2016 IEEE 55th Conference on Decision and Control (CDC)
ARIA Resort & Casino
December 12-14, 2016, Las Vegas, USA

To Observe or Not to Observe: Queuing Game Framework for Urban Parking

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Abstract— **We model parking in urban centers as a set of parallel queues and overlay a game theoretic structure. We model arriving drivers as utility maximizers and consider two games: one in which it is free to observe the queue length and one in which it is not. Not only do we compare the Nash induced welfare to the socially optimal welfare, confirming the usual result that Nash is worse for society, we also show that by other performance metrics more commonly used in transportation—such as occupancy and time spent circling—the Nash solution is suboptimal. We find that gains to welfare do not require everyone to observe. Through simulation, we explore a more complex scenario where drivers decide based the queueing game whether or not to enter a collection of queues over a network. Our simulated models use parameters informed by real-world data collected by the Seattle Department of Transportation.**

I. Introduction

In recent years, *congestion of surface streets* is becoming increasingly severe and is a major bottleneck of sustainable urban growth [1]. Studies indicate that parking-related congestion on arterials in U.S. cities can range from 8–74% [2]. This creates an unique opportunity for municipalities to mitigate congestion via parking. Yet, there seems to be a lack of understanding of the fundamental relationship between congestion and parking. Consequently, the problem of *smart parking* has received significant attention from both academia and government. Numerous forecasting models have been developed to predict parking availability at various timescales [3], [4] and different control stategies have been proposed to keep parking occupancy at target levels [5], [6].

Pricing, both static and dynamic, is the main tool used to control the parking system. A major difficulty in developing effective pricing strategies is the asymmetry of information between parking managers and drivers. The consequence of which is that price signals are often ignored by the drivers, leading to inefficiencies. A case in point is the parking pilot study, SFpark, conducted by the San Francisco Municipal Transportation Agency [7] in which drivers changed their behavior only after the *second* price adjustment because of a spike in awareness of the program [8]. Due to the replacement of coin-fed meters by smart meters, people are actually less cognizant of the cost of parking [9]. This motivates a key focus of this paper: in contrast to considering pricing as the main incentive, we study how information access impacts behaviors of drivers.

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This work is supported by NSF EAGER CNS:1634136

We model parking as system of parallel queues and impose a game theoretic structure on them. Each of the queues represents a street blockface. The queue itself is abstractly modeled as the roadways and circling behavior is the process of queueing. The parking spots along blockfaces are the servers in the queue model. Drivers are modeled as utility maximizers deciding whether to park based on the reward for parking versus its cost. We consider two game settings: in the first, drivers observe the queue length and thus, make an informed decision as to whether they should join the queue to find parking or balk, meaning they opt-out of parking and perhaps choose another mode of transit. In the second case, drivers choose to balk, join without observing, or pay to observe the queue after which they join or balk as in the setting of the first game.

We characterize the Nash equilibrium and the socially optimal solution in both cases and show, unsurprisingly, that there are inefficiencies in comparing the two. What is more interesting, is that we show that by other metrics—congestion (time circling) and occupancy—which are more often used by planners, the Nash solution is also inefficient and much more so than the socially optimal solution. We also develop a simulation tool that investigates how different parameter combinations such as network topology and utilization (occupancy) can impact wait time (congestion) and welfare.

The remainder the paper is organized as follows. In Section II, we outline the basic queuing framework applied to urban parking. In Sections III and IV, we describe the free observation and costly observation queuing game, respectively. We present a queue–flow network model in Section V and show through simulations the utilization and wait time for different Nash and socially optimal equilibria. Finally, in Section VI, we make concluding remarks and discuss future directions.

II. Queueing Framework

We use an M/M/c/n queue to represent a collection of block faces that collectively have an on-street parking supply of $c \geq 1$ (for background on queues see e.g. [10]). The number n represents the maximum number of customers in the system including those customers being served (i.e. parked) and those circling looking for parking. We make the following assumptions: The arriving customers form a stationary Poisson process with mean *arrival rate* $\lambda > 0$. The parking duration for a customer parks is assumed to be exponential. The servers are modeled as the $c \geq 1$ parking spots with mean *service rate* $\mu > 0$. Waiting customers are severed in the order of their arrival.

