Expansion Planning of Urban Electrified Transportation Networks: A Mixed Integer Convex Programming Approach

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***Abstract*—Electric vehicles (EVs) have been widely acknowledged as an effective solution to alleviate the fossil fuel shortage and environmental pressure in modern metropolises. To foster the large-scale integration of EVs, transportation electrification is becoming an emerging trend. This paper proposes a comprehensive model for the expansion planning of urban electrified transportation networks (ETNs), which determines the best investment strategies for the transportation network (TN) and the power distribution network (PDN) simultaneously, including the sites and sizes of new lanes, charging facilities, distribution lines, and local generators. The steady-state distribution of traffic flow in the TN is characterized by the Nesterov user equilibrium (NUE). The operating condition of the PDN is described by linearized branch power flow (BPF) equations. To consider the interdependency between the TN and PDN created by the charging behavior of EVs, the power demands of on-road charging facilities is assumed to be proportional to the road traffic flow. The expansion planning model is formulated as a mixed integer nonlinear program (MINLP) with NUE constraints. In order to retrieve a global optimal solution, it is further transformed into an equivalent mixed integer convex program (MICP) without exploiting approximation. Case studies on a test ETN corroborate the proposed model and method.**

***Index Terms*—electric vehicle, expansion planning, interdependency, power distribution network, electrified transportation network, Nesterov user equilibrium**

NOMENCLATURE

The major symbols and notations used throughout the paper are defined below for quick reference. Others are defined after their first appearance as required.

A. Sets

E_L	Set of distribution lines in PDN.
E_B	Set of electrical buses in PDN.
$C(i)$	Charging facilities served by bus i / E_N.
$\pi(i)$	Set of child buses of bus i / E_N.
T_A	Set of links (roads, or arcs) in TN.
T_N	Set of nodes (intersections) in TN.
T_N^R	Set of origin nodes, $T_N^R \leq T_N$.
T_N^S	Set of destination nodes, $T_N^S \leq T_N$.
D_T^{RS}	Set of origin-destination (O-D) pairs, which is defined as $D_T^{RS} = \}(r,s)\ r \ / \ T_N^R, s \ / \ T_N^S|$
K_{rs}	Set of available paths between O-D pair (r,s).

B. Parameters

a_i, b_i	Production cost parameters of generator at bus i.
c_a	Traffic flow capacity of existing link a.
n_a^{ce}	Number of existing charging facilities on link a.
P_l^u	Active power flow capacity of distribution line l.
Pr_a^r	Cost of building one additional road on link a.
Pr_a^c	Cost of building one charging facility on link a.
Pr_i^g	Cost of building one generator at bus i.
Pr_{ij}^l	Cost of building one additional distribution line between buses i and j.
p_i^{dc}	Traditional power demand at bus i.
p_i^+	Active power generation capacity at bus i.
q_i^d	Reactive power demand at bus i.
q_i^+	Reactive power generation capacity at bus i.
q_{rs}^t	Traffic demand (trip rate) between O-D pair (r,s).
r_{ij}^l	Resistance of line l connecting buses i and j.
S_l	Complex power flow capacity of line l.
t_a^0	Non-congested travel time on link a.
V_0	Voltage magnitude at the slack bus.
V_i^-	Lower bound of voltage magnitude at bus i.
V_i^+	Upper bound of voltage magnitude at bus i.
x_{ij}^l	Reactance of line l connecting buses i and j.
Λ	The link-path incidence matrix.
δ_{ak}^{rs}	Elements of Λ, if path k between O-D pair (r,s) passes link a, $\delta_{ak}^{rs}=1$, otherwise $\delta_{ak}^{rs}=0$.
κ	Discount factor, which leverages long-term investment cost and short-term operation cost.
ω	Monetary value of vehicle travel time.
η	Charging rate of traffic flow.
ρ	Contracted energy price.
Δc_a	Traffic flow capacity of the expanded link.
Δp_a^c	Power capacity of the expanded charging facility.
Δp_i^m	Active generation capacity of the expanded unit.
Δq_i^m	Reactive generation capacity of the expanded unit.

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C. Variables

f_k^{rs}	Traffic flow on path k between O-D pair (r, s).
n_a^c	Positive integer which indicates how many charging facilities should be invested in on link a.
n_i^g	Positive integer which indicates how many generation units should be invested in at bus i.
p_i^d	Total active power demand at bus i.
p_i^g	Total active power generation at bus i.
P_{ij}^l	Active power flow in distribution line l.
q_i^g	Total reactive power generation at bus i.
Q_{ij}^l	Reactive power flow in distribution line l.
V_i	Voltage magnitude at bus i.
v_{an}^t	Additional continuous variable for linearizing the total vehicle travel time function.
v_{ijn}^{dl1}	Additional continuous variable for linearizing bilinear terms in the voltage drop equation.
v_{ijn}^{dl2}	Additional continuous variable for linearizing bilinear terms in the voltage drop equation.
x_a	Aggregated traffic flow on link a.
z_{an}^t	Binary variable of road expansion; the number of new roads to be invested in on link a is expressed by a binary expansion $\sum_{n=0}^{N_R} 2^n z_{an}^t$.
z_{ijk}^{dl}	Binary variable of distribution line expansion between the head bus i and the tail bus j; the number of new lines to be invested in is given by a binary expansion $\sum_{n=0}^{N_L} 2^n z_{ij}^{dl}$.
π, μ, λ	Dual variables of the traffic assignment problem.

D. Abbreviations

BPR	Bureau of Public Roads.
BPF	Branch power flow.
ETN	Electrified transportation network.
EV	Electric vehicles.
KKT	Karush-Kuhn-Tucker.
LMP	Locational marginal price.
LP	Linear programming
MICP	Mixed integer convex program.
MINLP	Mixed integer nonlinear program.
MPEC	Mathematic program with equilibrium constraints.
NUE	Nesterov user equilibrium.
OPF	Optimal power flow.
PDN	Power distribution network.
SO	Social optimum.
TAP	Traffic Assignment Problem.
TN	Transportation network.
UE	User equilibrium.

I. Introduction

THE rapid commercialization of electric vehicles (EVs) [1] has created an emerging trend of transportation electrification [2]–[5], which calls for the installation of brand-new battery charging/swapping infrastructures on the urban transportation network (TN) to support the integration of EVs. Coordinated charging of EVs also helps the power system accommodate high penetration of renewable energy [6]–[8]. However, the inappropriate placement and operation of charging facilities may create negative impacts on the power distribution network (PDN) [9], [10]. In this regard, most existing research focuses on the optimal deployment of charging or swapping stations, following either of the two paradigms below.

In power system oriented studies, operating requirements of the PDN are usually considered in detail. Along this line, a two-stage procedure for planning EV charging stations in distribution systems is proposed in [11], in which the candidate sites for charging facilities are determined in the first stage subject to their service radius and environmental considerations, and the optimal capacity that minimizes the total life-cycle cost is calculated. A multi-objective model for charging station planning is suggested in [12]. By jointly optimizing a maximal vehicle flow capturing problem and an optimal power flow (OPF) problem, the resulting strategy is advantageous in covering a larger service area and minimizing power losses and voltage deviations. The model is further improved in [13] by incorporating the Traffic Assignment Problem (TAP), which captures the system-level vehicle flow distribution in the transportation network (TN). Another optimization model is devised in [14] for locating and sizing battery swapping stations in distribution systems, and the charging control strategy is also discussed. The charging station placement problem is formulated as an MINLP in [15]. Taking the special structure into account, four methods are suggested to solve the proposed model. An agent-based model for cost-effective siting of electric vehicle charging infrastructure is presented in [16], in which the travel survey data and traffic demand forecasts are used to produce specific mobility patterns. Recently, simultaneous expansion planning of charging stations and PDN infrastructures has been addressed in [17] and [18] encompassing financial, technical, and environmental considerations.

