\ln(\mathcal{L}) = K \sum_{k=1}^{M} \left(\Big[\sum_{\ell=1}^{p} \ln |\mathbf{Y}_k^{(\ell)}| \Big] - \ln(|\mathbf{Y}_k|) \right). \tag{29}$$

Theorem 1 allows us to calculate the test threshold analytically.

Theorem 1. The GLRT for (10) is given by $2\rho \, \ln(\mathcal{L}) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \tau$ where ρ and $\ln(\mathcal{L})$ are given by (24) and (29), respectively. The threshold τ is picked to achieve a pre-specified $P_{fa} = 1 - P\{2\rho \, \ln(\mathcal{L}) \le \tau \,|\, \mathcal{H}_0\}$ where $P\{2\rho \, \ln(\mathcal{L}) \le \tau \,|\, \mathcal{H}_0\}$ is given by (28) and the various needed parameters are specified in (24)-(27) •

5. SIMULATION EXAMPLES

First we investigate the efficacy of Theorem 1 in computing the GLRT threshold for a given P_{fa}. We consider p antennas (p=2,3 or 4). Let $\tilde{\mathbf{n}}_i(t)$, $i \in [1,p]$, denote p independent zero-mean white proper Gaussian sequences. We generate $\mathbf{n}_i(t) = a_I h_I(t) \circledast \tilde{\mathbf{n}}_i(t) + a_Q h_Q(t) \circledast \tilde{\mathbf{n}}_i^*(t)$ where $a_I = a_Q^* = (1+j1)/\sqrt{2}$, $\circledast$ denotes convolution, $h_I(t) = [0.3 \ 1 \ 0.3]$, $h_Q(t) = [0.4 \ 1 \ 0.5]$. Thus noise $\mathbf{n}(t)$ is spatially independent, improper Gaussian. To estimate the PSD of augmented $\mathbf{y}(t)$ for $N = 256$, we choose $m_t = 7$ leading to $K = 15$ and $M = 8$. In Fig. 1 we compare the actual P_{fa} and design P_{fa} based on 10,000 runs. It is seen that Theorem 1 is effective in accurately calculating the threshold value.

Next we show the receiver operating characteristic (ROC) curves. The noise $\mathbf{n}(t)$ is as in the previous example and the PU signal is given by $\mathbf{s}(t) = \sum_{l=0}^{4} \mathbf{h}(l) I(t-l)$ where $I(t)$ is a scalar BPSK sequence and vector channel $\mathbf{h}(l)$ is Rayleigh fading with 5 taps, equal power delay profile, mutually independent components. Thus both signal and noise are improper. The probability of detection P_d versus false-alarm rate P_{fa} results for three different SNR values and $p = 3$, based on 10,000 runs, is shown in Fig. 2; SNR is defined as ratio of the sum of signal powers at the p antennas to the sum of noise powers. In all cases we have N=256, K=15 and M=8. It is seen that performance improves with increasing SNR. The approach of [12] applied to this problem treats improper noise as signal, e.g., when designed with $P_{fa} = 0.005$, it detects the improper noise of Fig. 1 ($p = 2$) with probability 0.015; under $\mathcal{H}_0$ the test statistic of [12] is not invariant to changes in impropriety of noise.