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Define the *traffic intensity* $\rho = \frac{\lambda}{c\mu}$ and let $Q_n(t)$ be the number of customers in the system at time t. Then $\{Q_n(t)\}_{t\geq 0}$ is a continuous time, ergodic Markov chain with state space $\{0, \ldots, n\}$. The stationary probability distribution of having k customers in the system is given by

$$p_k(n) = \frac{d_k}{\sum_{k=0}^{n} d_k}, \; 0 \leq k \leq n, \tag{1}$$

where

$$d_k = \begin{cases} \frac{(\rho c)^k}{k!}, & 0 \leq k \leq c-1 \\ \frac{(\rho c)^c}{c!}\rho^{k-c}, & k \geq c \end{cases} \tag{2}$$

Let $Z_k = X + Y_k$ be a random variable that measures the time spent in the system when the state of the system is k and where X is a random variable representing the service time and Y_k is a random variable representing the time that the customer spends in the queue. The random variables X and Y_k are independent, X has an exponential distribution with density $f(t) = \mu e^{-\mu t}$, and Y_k (for $k \geq s$) has a gamma distribution with density

$$g_k(t) = \frac{(c\mu)^{k-c+1}}{(k-c)!} t^{k-c} e^{-c\mu t}. \tag{3}$$

If $h(t)$ is the waiting cost to a customer spending t time units in the system, then the expected waiting cost to a customer who arrives and finds the system in state k is given by $E[h(Z_k)]$. While we can consider non-linear waiting cost functions, for simplicity we will assume that it is a linear function with constant waiting cost parameter $C_w > 0$, i.e. $h(t) = C_w t$.

We consider two game theoretic formulations overlaid on the queuing system. First, we consider the game in which arriving customers can view the queue length and then decide whether or not to join or balk. We refer to this game as the *free observation queue game*. This setting represents a ideal situation where the entire state information is available to all of the users, which is not currently achievable in practice. In the second game, we consider the setting where arriving customers do not *a priori* know the queue length. Instead, they choose to either balk, join without knowing the queue length, or pay a price to observe the queue after which they balk or join. We refer to this game as the *costly observation queue game*.

III. Free Observation Queuing Game

We first consider the observable queue game in which arriving customers know the queue length and choose to join by maximizing their utility which is a function of the reward for having parked and the cost of circling and paying for parking. The *nominal expected utility* of an arriving customer to the system in state k is $\alpha_k = R - w_k$ where $R > 0$ is the reward for parking. The *total expected utility* of a customer arriving to the system in state k is given by

$$\beta_k = \alpha_k - \frac{C_p}{\mu} = R - \frac{C_w(k+1)}{\mu c} - \frac{C_p}{\mu} \tag{4}$$

where C_p is the cost for parking. If the customer balks, the expected utility is zero.

It can be easily verified that the sequence $\{\alpha_k\}$ is decreasing and as is $\{\beta_k\}$. Furthermore, the optimal strategy for a customer finding the queue in state k and deciding whether or not to join by maximizing their expected utility is to join the queue if and only if $\beta_k \geq 0$. In this case, if the decision to join the queue depends on the customer optimizing their individual utility, then the system will be a M/M/c/n_b where

$$n_b = \left\lfloor \frac{R\mu c - C_p c}{C_w} \right\rfloor \tag{5}$$

is the *balking level* and is determined by solving $\beta_{n_b-1} \geq 0 > \beta_{n_b}$. Let x denote the strategy of an arriving customer and suppose $x \in \{j, b\}$ where j represents *joining* and b represents *balking*. Hence, the equilibrium strategy for customers is

$$x = \begin{cases} j, & 0 \leq k < n_b \\ b, & \text{otherwise} \end{cases} \tag{6}$$

The socially optimal strategy, on the other hand, is determined by maximizing social welfare. For a M/M/c/n queue, the total expected utility per unit time obtained by the customers in the system is given by

$$U_{sw}(n) = \lambda \sum_{k=0}^{n-1} p_k(n)\beta_k \tag{7}$$

Theorem 1 ([11, Theorem 1]): There exists n_{so} maximizing $U_{sw}(n)$ and $n_{so} \leq n_b$ so that $U_{sw}(n_b) \leq U_{sw}(n_{so})$.

Ideally incentivize drivers to close the gap between the social optimum and the user-selected equilibrium. In order to obtain the socially optimal balking rate n_{so} we can adjust the price for parking $\hat{C}_p = C_p + \Delta C_p$.

Proposition 1: The pricing mechanism $\hat{C}_p$ that achieves the socially optimal balking level n_{so} is determined by solving $\alpha_{n_{so}} < \hat{C}_p/\mu \leq \alpha_{n_{so}-1}$.