In transportation system oriented studies, monitoring the vehicular flow going through each road is the main concern. Along this line, a multi-period planning model is proposed in [19] to expand EV charging stations, which incorporates topological dynamics of the TN, as well as the route choice of EVs and their limited travel range. To capture the selfish behavior when drivers choose their routes and recharging plans, a multi-class network equilibrium model is proposed in [20], based on which the charging station location problem is formulated as a mathematical program with equilibrium constraints (MPEC). Following a similar framework, the deployment of wireless charging facilities for capturing the maximum traffic flow is investigated in [21], in which the routing choice behavior of EVs is modeled by a stochastic user equilibrium. Locating multiple types of charging stations is studied in [22] using the maximum coverage concept. In addition, road capacity expansion planning is a classical problem in transportation research and has been extensively discussed, such as in [23]–[25]. It is usually formulated as a mathematical program subject to traffic flow equilibrium constraints, which has already been very challenging to solve. Joint planning of road and on-road charging infrastructure is attracting more and more attention in recent years, but published work is rare.

Most of the aforementioned research focuses on either the PDN or the TN. In the former category, the traffic system condition is usually simplified or ignored, and the driving patterns of EVs are assumed to be exogenously given, either in a deterministic or stochastic manner. Such an assumption is reasonable for instances where the local charging demand profile can be predicted accurately or its probability distribution is known, such as residential and office areas. However, this is may no longer be the case for the system-level study of the on-road charging infrastructure of the ETN. In transportation theory, although the origins and destinations of vehicles can be specified, multiple routes may exist for traveling between origins and destinations, leading to different traffic flow patterns and different charging demand patterns, and finally the power flow of the PDN as well as its operation is affected. Such interdependency has been well recognized and studied by the transportation community, but the operating details of the power system are usually ignored. Recently, a systematic modeling framework and a hybrid simulation platform for the interdependent transportation and power systems are developed in [26] to study the impacts of EV charging facilities on both networks. Analytical models for designing, operation and optimization of such coupled networks are still in great need. A comprehensive study on the expansion planning of the ETN calls for interdisciplinary research. Along this line, the traffic equilibrium constrained deployment of public charging stations is investigated in [27], where the stable distribution of traffic flows in the TN is determined from a combined distribution and assignment model, while the PDN is modeled by the direct-current OPF and offers electricity at the locational marginal price (LMP). A similar framework is adopted in [28] and [29] to study the pricing issue in ETNs.

This interdisciplinary research aims to comprehensively address the expansion planning problem of ETNs and close the gap in existing methods in three aspects:

1) The system-level modeling of the ETN which consists of coupled TN and PDN, and accounts for their interdependency. The mathematical formulations of both infrastructures are different from those in existing literatures, and more dedicated for the strategic planning research. In urban transportation system research, Beckmann's formulation [30] for the static TAP has been the reference model since 1960's [31], and widely adopted, such as in [13], [21], [23]–[25], [27]–[29]. Nesterov and de Palma have noticed that some assumptions in Beckmann's model may not be entirely consistent with reality. They propose a new TAP formulation in [32] and [33]. It is shown in [34] that Beckmann's model and Nesterov's model provide similar results when the TN is not heavily congested. In this paper, Nesterov's model is employed to describe the stable traffic flow in TNs, which is more suitable for addressing the planning issue, since it incorporates an explicit bound that the traffic flow on each road cannot exceed, and is also advantageous in computation because of its linearity. As for the PDN, we use the linearized branch power flow (BPF) model developed in [35]–[37] to determine the steady-state distribution of the bus voltage and line power flow. Unlike the direct-current power flow model used in [27], which assumes the voltage magnitudes at all buses are equal to 1 and is suitable for high-voltage power transmission systems, BPF treats the magnitudes of bus voltages as variables; this approach is more realistic for low-voltage PDNs, in which the line resistance is comparable to the line reactance in per unit value. The linearized BPF model provides satisfactory accuracy for PDN operation and planning applications, which has been justified in [38]–[40].
2) The expansion planning model of the ETN. As mentioned before, most current research only focuses on one, or at most a few, of the items which are in need of upgrading, such as the charging stations or roads, while neglects the interdependency between TN and PDN. It has already been acknowledged that the on-road charging system can introduce notable interdependency across TNs and PDNs [41], hence infrastructures of TN (roads or lanes) and PDN (on-road charging facilities, local generators and distribution lines) should be coordinately expanded. However, coordination is usually ignored or partly simplified in most existing research, in spite of the fact that distributed generation and charging stations are simultaneously modeled in [17], substation-feeder coordination is considered in [18]. Moreover, although road capacity planning and power system planning have long been studied separately for decades, they have seldom been considered together, except for the first attempt in [27], in which a network equilibrium model that jointly considers the interactions among charging opportunities, energy prices, and route choices of EVs in a coupled TN and PDN is proposed. More precisely speaking, although the impact of power system operation and LMP is considered in [27] for deploying charging stations, the work does not model the expansion planning of roads or other PDN facilities. This paper proposes a comprehensive model for the simultaneous expansion planning of ETN infrastructures, including roads, on-road charging facilities, generation units, and distribution lines, in a coordinated manner for the first time. Unlike [12], [13] and [21], in which the deployment of charging stations aims to cover the largest possible service area, this work assumes that the charging facilities can be built at the side of each road and meet the local charging demand on that link, or at pre-specified candidate sites.
3) The solution algorithm. Because of the presence of interdependency between TN and PDN, the traffic equilibrium, which is determined from an optimization problem, is incorporated as constraints in the proposed expansion planning model, giving rise to an MPEC which has a two-level optimization structure and is challenging to solve. In fact, even the instances which only consider charging station planning have proved to be very difficult; for example, the model studied in [11] is a non-convex programming problem, the proposed primal-dual interior point algorithm only finds a local optimal solution; the multi-objective models in [12]–[14] is solved by intelligent algorithms; and the MINLP model in [15] is solved by certain heuristic methods

that rely on the specific problem structure. The genetic algorithm used in [17] and the ordinal optimization approach adopted in [18] also rely on a large number of samples and may miss the global optimal solutions. By using the primal-dual optimality condition transformation and linearization techniques from integer algebra, the lower level TAP will be reduced to traditional nonlinear constraints, and the proposed optimization model will be further reformulated as an equivalent MICP without exploiting approximation; thereby a global optimal solution can be found by commercial solvers with affordable computational expense, since the time horizon of a planning problem is long.

The rest of this paper is organized as follows. The mathematical models of TN and PDN will be briefly introduced in Section II. The MPEC model of ETN expansion planning will be presented in Section III, where the traffic equilibrium of the TN is modeled through the NUE in the lower level, while all the investment strategies and BPF constraints are considered in the upper level. Its equivalent MICP will be formulated in Section IV. The effectiveness of the proposed method has been demonstrated in Section V through case studies on a test ETN consisting of a TN with 20 links and a PDN with 20 buses. Conclusions are drawn in Section VI.