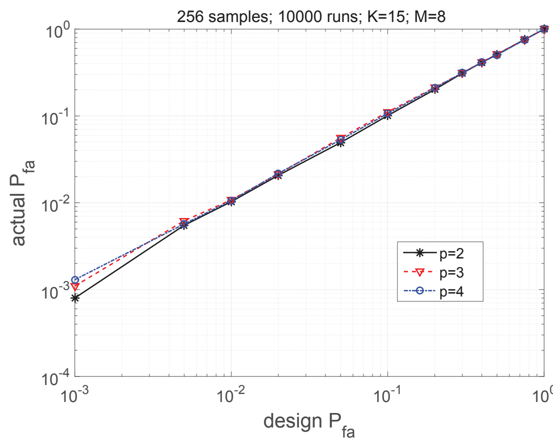

Fig. 1: Actual P_{fa} vs. design P_{fa}, $N = 256$, $K = 15$, $M = 8$

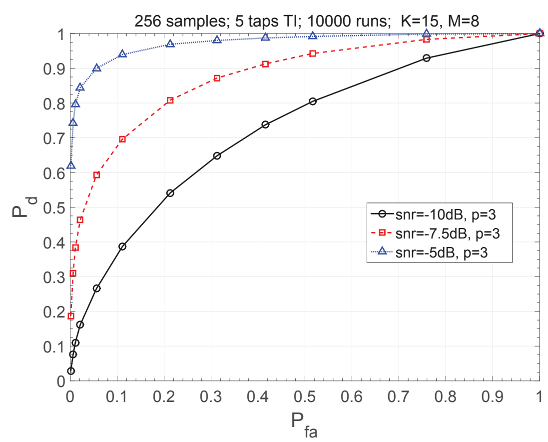

Fig. 2: ROC curve, $N = 256$, $K = 15$, $M = 8$

6. CONCLUSIONS

We investigated a PSD-based method for detection of common improper signal in improper noise. Our proposed approach is based on GLRT and it extends the approach of [12] to improper noise. A source of improper noise/signal is IQ imbalance during down-conversion of bandpass noise/signal to baseband [3]. An analytical method for calculation of the test threshold was provided and illustrated via simulations.

7. REFERENCES

[1] P.J. Schreier and L.L. Scharf, *Statistical Signal Processing of Complex-Valued Data*, Cambridge, UK: Cambridge Univ. Press, 2010.

[2] F.D. Neeser and J.L. Massey, "Proper complex random processes with applications to information theory," *IEEE Trans. Inform. Theory*, vol. 39, pp. 1293-1302, July 1993.

[3] D. Tandur and M. Moonen, "Efficient compensation of transmitter and receiver IQ imbalance in OFDM systems," *EURASIP J. Advances Signal Proc.*, Vol. 62, Article ID 106562, 14 pages, 2010.

[4] T. Adali, P.J. Schreier and L.L. Scharf, "Complex-valued signal processing: The proper way to deal with impropriety," *IEEE Trans. Signal Proc.*, vol. 59, pp. 5101-5125, Nov. 2011.

[5] D.P. Mandic and V.S.L. Goh, *Complex Valued Nonlinear Adaptive Filters*. New York: Wiley, 2009.

[6] N. Klausner, M.R. Azimi-Sadjadi and L.L. Scharf, "Detection of spatially correlated time series from a network of sensor arrays," *IEEE Trans. Signal Proc.*, Vol. 62, No. 6, pp. 1396-1407, March 2014.

[7] E. Axell, G. Leus, E.G. Larsson and H.V. Poor, "Spectrum sensing for cognitive radio: State-of-the-art and recent advances," *IEEE Signal Proc. Mag.*, vol. 29, no. 3, May 2012.

[8] R. Zhang, T.J. Lim, Y.-C. Liang and Y. Zeng, "Multi-antenna based spectrum sensing for cognitive radios: A GLRT approach," *IEEE Trans. Communications*, vol. 58, no. 1, pp. 84-88, Jan. 2010.

[9] D. Ramirez, J. Via, I. Santamaria and L.L. Scharf, "Detection of spatially correlated Gaussian time series," *IEEE Trans. Signal Proc.*, vol. 58, no. 10, pp. 5006-5015, Oct. 2010.

[10] J.K. Tugnait, "Multichannel spectrum sensing via multivariate power spectrum analysis," in *Proc. 12th IEEE Intern. Workshop on Signal Proc. Advances in Wireless Commun. (SPAWC 2011)*, pp. 106-110, San Francisco, CA, June 26-29, 2011.

[11] L. Huang, Y-H. Xiao and Q.T. Zhang, "Robust spectrum sensing for noncircular signal in multiantenna cognitive receivers," *IEEE Trans. Signal Proc.*, Vol. 63, No. 2, pp. 498-511, Jan. 2015.

[12] J.K. Tugnait, "Multiantenna spectrum sensing for improper signals over frequency selective channels," in *Proc. 2016 IEEE Intern. Conf. Acoustics, Speech & Signal Processing (ICASSP 2016)*, pp. 3781-3785, Shanghai, China, March 20-25, 2016.

[13] D.R. Brillinger, *Time Series: Data Analysis and Theory*, Expanded edition. New York: McGraw Hill, 1981.

[14] R.J. Muirhead, *Aspects of Multivariate Statistical Theory*. New York: John Wiley, 1982.

[15] T.W. Anderson, *An Introduction to Multivariate Statistical Analysis*, Third edition. Hoboken, NJ: John Wiley, Wiley-Interscience, 2003.

[16] G.E.P. Box, "A general distribution theory for a class of likelihood criteria," *Biometrika*, vol. 36, pp. 317-246, Dec. 1949.

[17] R.A. Horn and C.R. Johnson, *Matrix Analysis*. Cambridge, UK: Cambridge Univ. Press, 1985.

[18] H.H. Andersen, M. Hojbjerre, D. Sorensen, and P.S. Eriksen, *Linear and Graphical Models for the Multivariate Complex Normal Distribution*, Lecture Notes in Statistics, vol. 101. New York: Springer-Verlag, 1995.