Proof: The goal is to find ΔC_p such that n_{so} is the balking rate. Let the reward under the new price of parking $\hat{C}_p = C_p + \Delta C_p$ be

$$\hat{\beta}_k = R - \frac{C_w(k+1)}{\mu c} - \frac{C_p + \Delta C_p}{\mu} \tag{8}$$

We know that n_{so} will be the balking rate if and only if $\hat{\beta}_{n_{so}-1} \geq 0 > \hat{\beta}_{n_{so}}$. Hence,

$$\hat{\beta}_{n_{so}-1} = R - \frac{C_w(n_{so}-1)}{\mu c} - \frac{C_p + \Delta C_p}{\mu} > 0 \tag{9}$$

$$\geq R - \frac{C_w n_{so}}{\mu c} - \frac{C_p + \Delta C_p}{\mu} \tag{10}$$

Rearranging, we get $\alpha_{n_{so}} < \hat{C}_p/\mu \leq \alpha_{n_{so}-1}$. ■

A. Congestion–Limited Balking Rate

For many municipalities, congestion is a primary objective. Hence, we consider the problem of designing the balking rate to achieve a particular level of parking–related congestion. In order to meet this objective, we can adjust the price of parking by selecting ΔC_p so that the balking level n_b, being the number of cars in the queuing system after which arriving customers decide to balk, is set to be the desired number of vehicles n_{cl} equaling some percentage of the total volume over the period of interest.

Proposition 2: The pricing mechanism $\hat{C}_p$ that achieves the congestion–limited balking level n_{cl} is determined by solving $\alpha_{n_{cl}} < \hat{C}_p/\mu \leq \alpha_{n_{cl}-1}$.

The above proposition is proved in the same way as Proposition 1; hence, we omit it.

Note that the value of n_{cl} may not be equal to n_{so} since the objectives that produce these values may not be aligned. Thus, designing the price of parking to maintain a certain level of congestion in a city may not be socially optimal. Similar results have been shown in the classical queuing game literature with regards to designing a toll that maximizes revenue (see, e.g., [11, Section 6]).

Proposition 3: If $n_{cl} \leq n_{so}$ or $n_{cl} > n_{so}$, $U_{sw}(n_{cl}) \leq U_{sw}(n_{so})$. Furthermore, if $n_{cl} \leq n_{so}$, then $U_{sw}(n_{cl}) = U_{sw}(n_{so})$.
The proof of the above proposition is due to the fact that n_{so} is the maximizer of U_{sw}. It tells us that selecting the balking rate to limit congestion may result in a decrease in social welfare.

Proposition 4: If $n_{cl} \leq n_b$, where n_b is the user-selected balking rate, then $U_{sw}(n_b) \leq U_{sw}(n_{cl})$ and vice versa.

Proof: The result is implied by the fact that that $U_{sw}(n)$ is *unimodal*, i.e. $U_{sw}(n) - U_{sw}(n-1) \leq 0$ implies that $U_{sw}(n+1) - U_{sw}(n) < 0$. Barring a little algebra, this is almost trivially true since $\{\beta_k\}$ is a decreasing sequence; indeed,

$$\begin{aligned} U_{sw}(n+1) - U_{sw}(n) =& \rho \tfrac{D_{n-1}}{D_{n+1}}(U_{sw}(n) - U_{sw}(n-1)) \\ & - \tfrac{d_{n-1}}{D_{n+1}}(\beta_{n-1} - \beta_n) \end{aligned} \tag{11}$$

Since $U_{sw}(n) - U_{sw}(n-1) \leq 0$ by assumption and $\{\beta_k\}$ is decreasing, U_{sw} is unimodal. ∎

The preceding propositions tell us that we can design the balking level by adjusting the price to match a particular desired level of congestion, we must be careful about how this level of congestion is selected since will impact social welfare. In particular, selecting n_{cl} will result in a decrease in the social welfare as compared to the socially optimal balking rate; on the other hand, it can result in an increase in social welfare if selected to be less than the user-selected balking rate n_b.

B. Example: Off-Street vs. On-Street Parking

Suppose customers have two alternatives. They can either choose on-street parking by selecting to enter a M/M/*c*/*n* queue as above with service time $1/\mu$ or they can choose off-street parking which we model as a M/M/∞ queue (infinitely available spots) with expected service time per customer $1/\mu$. We assume the reward R is the same for both cases. The utility for off-street parking is

$$U_{off} = R - \tfrac{C_{off}}{\mu} \tag{12}$$

where C_{off} is the cost for off-street parking per unit time. The utility for joining the on-street parking queue is

$$U_{on}(k) = R - \tfrac{C_{on}}{\mu} - \tfrac{C_w(k+1)}{c\mu} \tag{13}$$

where C_{on} is the cost per unit time for on-street parking, C_w is the cost per unit time for waiting in the queue (circling for parking), and k is the state of the queue. In essence, we consider that, when a customer balks, they choose off-street parking which represents the *outside option*. Hence, we can determine the rate at which people choose off-street in the same way as we determined the balking rate above. In particular, we find the off-street balking level n_{off} for which $U_{on}(n_{off}-1) \geq U_{off} > U_{on}(n_{off})$. Hence, we have that

$$n_{off} = \left\lfloor c\tfrac{C_{off} - C_{on}}{C_w} \right\rfloor. \tag{14}$$