II. Mathematical Model of the TN and PDN

A. Transportation System Model

A brief introduction to transportation network model is presented here. More details can be found in textbook [31]. The TN is represented by a connected graph $G_T = [T_N, T_A]$ where T_N is the set of nodes (intersections) and T_A is the set of links (roads or arcs). Each link $a \,/\, T_A$ is associated with a capacity limit c_a (the maximal number of vehicles that can pass this link per unit time) and a non-congested travel time t_a^0 (the travel time across this link at the speed limit). Given a set of O-D pairs D_T^{RS}, we are aware of the total traffic flow q_{rs}^t (also called the trip rate) leaving from the origin r and traveling to its destination s, but at the current stage it is not clear which route will be used. Each O-D pair $(r, s) \,/\, D_T^{RS}$ is connected by a set of routes, which is denoted by K_{rs}. The traffic flows on link $a \,/\, T_A$ and path $k \,/\, K_{rs}$ are denoted by x_a and f_k^{rs}, respectively. We define the indicator variable $\delta_{ak}^{rs} = 1$ if link a is a part of path k, otherwise $\delta_{ak}^{rs} = 0$; then the link flow x_a and path flow f_a^{rs} have the following relationship:

$$x_a = \sum_{rs} \sum_{k} f_k^{rs} \delta_{ak}^{rs}, \ \{a \,/\, T_A \tag{1.1}$$

or, in a compact form,

$$x = \Lambda f \tag{1.2}$$

where coefficient matrix $\Lambda = [\delta_{ak}^{rs}]$, $\{k \,/\, K_{rs}, \{(r, s) \,/\, D_T^{RS}$, $\{a \,/\, T_A$ is the link-path incidence matrix, if path k between O-D pair (r, s) passes link a, $\delta_{ak}^{rs} = 1$, otherwise, $\delta_{ak}^{rs} = 0$; vector $x = [x_a]$, $\{a \,/\, T_a$; vector $f = [f_k^{rs}]$, $\{k \,/\, K_{rs}$, $\{(r, s) \,/\, D_T^{RS}$.

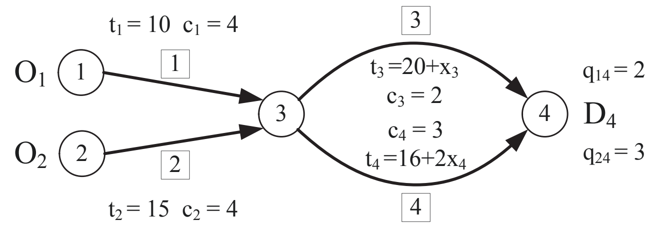

Fig. 1. A simple TN modified from [31].

Moreover, the path flow should meet the traffic demand, i.e.,

$$\sum_{k \in K_{rs}} f_k^{rs} = q_{rs}^t, \ \{r, s \tag{1.3}$$

or, in a compact form,

$$Ef = q^t \tag{1.4}$$

where $q^t = [q_{rs}^t], \{(r, s) \,/\, D_T^{RS}$, and E is a matrix consist of 0 and 1 corresponding to the coefficients in (1.3).

An example modified from [31] is used to explain above concepts and notations more intuitively. A simple TN is shown in Fig. 1. The network includes 4 nodes and 4 links. O-D pair O_1-D_4 is connected by paths $\boxed{1} \Rightarrow \boxed{3}$ and $\boxed{1} \Rightarrow \boxed{4}$; the trip rate is $q_{14} = 2$. O-D pair O_2-D_4 is connected by paths $\boxed{2} \Rightarrow \boxed{3}$ and $\boxed{2} \Rightarrow \boxed{4}$; the trip rate is $q_{24} = 3$. The link flows can be expressed by path flows as

$$\begin{aligned} x_1 &= f_1^{14} + f_2^{14} \\ x_2 &= f_1^{24} + f_2^{24} \\ x_3 &= f_1^{14} + f_1^{24} \\ x_4 &= f_2^{14} + f_2^{24} \end{aligned}$$

and the flow conservation equations are given by

$$\begin{aligned} f_1^{14} + f_2^{14} &= 2 \\ f_1^{24} + f_2^{24} &= 3 \end{aligned}$$

In transportation engineering, road travel time is the most important metrics to measure congestions. Let t_a denote the travel time on link a; it is a function of the traffic flow. The different descriptions of road travel latency lead to completely different TAP models: Beckmann's model and Nesterov's model, which will be briefly introduced below.

Assumptions in Beckmann's model

The travel time on link a is assumed to be a strictly increasing function that depends only on its own vehicle flow x_a, such as the following most widely used BPR function [42]

$$t_a(x_a) = t_a^0 \left[1 + 0.15 \left(\frac{x_a}{c_a} \right)^4 \right], \ \{a \,/\, T_A$$

where c_a is the vehicle flow on link a when $t_a = 1.15 t_a^0$. In some literature, it is also called the capacity of link a. However, the capacity limit is not a mandatory constraint, but is penalized by a quickly growing travel time if such a constraint is violated. Some inconsistencies between the BPR function and the reality of traffic congestion have been pointed out in [32], [33].

1) A less important criticism relates to the strictly increasing assumption for the latency function $t_a(x_a)$. Clearly, the travel time is a constant when the road is not congested and every vehicle can move at the maximal allowed speed.
2) It is clear that the vehicle flow on link a cannot be arbitrarily large. Ignoring the mandatory road capacity constraint may lead to infeasible solutions when the traffic flow on a link is greater than its capacity.
3) Most important of all, there is a physical contradiction to the monotonicity of $t_a(x_a)$. According to the definition of traffic flow: flow $=$ speed $\subseteq$ density, if the flow x_a is large, neither the speed nor the density can be very small (it is not realistic to compensate for the drop in speed with a dramatic increase in the density, because every vehicle in motion has to maintain a safe distance from the vehicle in front of it). As a result, by assuming a maximal density of vehicles, a further increment of x_a must lead to a rise in the speed, and in turn result in a drop in the travel time.

Assumptions in Nesterov's model

To mitigate the discrepancy in the road latency function, Nesterov and de Palma abandon making further assumptions on the functional form of t_a. Instead, they establish a new TAP model based on two weaker assumptions:

A1 Traffic flow on a link cannot exceed its capacity, i.e.,

$$x_a \geq c_a, \ \{a \ / \ T_A \tag{2.1}$$

It should be mentioned that the capacity parameter c_a in Nesterov's model is not necessarily the same as that in Beckmann's model.

A2 There is no delay on a link where the traffic flow does not reach its capacity; slowdown only occurs when the traffic flow reaches its capacity, i.e.

$$\begin{aligned} x_a < c_a \in \ t_a = t_a^0 \\ x_a = c_a \in \ t_a \approx t_a^0 \end{aligned} \tag{2.2}$$

We say link a is congested if $t_a > t_a^0$. Congestion occurs in the following manner: when the traffic flow x_a on certain link a reaches its capacity c_a, any additional vehicle will slow down the speed of existing vehicles, thus the travel time t_a increases, while the total flow remains the same (but car density increases). A1 and A2 are weaker than the functional assumption on the link travel time in Beckmann's model.

Before proceeding to Nesterov's TAP model, we clarify two important concepts in traffic theory:

1) Social Optimum (SO) [43]. A traffic flow pattern reaches an SO if the total travel time is minimized (also referred to as the second Wardrop Principle). This setting requires a central agency which is eligible to decide a travel plan for every driver, while all the drivers are required to behave cooperatively so as to guarantee the most efficient utilization of the entire transportation system. SO is an ideal state for theoretical study, but is unlikely to happen or implement in reality.
2) User Equilibrium (UE) [43]. Each driver selects his route in order to minimize his own travel time (also referred to as the first Wardrop Principle). In the UE pattern, no traveler has the incentive to change his current route unilaterally. UE captures the selfish behavior of vehicles in urban transportation systems, and will be used in the proposed ETN expansion planning model, since it better fits the reality.

Nesterov's traffic assignment model

Define vectors $c = [c_a]$, $\{a \ / \ T_A$, $t^0 = [t_a^0]$, $\{a \ / \ T_A$, and $t = [t_a]$, $\{a \ / \ T_A$. Suppose there is a central coordinator who can manage the behavior of vehicles to reach an SO pattern. In accordance with the definition of SO, the traffic flow vector x and travel time vector must solve the following nonlinear programming problem

$$\min_{x,f,t} \ x^T t \tag{3.1}$$
$$s.t. \ x \quad \Lambda f = 0 \tag{3.2}$$
$$Ef = q^t \tag{3.3}$$
$$x \geq c \tag{3.4}$$
$$f \approx 0, \ t \approx t^0 \tag{3.5}$$

where $x^T t = \sum_a x_a t_a$ is the total travel time, constraints (3.2) and (3.3) are the compact forms of (1.1) and (1.3), respectively. Inequalities (3.4) and (3.5) are the boundary constraints of decision variables. Constraints (1.1) and (3.5) naturally suggest $x \approx 0$. Based on this fact, it is clear that at the optimal solution, $t^* = t^0$ whatever x appears to be. In view of this, we arrive at *Nesterov's SO model*, which is a linear program (LP):

$$\min_{x,f} \ x^T t^0 \tag{4.1}$$
$$s.t. \ x \quad \Lambda f = 0 \tag{4.2}$$
$$Ef = q^t \tag{4.3}$$
$$x \geq c \tag{4.4}$$
$$f \approx 0 \tag{4.5}$$

LP (4) indicates that no congestion happens in the SO pattern. Please keep in mind that if the traffic demand exceeds the transit capacity of the TN, constraints (4.2)-(4.5) will be contradictive, and TAP (4) will be infeasible. In this paper, the associating TAP will be always feasible because the road capacity can be enlarged.

Next, we associate the capacity constraint (4.4) with respect to a dual variable $\lambda = [\lambda_a], \{a \ / \ T_A$, which can be interpreted as a delay that the user would experience when using a congested road, and formulate the Lagrange relaxation of SO (4) as

$$\max_{\lambda \geq 0} \min_{f} \ f^T \Lambda^T t^0 + \lambda^T (\Lambda f \quad c) \tag{5.1}$$
$$s.t. \ Ef = q^r, f \approx 0 \tag{5.2}$$

For fixed $\bar{\lambda} \approx 0$, problem (5.1)-(5.2) can be decoupled into the subproblems shown in (5.3)-(5.4) with respect to each each O-D pair (r, s), since the capacity limitation is relaxed. Moreover, the constant term $\bar{\lambda}^T c$ in the objective function can be omitted, then the relaxed SO problem for O-D pair (r, s) / R_T^{RS} can be written as

$$\min_{f^{rs}} \ (t^0 + \bar{\lambda})^T \Lambda^{rs} f^{rs} \tag{5.3}$$

$$s.t. \ E^{rs} f^{rs} = q_{rs}^t, f^{rs} \approx 0 \tag{5.4}$$

where $f^{rs} = [f_k^{rs}], \{k \ / \ K_{rs}$, notations Λ^{rs} and E^{rs} represent the sub-matrices in Λ and E corresponds to O-D pair (r, s); Problem (5.3)-(5.4) can be interpreted as follows: when each driver between O-D pair (r, s) makes his own decision given the delay vector $\bar{\lambda}$ without a central coordinator, he must pick up the path with minimal travel time, i.e., $f_K^{rs} = q_{rs}^t$, where $K = \min_k [t_k^{rs}, \{k]$, and $f_k^{rs} = 0$, $\{k \not\models K$. The total travel time between O-D pair (r, s) is $t^{rs} = (t^0 + \bar{\lambda})^T \Lambda^{rs}$. It is apparent that SO and UE give the same traffic flow pattern in the absence of capacity constraints. Now suppose the optimal solution of LP (4.1)-(4.5) is x^*, and the corresponding dual variable of (4.4) is λ^*. Because strong duality holds for LPs, which means the duality gap is 0 at the optimal primal-dual pair (x^*, λ^*), we can arrive at an important conclusion [32], [33]:

$$\begin{array}{ll} (x^*, t^0) & \text{is a traffic assignment at SO} \\ (x^*, t^0 + \lambda^*) & \text{is a traffic assignment at UE} \end{array}$$

Now we can see that the travel delay caused by congestion is characterized by the Lagrange dual multipliers, which depends on the traffic flow condition in the whole system, rather than a univariate function $t_a(x_a)$ which solely depends on x_a in a particular link. The way of defining travel latency distinguishes Beckmann's model and Nesterov's model. It is worth mentioning that the UE and SO in Nesterov's model share the same traffic flow pattern. The two states only differ in the travel times, indicating different vehicle densities.

Comparing the Beckmann's model and Nesterov's model

The traffic flow distributions provided by the Beckmann's model and the Nesterov's model are compared on the system shown in Fig. 1. For simplicity, travel time functions on link $\boxed{1}$ and link $\boxed{2}$ are constants; while those corresponding to link $\boxed{3}$ and link $\boxed{4}$ are linear functions, as shown in Fig. 1. The link capacity is also given in Fig. 1.

Owing to the flow conservation equations, there must be $x_1^* = 2$ and $x_2^* = 3$ for both models. The traffic flow solution offered by Beckmann's TAP satisfies the following condition [31], which is equivalent to the definition of UE:

Wardrop UE condition [31], [43]: the travel times (between a certain O-D pair) on all used paths are equal, and no greater than that would be experienced on any unused paths.

In this regard, in Beckmann's model, the link flow x_3 and x_4 must satisfy

$$\begin{aligned} 20 + x_3 &= 16 + 2x_4 \\ x_3 + x_4 &= 5 \end{aligned}$$

which gives $x_3^* = 2$ and $x_4^* = 3$, so the corresponding path travel time is given as

$$\begin{aligned} t_1^{14} &= t_1(x_1^*) + t_3(x_3^*) = 32 \\ t_2^{14} &= t_1(x_1^*) + t_4(x_4^*) = 32 \\ t_1^{24} &= t_2(x_2^*) + t_3(x_3^*) = 37 \\ t_2^{24} &= t_2(x_2^*) + t_3(x_4^*) = 37 \end{aligned}$$

In accordance with LP (4), the link flow x_3 and x_4 offered by Nesterov's TAP must solve the following LP

$$\begin{aligned} \min \ & x_3 + 2x_4 \\ s.t. \ & x_3 + x_4 = 5 \\ & x_3 \geq 2 : \ \lambda_3 \\ & x_4 \geq 3 : \ \lambda_4 \end{aligned}$$

The optimal solution is $x_3^* = 2$, $\lambda_3^* = 0$ and $x_4^* = 3$, $\lambda_4^* = 4$, indicating that link $\boxed{4}$ is congested. The link travel times at the UE pattern are given by

$$\begin{aligned} t_1^{14} &= t_1^0 + t_3^0 + \lambda_3^* = 10 + 20 + 0 = 30 \\ t_2^{14} &= t_1^0 + t_4^0 + \lambda_4^* = 10 + 16 + 4 = 30 \\ t_1^{24} &= t_2^0 + t_3^0 + \lambda_3^* = 15 + 20 + 0 = 35 \\ t_2^{24} &= t_2^0 + t_4^0 + \lambda_4^* = 15 + 16 + 4 = 35 \end{aligned}$$

which also meet the Wardrop principle: no road user can reduce travel time by changing his route unilaterally.

In this particular case, Backmann and Nesterov TAPs offer consistent UE in term of link traffic flow. Comparative studies on well-studied TNs and real-world large-scale networks in Switzerland suggest that both TAP models provide similar UEs, especially when capacity constraints are imposed in Backmann's model [34]. It is also suggested in [34] that the travel times of the two models are generally not comparable, as they are based on different assumptions.