IV. Costly Observation Queuing Game

We now relax the above framework so that arriving customers do not observe the queue length without paying a price. More specifically, suppose now that we have a M/M/c/n queue and that when customers arrive they can either balk, join, or pay a cost to observe the queue length after which they decide to balk or join. For on-street parking where there is an smart phone app to which a customer can pay a subscription fee to gain access to information or choose not to, this model makes sense. We take the theoretical model from [12].

Assume that each customer chooses to observe the queue with probability P_o at a cost C_o, balks without observing with probability P_b, and joins with out observing with probability P_j. We use the notation $P = (P_o, P_b, P_j) \in \Delta_2$ for the strategy of arriving drivers where $\Delta_2 = \{P = (P_o, P_b, P_j) | \ P_i \geq 0, i \in \{o, b, j\},\ P_o + P_j + P_b = 1\}$ is the strategy space, i.e. the 2–simplex. The *effective arrival rate* for this queue is then

$$\tilde{\lambda} = \begin{cases} (1-P_b)\lambda, & k < n_b \\ P_j\lambda, & k \geq n_b \end{cases} \tag{15}$$

where k is the queue length and $n_b = \lfloor \frac{R\mu c - C_p c}{C_w} \rfloor$ is the selfish balking level for the observable case. Of course, as before, we assume that $n_b \geq 1$ to avoid the trivial solution where $P_b = 1$ is a dominant strategy. In addition, we assume $n \geq n_b > c$ since if $c \leq n < n_b$ then users would be forced to balk n and we would just replace n_b in the above equations with n. The only other case is $n < c \leq n_b$ and it is non-sensical since c is the number of servers. We remark that if $C_o = 0$, then the game reduces to the observable game since $P_o = 1$. Hence, we investigate the case when $C_o > 0$.

The stationary probability distribution is as before (see (1)) except we use the *effective traffic intensity* $\rho = \frac{\tilde{\lambda}}{c\mu}$. In particular, we write the balance equations

$$\begin{aligned} (1-P_b)\lambda p_k^n &= (k+1)\mu p_{k+1}^n,\ 0 \leq k < c && (16) \\ (1-P_b)\lambda p_k^n &= c\mu p_{k+1}^n,\ c \leq k < n_b && (17) \\ P_j\lambda p_k^n &= c\mu p_{k+1}^n,\ n_b \leq k \leq n && (18) \end{aligned}$$

and we let $\eta = P_j\rho$, $\xi = (1-P_b)\rho$. Then,

$$p_k^n = \begin{cases} \frac{c^k\xi^k}{k!}p_0^n, & 0 \leq k < c \\ \frac{c^c\xi^k}{c!}p_0^n, & c \leq k < n_b \\ \eta^{k-n_b}\xi^{n_b}\frac{c^c}{c!}p_0^n, & n_b \leq k \leq n \end{cases} \tag{19}$$

so that

$$p_0^n = \left(\sum_{k=0}^{c-1}\frac{(c\xi)^k}{k!} + \sum_{k=c}^{n_b-1}\frac{c^c\xi^k}{c!} + \frac{\xi^{n_b}c^c}{c!}\frac{1-\eta^{n-1-n_b}}{1-\eta}\right)^{-1} \tag{20}$$

Note that we now use the more compact notation $p_k(n) \equiv p_k^n$ and similarly, we use the notation $U(n) \equiv U^n$ for utilities.

Once the customer knows the queue length then their reward is the same as in the observable case, i.e. $\beta_k = R - w_k - C_p/\mu$. However, since they do not know *a priori* the queue length, the customer must make the decision as to joining, balking, or observing by maximizing their expected utility.