B. Distribution System Model

The charging facilities in the ETN are served by a PDN, which is represented by a connected graph $G_E = [E_N, E_L]$, where E_N and E_L represent the sets of electrical buses and distribution lines, respectively. In power system engineering practice, a PDN is intentionally operated with a tree topology. Without loss of generality, we assume that each electrical bus connects to one generator and serves one load with constant traditional demand and possible EV charging requests which may depend on the traffic flow pattern. In this regard, the interface equation between TN and PDN can be expressed by

$$p_i^d = p_i^{dc} + \eta \sum_{a \in C(i)} x_a, \ \{i \ / \ E_N \tag{6}$$

where $C(i)$ denotes the set of charging facilities on link a that are served by bus i. We assume the average charging request on link a is a linear function of the vehicle flow x_a. The charging rate η can be determined from an intuitive forecast according to the penetration level of EV or the operating experience from existing ETNs. It is a key parameter for modeling the grid impact of the EV load. This assumption is reasonable for a system-level study of the ETN and also

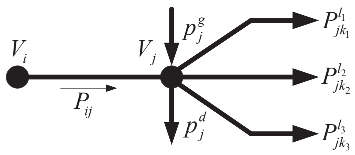

Fig. 2. Typical topology in a radial distribution network.

adopted by [27] and [28], although it seems simple and may be different from the vehicular arrive rate functions or probability distributions for studying a single charging station. Moreover, it provides a versatile and convenient way to model the interdependency between TN and PDN.

The typical connection of distribution lines (each line is denoted by a tuple (l, i, j), where i is the head bus and j is the tail bus) of a radial PDN is shown in Fig. 2. The voltage magnitudes V_i and V_j, and the power flows P_{ij}^l and Q_{ij}^l delivered through line l, are amenable to BPF equations developed in [35]–[37]:

$$P_{ij}^l + p_j^g \quad r_{ij}^l \frac{(P_{ij}^l)^2 + (Q_{ij}^l)^2}{V_i^2} = \sum_{k \in \pi(j)} P_{jk}^l + p_j^d \tag{7.1}$$

$$Q_{ij}^l + q_j^g \quad x_{ij}^l \frac{(P_{ij}^l)^2 + (Q_{ij}^l)^2}{V_i^2} = \sum_{k \in \pi(j)} Q_{jk}^l + q_j^d \tag{7.2}$$

$$\begin{aligned} V_j^2 = V_i^2 \quad & 2(r_{ij}^l P_{ij}^l + x_{ij}^l Q_{ij}^l) \\ & + \left[(r_{ij}^l)^2 + (x_{ij}^l)^2\right] \frac{(P_{ij}^l)^2 + (Q_{ij}^l)^2}{V_i^2} \end{aligned} \tag{7.3}$$

where $\pi(j)$ is the set of child buses of bus j; it contains more than one element only when multiple distribution lines start from bus j. The left-hand side of equation (7.1)/(7.2) is the active/reactive power injected into bus j; the right-hand side of equation (7.1)/(7.2) is the active/reactive power withdrawn from bus j. In engineering practice, line losses account for only a small part of the total delivered power. By ignoring the loss terms in (7.1) and (7.2), we arrive at the following simplified power balancing equations:

$$P_{ij}^l + p_j^g = \sum_{k \in \pi(j)} P_{ik}^l + p_j^d \tag{8.1}$$

$$Q_{ij}^l + q_j^g = \sum_{k \in \pi(j)} Q_{ik}^l + q_j^d \tag{8.2}$$

It was justified in [35] that the third term on the right-hand side of (7.3) should generally be much smaller than the second term, and thus can be ignored. Moreover, since the voltage magnitude at every bus is close to the reference voltage at the slack bus, we have $(V_j \quad V_0)^2 \to 0$, indicating that $V_j^2 \to V_0^2 \quad 2V_0(V_0 \quad V_j)$. Substituting it into (7.3) yields

$$V_j = V_i \quad \frac{r_{ij}^l P_{ij}^l + x_{ij}^l Q_{ij}^l}{V_0} \tag{8.3}$$

The linearized BPF equations (8.1)-(8.3) have been justified and extensively used in distribution system studies, such as in [38]–[40].

III. Mathematical Formulation

A. Expansion Planning Model

Some basic settings for ETN expansion planning are summarized below.

1) New roads, whose number is given by $\sum_{n=0}^{N_R} 2^n z_{an}^t, \{a,$ where z_{an}^t / $\}0, 1|$, can be built in parallel with existing links. They have the same capacity Δc_a.
2) New distribution lines, whose number is expressed by $\sum_{n=0}^{N_L} 2^n z_{ijn}^{dl}$, $\{(i, j, l)$, where z_{ijn}^{dl} / $\}0, 1|$, can be built in parallel with existing distribution lines. For simplicity, we assume that every distribution line between buses i and j has the same parameter. In this regard, the resistance and reactance are reduced to $1/(1 + \sum_{n=0}^{N_L} 2^n z_{ijn}^{dl})$ compared to their original values. Meanwhile, the maximal allowed active and complex power flow become $(1 + \sum_{n=0}^{N_L} 2^n z_{ijn}^{dl}) P_l^u$ and $(1 + \sum_{n=0}^{N_L} 2^n z_{ijn}^{dl}) S_l$, respectively. From another perspective, the power flow in line l is reduced to $1/(1 + \sum_{n=0}^{N_L} 2^n z_{ijn}^{dl})$, while the line parameters remain the same.
3) New charging facilities, whose number is given by a positive integer variable n_a^c, and whose capacity is Δp_a^c, can be built beside link a. Unlike existing studies which aim to enlarge the service area of charging stations, we assume these facilities can be built beside any link. This assumption is suitable for main roads, such as the ring expressway system of metropolitan areas. Nevertheless, it can be relaxed by specifying candidate sites where the charging facilities can be built.
4) New generators, whose number is given by a positive integer variable n_i^g, can be built at each electrical bus of the PDN. We can also specify candidate locations for generators. For example, in case study, the available site for generators is a subset of E_B.

We use binary variables to represent the number of lanes and distribution lines because these variables will be multiplied by other continuous variables. These nonlinear terms involving the production of a binary variable and a continuous variable will be linearized by using techniques from integer algebra. More importantly, this expression uses only a logarithmic number of binary variables. For example, if at most 7 additional lanes can be invested in on link a, we use $\log_2(7 + 1) = 3$ binary variables for link expansion.

Given the desired traffic and electrical demands to be served, the mathematical model of ETN expansion planning is presented as follows:

$$\begin{aligned} \min \;& F_{TN} + F_{PDN} \\ s.t. \;& \text{Cons-PDN} \\ & \text{Cons-Couple} \\ & \}x, \lambda| \;\; \text{solves (4)} \end{aligned} \tag{9.1}$$

The first term F_{TN} in the objective function is the total cost of the TN, which is given by

$$F_{TN} = \sum_{a \in T_A} \left[\omega(t_a + \lambda_a) x_a + \frac{\mathrm{Pr}_a^r}{\kappa} \sum_{n=0}^{N_R} 2^n z_{an}^t \right] \tag{9.2}$$

where $\sum_{a\in T_A}(t_a+\lambda_a)x_a$ is the total vehicle travel time under the UE pattern, ω is the monetary value of time, the first term in F_{TN} is the equivalent travel cost of TN, the second term represents the investment cost on new roads, and κ is the discount factor, which leverages long-term investment cost and short-term operation cost.

The second term F_{PDN} in the objective function is the total cost of the PDN, which is given by

$$
\begin{aligned}
F_{PDN} = \sum_{i\in E_B}\left[a_i(p_i)^2 + b_i p_i\right] + \rho \sum_{j\in\pi(0)} P_{0j}^l + \\
\sum_{a\in T_A}\frac{\mathrm{Pr}_a^c}{\kappa}n_a^c + \sum_{l\in E_L}\sum_{n=0}^{N_L}\frac{\mathrm{Pr}_{ij}^l}{\kappa}2^n z_{ijn}^{dl} + \sum_{i\in E_N}\frac{\mathrm{Pr}_i^g}{\kappa}n_i^g
\end{aligned}
\tag{9.3}
$$

where the first term is the generator production cost, which is a convex quadratic function; P_{0j}^l is the active power delivered through distribution line l that connects to the slack bus, $\pi(0)$ is the set of child buses of the slack bus, and such that the second term is the purchasing cost paid to the power market; the third to fifth terms represent investments on the new charging facilities, distribution lines and generation units, respectively, which are discounted into short-term operation cost.

The operation constraint on the PDN is expressed as

$$
\text{Cons-PDN} = \text{Cons-BPF} + \text{Cons-BND}
$$

where Cons-BPF represents linearized BPF constraints, and is given by

$$
\text{Cons-BPF} =
\left\{
\begin{aligned}
& P_{ij}^l + p_j^g = \sum_{k\in\pi(j)} P_{jk}^l + p_j^d,\ \{(i,j,l) && (9.4.1)\\
& Q_{ij}^l + q_j^g = \sum_{k\in\pi(j)} Q_{jk}^l + q_j^d,\ \{(i,j,l) && (9.4.2)\\
& V_j = V_i \quad \frac{r_{ij}^l P_{ij}^l + x_{ij}^l Q_{ij}^l}{V_0\left(1+\sum_{n=0}^{N_L}2^n z_{ijn}^{dl}\right)}, \{(i,j,l) && (9.4.3)
\end{aligned}
\right\}
$$

and Cons-BND represents physical limitations of decision variables, which is defined as

$$
\text{Cons-BND} =
\left\{
\begin{aligned}
& 0 \geq p_i^g \geq p_i^+ + \Delta p_i^m n_i^g,\ \{i && (9.4.4)\\
& q_i^+ \quad \Delta q_i^m n_i^g \geq q_i^g \geq q_i^+ + \Delta q_i^m n_i^g, \{i && (9.4.5)\\
& V_i^- \geq V_i \geq V_i^+,\ \{i,\ Q_{ij}^l \approx 0, \{l && (9.4.6)\\
& 0 \geq P_{ij}^l \geq \left(1+\sum_{n=0}^{N_L}2^n z_{ijn}^{dl}\right) P_l^u && (9.4.7)\\
& \left\|\begin{matrix} P_{ij}^l \\ Q_{ij}^l \end{matrix}\right\|_2 \geq S_l\left(1+\sum_{n=0}^{N_L}2^n z_{ijn}^{dl}\right), \{l && (9.4.8)
\end{aligned}
\right\}
$$

The active power generation boundaries and reactive power generation boundaries of units, voltage magnitude boundaries of buses, and power flow bounds of distribution lines are included through (9.4.4)-(9.4.8), respectively. The coupling constraints are given by

$$
\text{Cons-Couple} =
\left\{
\begin{aligned}
& p_i^d = p_i^{dc} + \eta \sum_{a\in C(i)} x_a,\ \{i \\
& \eta x_a \geq \Delta p_a^c(n_a^c + c_a^{ce}),\ \{a
\end{aligned}
\right\}
\tag{9.5}
$$

where the first constraint is the interface equation defined by (6), and the second constraint requires that the charging demand on each road does not exceed the service capability that the charging facilities in service can offer.

The traffic flow x and possible delay λ are determined by the lower-level problem, which represents the Nesterov's TAP (4), where the road capacity is

$$
c_a = c_a^0 + \Delta c_a \sum_{n=0}^{N_R} 2^n z_{an}^t,\ \{a \tag{9.6}
$$

Therefore, expansion planning problem (9.1) with the UE constraint, which is an embedded optimization problem, comes down to an MPEC, which is challenging to solve. We will leave the solution method to the next section. It should be pointed out that the proposed modeling framework is readily extendable to incorporate multiple demand scenarios and multiple horizons. For the purposes of clarity, we only consider a fixed demand and a single period in the current version.

IV. An MICP Reformulation

Three bottlenecks prevent problem (9.1) from being solved efficiently:

1) The lower-level TAP (4) as a constraint.
2) The production term $\lambda_a x_a$ in F_{TN}.
3) The nonlinear voltage drop equation (9.4.3).

We will derive equivalent linear expressions for them by exploiting the special problem structure.

A. Linearizing the UE Constraints

Because the TAP (4) is an LP, its solution can be characterized by either the KKT optimality condition or the primal-dual optimality condition. We adopt the latter because it involves fewer constraints and a simpler structure without complementarity-slackness conditions. The primal-dual optimality condition of TAP (4) is given as follows:

$$
x \quad \Lambda f = 0,\ Ef = q^t,\ x \geq c,\ f \approx 0 \tag{10.1}
$$

$$
\lambda \approx 0,\ \mu \quad \lambda = t^0,\ E^T\pi \quad \Lambda^T\mu \geq 0 \tag{10.2}
$$

$$
\pi^T q^t \quad \lambda^T c = x^T t^0 \tag{10.3}
$$

where (10.1) and (10.2) are the feasible regions of primal and dual variables, respectively, and (10.3) is the strong duality condition, which enforces equal values on the primal and dual objectives. The feasible solution of (10.1)-(10.3) is also the UE of TN. In view of equation (9.6), the nonlinearity only exists in the term $\lambda^T c = \sum_a \lambda_a(c_a^0 + \Delta c_a \sum_{n=0}^{N_R} 2^n z_{an}^t)$.

We introduce a matrix variable $V_R^t = [v_{an}^t], \{a, \{n$, and define the vector of incremental capacity $\Delta c = [\Delta c_a], \{a$, the vector of initial capacity $c^0 = [c_a^0], \{a$, and a constant vector $b_R^t = [2^n], \{n$. If the relationship

$$v_{an}^t = \lambda_a z_{an}^t, \ \{a, \ \{n \tag{10.4}$$

holds, then we have a linear expression

$$\lambda^T c = \lambda^T c^0 + (\Delta c)^T V_R^t b_R^t \tag{10.5}$$

Since v_{an}^t represents the product of a binary variable z_{an}^t and a positive continuous variable λ_a, constraint (10.4) can be replaced by the following linear constraints:

$$\begin{aligned} 0 \geq v_{an}^t \geq M z_{an}^t, \ \{a, n \\ 0 \geq \lambda_a \quad v_{an}^t \geq M(1 \quad z_{an}^t), \ \{a, n \end{aligned} \tag{10.6}$$

where M is a big enough constant, which can be interpreted as the maximal possible delay. When $z_{an}^t = 0$, (10.6) enforces $v_{an}^t = 0$ and $0 \geq \lambda_a \geq M$; otherwise, when $z_{an}^t = 1$, (10.6) enforces $v_{an}^t = \lambda_a$ and $0 \geq v_{an}^t \geq M$. This is equivalent to the original expression in (10.4). The tightest value of M should be $M^* = \min_a \}\lambda_a^*|$, where $\lambda^* = [\lambda_a], \{a$ is the optimal dual variable of problem (9.1) and unknown in advance. Any $M \approx M^*$ without causing numeric issue is valid for (10.6). However, a smaller M will yield more efficient computation. A possible value of M is the estimated maximal delay on the link of TN from certain heuristic.

Now we can express the linearized UE constraints as

$$\text{Cons-TAP-UE-Lin} = \left\{ \begin{matrix} x \quad \Lambda f = 0, \ Ef = q^r, f \approx 0 \\ x \geq c^0 + \Delta C Z_R^t b_R^t, \quad (10.6) \\ \lambda \approx 0, \mu \quad \lambda = t^0, H^T \pi \quad \Lambda^T \mu \geq 0 \\ \pi^T q^r = x^T t^0 + \lambda^T c^0 + (\Delta c)^T V_R^t b_R^t \end{matrix} \right\} \tag{11}$$

where ΔC is a diagonal matrix whose elements are Δc_a, and the binary variable matrix Z_R^t is defined as $Z_R^t = [z_{an}^t], \{a, n$.

B. Linearizing the Objective Function

The production term $\lambda_a x_a$ in F_{TN} involves two continuous variables. There is no general methodology for linearizing this term without exploiting approximation. We derive an exact linear reformulation based on the special structure of TAP (4).