The utility for observing the queue is given by

$$U_o^n(P) = \sum_{k=0}^{n_b-1} p_k^n \beta_k - C_o \tag{21}$$

$$= p_0^n \Big[\left(R - \frac{C_p}{\mu}\right) \left(\sum_{k=0}^{c-1} \frac{c^k \xi^k}{k!} + \sum_{k=c}^{n_b-1} \frac{c^c \xi^k}{c!} \right) - \frac{C_w}{c\mu} \left(\sum_{k=0}^{c-1} \frac{c^k \xi^k (k+1)}{k!} + \sum_{k=c}^{n_b-1} \frac{c^c \xi^k (k+1)}{c!} \right) \Big] - C_o, \tag{22}$$

the utility for joining without observing is given by

$$U_j^n(P) = \sum_{k=0}^{n-1} p_k^n \beta_k \tag{23}$$

$$= p_0^n \Big[\left(R - \frac{C_p}{\mu}\right) \Big(\sum_{k=0}^{c-1} \frac{c^k \xi^k}{k!} + \sum_{k=c}^{n_b-1} \frac{c^c \xi^k}{c!} + \sum_{k=n_b}^{n-1} \eta^{k-n_b} \xi^{n_b} \frac{c^c}{c!} \Big) - \frac{C_w}{c\mu} \Big(\sum_{k=0}^{c-1} \frac{c^k \xi^k (k+1)}{k!} + \sum_{k=c}^{n_b-1} \frac{c^c \xi^k (k+1)}{c!} + \sum_{k=n_b}^{n-1} \eta^{k-n_b} \xi^{n_b} \frac{c^c (k+1)}{c!} \Big) \Big], \tag{24}$$

and the utility for balking is $U_b^n(P) \equiv 0$.

Proposition 5: A symmetric, mixed Nash equilibrium exists for the game (U_o^n, U_b^n, U_j^n).

The above proposition is a direct consequence of Nash's result for finite games [13]. On the other hand, if we were to consider a queue where $n \to \infty$ (i.e. with an infinite number of players), then Nash's result would no longer hold. This framework is explored in the working paper [12].

Customers are assumed to be homogeneous which is to say they all have the same utility function. In particular, let us use the notation $u_i(P) = \sum_{k \in \{o,b,j\}} P_k U_k^n(P)$ for the expected utility of player i as a function of the mixed strategy $P = (P_o, P_b, P_j)$ where $P_k = \frac{1}{n} \sum_{i=1}^n P_k^i$ and $P^i = (P_o^i, P_b^i, P_j^i)$ denotes the strategy of player i. Then $u_i(P) = u_j(P)$ for $i, j \in \{1, \ldots, n\}$. Thus, we seek a *symmetric Nash equilibrium* which means that it is a best response against itself.

Definition 1: A $\{P^i | \ P^i \in \Delta_2\}_{i=1}^n$ is a symmetric Nash equilibrium if $P^i = P^j$ for all $i, j \in \{1, \ldots, n\}$.

Depending on the relative values of the utility functions U_b^n, U_j^n and U_o^n, an equilibrium (P_o, P_j, P_b) satisfies

$$P_o = 1, P_b = P_j = 0, \quad U_o^n > \max\{U_j^n, U_b^n\} \tag{25a}$$
$$P_b = 1, P_o = P_j = 0, \quad U_b^n > \max\{U_o^n, U_j^n\} \tag{25b}$$
$$P_j = 1, P_o = P_b = 0, \quad U_j^n > \max\{U_o^n, U_b^n\} \tag{25c}$$
$$P_o = 0, 0 \leq P_j, P_b \leq 1, \quad U_b^n = U_j^n > U_o^n \tag{25d}$$
$$P_j = 0, 0 \leq P_o, P_b \leq 1, \quad U_b^n = U_o^n > U_j^n \tag{25e}$$
$$P_b = 0, 0 \leq P_j, P_o \leq 1, \quad U_j^n = U_o^n > U_b^n \tag{25f}$$
$$0 \leq P_b, P_j, P_o \leq 1, \quad U_o^n = U_j^n = U_b^n \tag{25g}$$

Using the above equations and for the purpose of computing the Nash equilibrium, we adapt the best response algorithm in [12] to the case where the utility of the outside option U_b—which may be balking to other modes of transit or selecting off–street parking—is not necessarily non-zero (see Algorithm 1). We conjecture that the Nash equilibrium is unique and empirically observe this in the simulations. This conjecture is true when the number of players is infinite and $U_b = 0$ as shown in [12].