Keep in mind that x and λ should be the optimal primal and dual solution for LP (4). Recall the KKT optimality condition, complementarity and slackness hold for the inequality constraint and its dual variable, leading to the following equation:

$$\lambda^T (x \quad c) = 0$$

Hence, we have $\lambda^T x = \lambda^T c$. This equality is easy to understand. Recall Nesterov's assumption A2 and (2.2): if $x_a < c_a$, indicating that road a is not congested, then we must have $t_a = t_a^0$ and $\lambda_a = 0$, thus $\lambda_a x_a = 0$; if road a is congested and $\lambda_a > 0$, we must have $x_a = c_a$ and $\lambda_a x_a = \lambda_a c_a$. In summary, $\lambda^T x = \lambda^T c$ holds at the optimal solution.

Using (10.5) in the previous subsection, we can arrange F_{TN} as a linear form, i.e.,

$$\begin{aligned} F_{TN} = \omega(x^T t^0 + \lambda^T c^0 + (\Delta c)^T V_R^t b_R^t) \\ + \sum_{a \in T_A} \sum_{n=0}^{N_R} \frac{\text{Pr}_a^r}{\kappa} 2^n z_{an}^t \end{aligned} \tag{12}$$

The expressions of F_{TN} in (9.2) and (12) are exactly equivalent, as (10.6) has already been considered in (11), no approximation error is involved.

C. Linearizing the Voltage Drop Constraint

Multiplying both sides of (9.4.3) by $1 + \sum_{n=0}^{N_L} 2^n z_{ijn}^{dl}$ gives the following equation

$$V_j \left(1 + \sum_{n=0}^{N_L} 2^n z_{ijn}^{dl}\right) = V_i \left(1 + \sum_{n=0}^{N_L} 2^n z_{ijn}^{dl}\right) \\ \frac{r_{ij}^l P_{ij}^l + x_{ij}^l Q_{ij}^l}{V_0}, \quad \{(i, j, l)$$

The production terms involving the continuous variable V_i/V_j and the binary variable z_{ijn}^{dl} can be linearized following the method similar to that in subsection A. By introducing continuous variables $v_{ijn}^{dl1} = V_i z_{ijn}^{dl}$, $v_{ijn}^{dl2} = V_j z_{ijn}^{dl}$, $\{(i, j, l), \{n$, constraint (9.4.3) is equivalent to

$$\left(V_j + \sum_{n=0}^{N_L} 2^n v_{ijn}^{dl2}\right) = \left(V_i + \sum_{n=0}^{N_L} 2^n v_{ijn}^{dl1}\right) \\ \frac{r_{ij}^l P_{ij}^l + x_{ij}^l Q_{ij}^l}{V_0}, \quad \{(i, j, l) \tag{13.1}$$

$$0 \geq v_{ijn}^{dl1} \geq V_i^+ z_{ijn}^{dl}, \quad \{(i, j, l) \tag{13.2}$$

$$0 \geq V_i \quad v_{ijn}^{dl1} \geq V_i^+ (1 \quad z_{ijn}^{dl}), \quad \{(i, j, l) \tag{13.3}$$

$$0 \geq v_{ijn}^{dl2} \geq V_j^+ z_{ijn}^{dl}, \quad \{(i, j, l) \tag{13.4}$$

$$0 \geq V_j \quad v_{ijn}^{dl2} \geq V_j^+ (1 \quad z_{ijn}^{dl}), \quad \{(i, j, l) \tag{13.5}$$

Now we can express the convex PDN constraints as

$$\text{Cons-PDN-CVX} = \left\{ \begin{matrix} (9.4.1), & (9.4.2) \\ (13.1) & (13.5) \\ (9.4.4) & (9.4.8) \end{matrix} \right\} \tag{14}$$

When the integrality of binary variable is relaxed, Cons-PDN-CVX yields linear and second-order cone constraints.

D. The Final MICP Formulation

On the basis of the previous discussion, the ETN expansion planning problem can be cast as the following MICP with a convex quadratic objective as well as mixed integer linear and second-order cone constraints:

$$\begin{aligned} \min \quad & F_{TN} + F_{PDN} \\ s.t. \quad & \text{Cons-PDN-CVX} \\ & \text{Cons-TAP-UE-Lin} \\ & \text{Cons-Couple} \end{aligned} \tag{15}$$

where F_{TN} is defined in (12), F_{PDN} is defined in (9.3), Cons-PDN-CVX is defined in (14), Cons-TAP-UE-Lin is defined in (11), and Cons-Couple is defined in (9.5).

It should be pointed out that the MICP reformulation relies on the linear models of the TAP and PDN. The linearized BPF model for PDN is only appropriate for radial networks, because radiality is crucial for constructing the voltage drop equation (8.3). In current engineering practices, PDNs are intentionally operated with a tree topology. However, there may be meshed distribution networks in the future. In such circumstance, the proposed MICP algorithm is readily compatible if the network is weakly meshed and the power flow direction in each distribution line can be easily determined in advance. Otherwise we can use the direct current power flow model [44] which preserves the model linearity, while sacrificing the accuracy on bus voltage. Another choice is to use the traditional bus injection based power flow model [44], which is nonlinear and non-convex, and directly solve an MINLP, at the expense of extremely high computational burden.

Finally, the proposed method aims to help government agency make better city planning decisions, and may be less accurate for real-time operation of the transportation system and power distribution system, which is not the main focus of this paper.

V. Case Studies

A. Basic settings

The proposed model and method are applied to a test ETN in this section. The TN topology shown in the left of Fig. 3 with outer-loop ring expressways is highly emblematic among modern metropolises. A radial PDN is contrivedly created just for the purpose of study, whose topology is shown in the right of Fig. 3. The nodes of the TN are denoted by circles and numbered T1, T2, ×××, T12; the electrical buses of the PDN are represented by blue blocks and numbered E1, E2, ×××, E20. The coupling structure is also illustrated in the TN part of Fig. 3. The parameters of both TN and PDN infrastructures are provided in Tables I-III (the values are given in p.u. without particular mention). We simulate the scenario of evening rush hour when the majority of traffic leaves from the northwest and travels to the east and the south. Details of the O-D pairs and their trip rates are given in Table II. The rest of the system parameters are given separately below (in p.u. without particular mention).

The active and complex power flow limit of existing distribution lines are $P_l^u = 1.0, \{l$ and $S_l = 1.2, \{l$, respectively. The capacity of new road is assumed to be $\Delta c_a = 0.5 c_a$, $\{a$. The capacity of every new charging facility and generator is $\Delta p_a^c = 0.1$, $\{a$ and $\Delta p_i^m = 1.0$, $\Delta q_i^m = 0.2$, $\{i$, respectively. The discounted investment cost for each charging facility and generation unit is $\mathrm{Pr}_a^c/\kappa = \$10$ and $\mathrm{Pr}_a^g/\kappa = \$100$, respectively. The fixed electrical demand at each bus of the PDN is $p_i^{dc} = 0.02$ and $q_i^d = 0.01$. Most of the active power loads originates from the charging requests of EVs. This corresponds to the situation in which the PDN only serves on-road charging facilities. This setting highlights the role of EV demand, which couples the TN and PDN, in this particular test. The lower bound and upper bound of bus voltage magnitude are $V_i^f = 0.93$ and $V_i^r = 1.05$, respectively, and the voltage magnitude at the slack bus is $V_0 = 1.04$. The electricity price at the slack bus is $\rho = \$1600$; the monetary value of unit travel time is $\omega = \$0.3333/\mathrm{min}$; and the charging rate of unit traffic flow is $\eta = 0.02$. We assume that every existing road of the TN and existing distribution line of the PDN can be considered for expansion. The charging facilities can be built at the roadside and connected to one electrical bus of the PDN. The available sites for generators are located at buses E7, E10, E11, and E14. Four identical units, whose parameters are $p_i^+ = 2.0$, $q_i^+ = 0.4$, $a_i = \$30$, and $b_i = \$400$, have already been connected to these buses. All simulations are implemented on a laptop computer with Intel i5-3210M CPU and 4 GB memory. MICP is coded in MATLAB environment with YALMIP toolbox [45] and solved by calling CPLEX [46].