Algorithm 1 Best Response Algorithm

function GETBESTRESPONSE($P_o, P_b, P_j, \varepsilon, \delta, \gamma$)
  while $|P_o^* - P_o| + |P_b^* - P_b| < \delta$
    $U_j \leftarrow U_j^n(P_j, P_o)$, $U_o \leftarrow U_o^n(P_j, P_o)$
    if $U_o > \max\{U_j, U_b\} + \varepsilon$:
      $(P_o^*, P_b^*, P_j^*) \leftarrow (1, 0, 0)$
    elif $U_j > \max\{U_o, U_b\} + \varepsilon$
      $(P_o^*, P_b^*, P_j^*) \leftarrow (0, 0, 1)$
    elif $U_b > \max\{U_o, U_j\} + \varepsilon$
      $(P_o^*, P_b^*, P_j^*) \leftarrow (0, 1, 0)$
    elif $|U_o - U_b| < \varepsilon$ & $\min\{U_o, U_b\} > U_j + \varepsilon$
      $(P_o^*, P_b^*, P_j^*) \leftarrow (P_o/(P_o + P_B), P_b/(P_o + P_b), 0)$
    elif $|U_j - U_b| < \varepsilon$ & $\min\{U_j, U_b\} > U_o + \varepsilon$
      $(P_o^*, P_b^*, P_j^*) \leftarrow (0, P_b, 1 - P_b)$
    elif $|U_j - U_o| < \varepsilon$ & $\min\{U_j, U_o\} > \varepsilon + U_b$
      $(P_o^*, P_b^*, P_j^*) \leftarrow (P_o, 0, 1 - P_o)$
    elif any two $\{|U_j - U_b| < \varepsilon, |U_o - U_b| < \varepsilon, |U_j - U_o| < \varepsilon\}$:
      $(P_o^*, P_b^*, P_j^*) \leftarrow (P_o, P_b, P_j)$
    end if
  end while
  if $|P_o^* - P_o| + |P_b^* - P_b| \geq \delta$:
    $P_o^+ \leftarrow (1 - \gamma) P_o^* + \gamma P_o$
    $P_b^+ \leftarrow (1 - \gamma) P_b^* + \gamma P_b$
    $P_j^+ \leftarrow (1 - \gamma) P_j^* + \gamma P_j$
    $(P_o, P_b, Pj) \leftarrow (P_o^+, P_b^+, P_j^+)$
end function

On the other hand, the socially optimal strategy $(P_o^{so}, P_b^{so}, P_j^{so}) \in \Delta_2$ is determined by maximizing the social welfare which is given by

$$U_{so}^n(P) = P_j U_j^n(P) + P_o U_o^n(P) + P_b U_b^n(P) \tag{26}$$

$$= P_j \lambda \sum_{k=0}^{n-1} p_k^n \beta_k + P_o \lambda \left(\sum_{k=0}^{n_b-1} p_k^n \beta_k - C_o \right). \tag{27}$$

As we stated in the previous section, it is well known that, in general, the social welfare is not maximized by the Nash equilibrium.

A. Example: On-Street vs. Off-Street Parking

We now consider that the balking option is to select off-street parking as we did in Section III-B. In particular, we define $U_b^n = U_{off} = R - C_{off}/\mu$. The Nash equilibrium can be computed using Algorithm 1 using $U_b^n = U_{off} = R - C_{off}/\mu$ instead of $U_b^n = 0$. On the other hand, the social welfare is now given by

$$U_{so}^n(P) = P_j U_j^n(P) + P_o U_o^n(P) + P_b U_b^n(P) \tag{28}$$

$$= P_j \lambda \sum_{k=0}^{n-1} p_k^n \beta_k + P_o \lambda \left(\sum_{k=0}^{n_b-1} p_k^n \beta_k - C_o \right) + P_b \lambda \left(R - \frac{C_{off}}{\mu} \right). \tag{29}$$

In Figure 1, we show the Nash equilibrium and the socially optimal strategy.

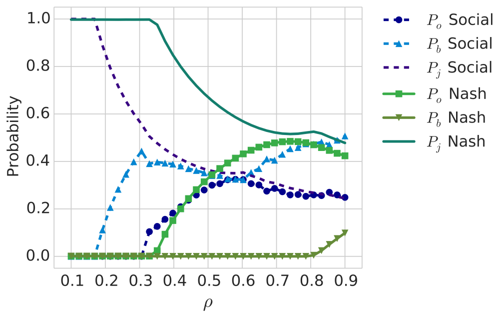

Fig. 1. Nash and socially optimal equilibria. The game we consider is on-street parking vs. off-street parking and we varied $\rho = \frac{\lambda}{c\mu}$ by keeping $\mu = 1/120$ and $c = 30$ constant and allowing $\lambda \in [0.025, 0.225]$. The other parameter values are $C_p = 0.05$, $C_o = 3.85$, $R = 95$, $C_w = 1.5$, $n = 100$, $C_{off} = 0.962$. The social welfare is consistently greater than the Nash induced welfare for all values of C_o as expected.

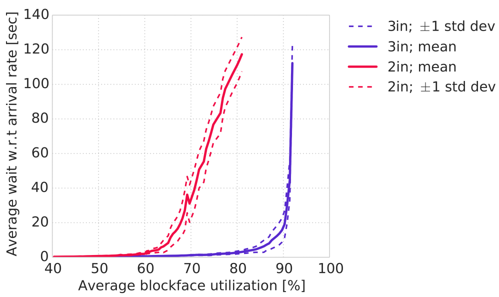

Fig. 2. Average wait time (proxy for congestion) with respect to the exogenous arrival rate $\lambda \in [0.6, 1.3]$ and fixed service rate per blockface ($c * \mu = 1.0$) plotted against the average block face utilization (proxy for occupancy) for a three node queue–flow network with arrivals injected at all three nodes (green) and only at two nodes (blue). There is a distinct difference in the occupancy vs. congestion curves depending on the network structure and average waits grow unboundedly as $\rho \to 1$.

V. Queue–Flow Network Simulations

To examine the congestion–occupancy relationship as a function of network topology and information access among other factors, we constructed a queue–flow network simulator[1]. The simulator constructs a syncrhonized list of blockface (drivers in service) and street (drivers waiting/circling) timers linked according to the street topology.

A. Congestion vs. Occupancy

The congestion–occupancy relationship is an important one to understand when it comes to designing the price of parking or information aimed at reducing congestion. Many municipalities and researchers design pricing schemes to target a single occupancy level—typically %80—for all blockfaces in a city despite network topology. In Figure 2, we simulate a three block queue–flow network and show that the congestion–occupancy relationship can be drastically different depending on how many nodes are treated as sources for injections. In particular, the upper bound for utilization (before wait time exponentially increases) for the 3-node injection case is around 88% while the 2-node injection case is around 65%.

[1]`https://github.com/cpatdowling/net-queue`

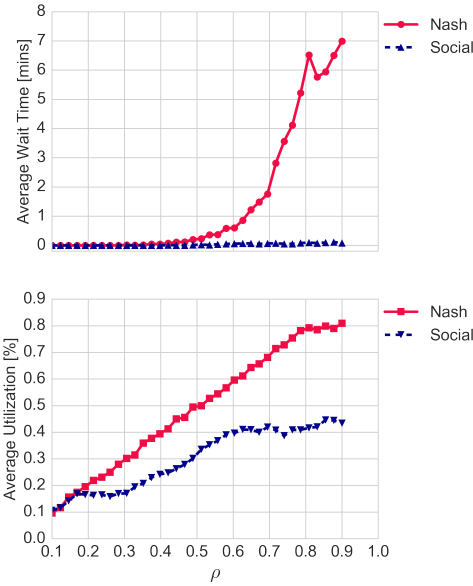

Fig. 3. (upper) Average wait time and (lower) average blockface utilization as a function of $\rho = \frac{\lambda}{c\mu}$ where $c = 30$ and $\mu = 1/120$ are fixed and $\lambda \in [0.025, 0.225]$ for a three node system. The other parameter values are $C_p = 0.05$, $C_o = 3.85$, $R = 95$, $C_w = 1.5$, $n = 100$, $C_{off} = 0.962$. The Nash equilibrium and the socially optimal equilibrium varies with ρ and is depicted in Figure 1. The discontinuity in the wait time for Nash around $\rho = 0.8$ is due to the fact that at that point the probability of balking P_b becomes non-zero (see Figure IV).

B. Costly Observation Queuing Game Simulations

Coupling the queue-flow network with the game theoretic models of the previous sections, we simulate the queueing game and its impact on network flow (average wait time) and on-street parking utilization (occupancy). Given a queue-flow network topology, for simplicity, we assume that each of the queues has the same service rate μ. In addition, we suppose that the total number of parking spots across all queues in the network is c, the arrival rate to the queuing network is λ, and the capacity of queue-flow network is n. This allows us to model the whole queuing system as a M/M/c/n queue.

We execute our simulation as follows. First, we determine the equilibrium of the game (resp. the socially optimal strategy) and then, we use the simulator described above to determine the average wait time and utilization. The game only effects the arrival process of the queue–flow network; once arriving drivers enter the network, the queue–flow simulator determines the drivers impact on the system and the wait time they experience. Given a strategy (P_o, P_b, P_j), we sample from a Poisson distribution with parameter $1/\lambda$ to determine the arrival time of the next driver. Then, we sample from the distribution determined by (P_o, P_b, P_j) to decide if the arriving driver will balk, join without observing, or pay to observe. If the driver balks, then we discard this arriving car. If the driver joins without observing, then we determine which node the driver enters by randomly choosing (using a uniform distribution) a queue in the network. If the driver pays to observe, then we examine the length of each queue in the system and the driver joins the queue with the shortest length as long as it is less than the balking rate.

On-Street Parking vs. Other Modes of Transit	Type	(P_o, P_b, P_j)	Utilization	Avg. Wait	Welfare
$\lambda = 1/5$, $C_o = 0.25$, $R = 75$, $C_w = 0.8$, $C_p = 0.05$	SO	$(0.00, 0.58, 0.42)$	33.2%	0.002	2.80
	N	$(0.85, 0.13, 0.02)$	69.3%	0.359	0.00
$\lambda = 1/4.85$, $C_o = 0.5$, $R = 75$, $C_w = 0.75$, $C_p = 0.05$	SO	$(0.00, 0.56, 0.44)$	34.9%	0.002	3.02
	N	$(0.84, 0.09, 0.07)$	77.9%	0.901	0.00
$\lambda = 1/4.5$, $C_o = 2.0$, $R = 75$, $C_w = 0.5$, $C_p = 0.075$	SO	$(0.00, 0.4, 0.6)$	52.3%	0.04	4.27
	N	$(0.55, 0.00, 0.45)$	88.0%	3.69	2.68
On-Street vs. Off-Street Parking	Type	(P_o, P_b, P_j)	Utilization	Avg. Wait	Welfare
$\lambda = 1/4.5$, $C_o = 3.85$, $R = 65$, $C_w = 1.5$, $C_{off} = 0.962$, $C_p = 0.05$	SO	$(0.47, 0.19, 0.34)$	69.9%	1.99	6.58
	N	$(0.49, 0.00, 0.51)$	84.0%	7.77	1.85
$\lambda = 1/4.75$, $C_o = 3.85$, $R = 65$, $C_w = 1.5$, $C_{off} = 0.962$, $C_p = 0.05$	SO	$(0.5, 0.14, 0.36)$	70.6%	2.23	9.23
	N	$(0.53, 0.00, 0.47)$	81.0%	5.96	7.19

TABLE I

QUEUE–FLOW NETWORK GAME SIMULATION RESULTS: FOR EACH OF THE SIMULATIONS WE SET THE TOTAL NUMBER OF PARKING SPACES TO BE $c = 30$, THE AVERAGE PARKING DURATION IS 120 MINUTES ($\mu = 1/120$) WHICH IS CONSISTENT WITH THE SEATTLE DEPARTMENT OF TRANSPORTATION DATA. WE USE THE SHORTHAND SO FOR SOCIALLY OPTIMAL AND N FOR NASH.

Table I contains the results of simulations for both the costly observation game simulations for the costly observation queuing game and the on-street vs. off-street example. The social welfare is always higher than the Nash welfare, as expected. The more interesting result is that the utilization rate (occupancy) and average wait time (time spent circling) are always less under the socially optimal strategy than the Nash equilibrium. These are metrics that are typically used by transportation planners. Moreover, in Figure 3, we show the result of simulating both the Nash equilibrium and the socially optimal strategy for various values of the traffic intensity ρ (holding all other parameters fixed). These simulations are for the same games depicted in Figure 1. As the traffic intensity increases, we see that both the Nash and socially optimal utilization increase almost linearly with the Nash utilization remaining greater. Again, what is interesting is that by metrics (wait time and occupancy) other than the social welfare the Nash performance is also worse than the socially optimal solution.

Another somewhat surprising finding is that only partial information availability amongst the users—as seen in Table I and Figure 1 where $P_o \neq 1$ for the socially optimal solution—is required to increase social welfare. Moreover, it seems the socially optimal equilibrium strategy requires *less* information availability. From a municipality's perspective, this is a useful result when designing a socially optimal parking infrastructure. Not everyone will know information about parking availability in the first place (e.g. tourists vs. residents).

VI. DISCUSSION AND FUTURE WORK

We presented a framework for modeling parking in urban environments as parallel queues and we overlaid a game theoretic structure on the queuing system. We investigated both the case where drivers have full information—i.e. observe the queue length—and where drivers have to pay to access this information. We show only partial information is required to increase social welfare. Finally, through simulations we connect the queuing game to a flow network model in order to characterize wait time (congestion) versus utilization (occupancy).

We view this paper as the first steps towards formally understanding the congestion–parking relationship and how information affects effeciency. We are actively working on a number of extensions. To name a few: first, we are working on designing information based mechanisms to close the gap between the user-selected and socially optimal solution and reduce the parking-related congestion by targeting balking rates. Second, we are working on relaxing the homogenity assumption by considering different preferences such as walking time to destination and different priority levels such as disabled placard holders.

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