TABLE I
Parameters of the Links

Link	c_a	Pr_a^r/κ	t_a^0 (min)	n_a^{ce}
T1-T3	18.0	50	6	2
T1-T2	20.0	80	10	4
T2-T6	17.0	50	6.5	4
T1-T4	9.8	35	5	1
T2-T5	7.9	35	5.5	1
T3-T4	8.5	35	6	2
T4-T5	13.5	70	12	3
T5-T6	8.2	40	6.5	2
T3-T7	19.0	80	10.2	4
T4-T8	14.0	75	11.5	3
T5-T9	13.8	75	12.5	2
T6-T10	20.0	80	10.5	3
T7-T8	8.9	35	5.8	1
T8-T9	13.2	70	11	2
T9-T10	9.15	35	5.9	2
T7-T11	17.5	50	6.3	4
T8-T11	9.76	35	5.7	2
T9-T12	8.97	35	5.8	1
T12-T10	18.2	50	6.1	0
T11-T12	20.0	80	9.8	3

TABLE II
O-D pairs and their trip rates

O-D pair	q_{rs}	O-D pair	q_{rs}	O-D pair	q_{rs}
T1-T6	5	T3-T6	7	T4-T9	6
T1-T10	6	T3-T10	7	T4-T10	7
T1-T12	6	T3-T12	6	T4-T12	5
T1-T11	5	T3-T11	5		

B. Results

The ETN expansion planning problem (15) is solved under four traffic demand scenarios. The reference demand is given in Table II. In each scenario, we increase the trip rates of all the O-D pairs according to the same portion. More precisely, parameter $q_{rs}^t, \{(r,s) \;/\; D_T^{RS}$ is increased by 25%, 50%, 75%, and 100%, respectively. The optimal expansion planning strategies under different demand scenarios are given in Tables IV-VII.

From Table IV and Table V we can see that more roads and charging facilities are invested in when the traffic demand increases. Table VI demonstrates that one generation unit will be built at bus E10 in the last two scenarios; no generator will be invested in for the other two scenarios with lower traffic

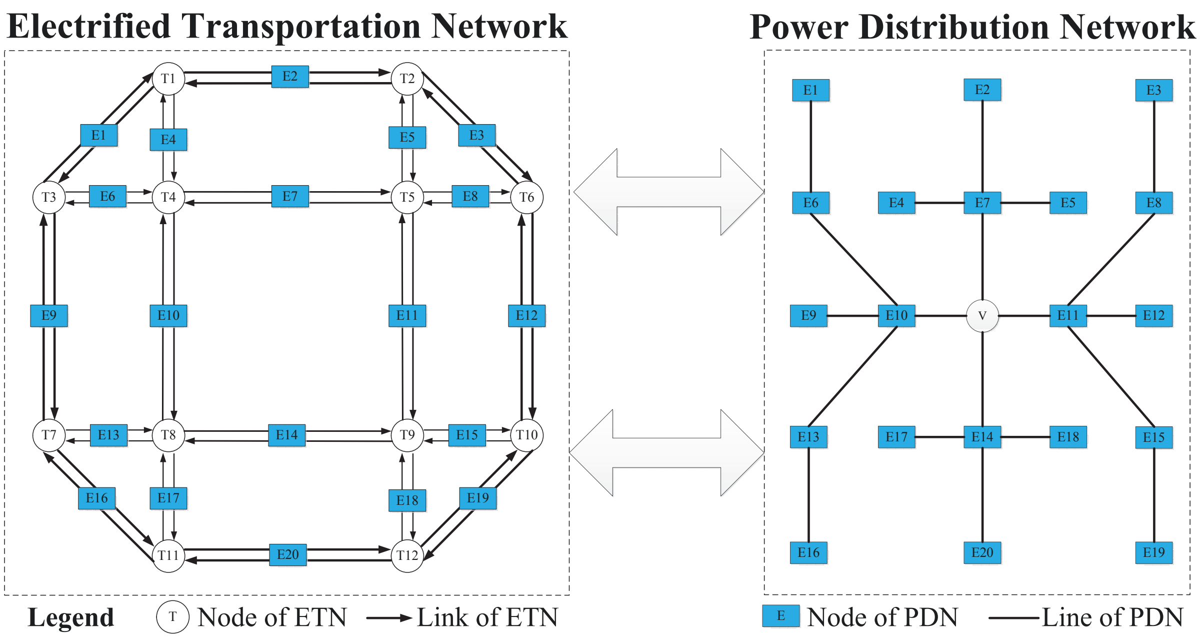

Fig. 3. Topology of the test ETN.

TABLE III
PARAMETERS OF THE DISTRIBUTION LINES

Line	r	x	Pr^l_{ij}/κ ($)
E0-E7	0.081	0.061	140.8
E0-E10	0.066	0.042	185.2
E0-E11	0.061	0.041	196.1
E0-E14	0.079	0.063	140.8
E7-E2	0.115	0.08	102.6
E7-E4	0.131	0.084	93.0
E7-E5	0.123	0.077	100.0
E10-E9	0.135	0.081	92.6
E10-E6	0.107	0.073	111.1
E10-E13	0.111	0.075	107.5
E6-E1	0.127	0.083	95.2
E13-E16	0.119	0.078	101.5
E11-E12	0.132	0.079	94.8
E11-E8	0.105	0.07	114.3
E11-E15	0.115	0.082	101.5
E8-E3	0.122	0.083	97.6
E15-E19	0.12	0.08	100.0
E14-E20	0.119	0.077	102.0
E14-E17	0.133	0.09	89.7
E14-E18	0.13	0.088	91.7

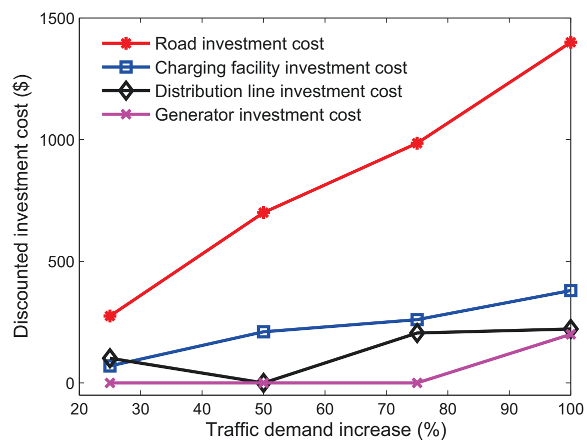

Fig. 4. Investment costs for different load scenarios.

demands. Table VII shows that distribution lines E10-E13, E13-E16, E11-E8, and E8-E3 are candidates for expansion in this problem, while others remain unchanged. As a result, the growing trends of expansion costs corresponding to different components are plotted in Fig. 4, showing the dominant status of road investments over others.

The reason can be explained by exploring the system topology and the equilibrium traffic flow pattern shown in Fig. 5. We can observe that links T4-T8, T7-T11, T2-T6, and T5-T6 are carrying heavy traffic flows, leading to higher charging demands at buses E10, E16, E3 and E8. Moreover, E3 and E16 are the terminal buses of the PDN. Delivering power to these buses introduces higher voltage drops on distribution lines. In this regard, to enhance the voltage magnitude at these buses, distribution lines E10-E13, E13-E16, E11-E8, and E8-E3 should be given higher priorities for upgrading. It is also interesting to notice that although the traffic demand is increased by 50% in the second scenario, which is higher than that in the first scenario, no generator or distribution lines will be built from a social optimal perspective, while one new distribution line is put into service in the first scenario. This apparently shows the importance of considering the interdependency in system planning.

To highlight the impact of EV charging request on both TN and PDN, we assume the PDN solely serves on-road charging

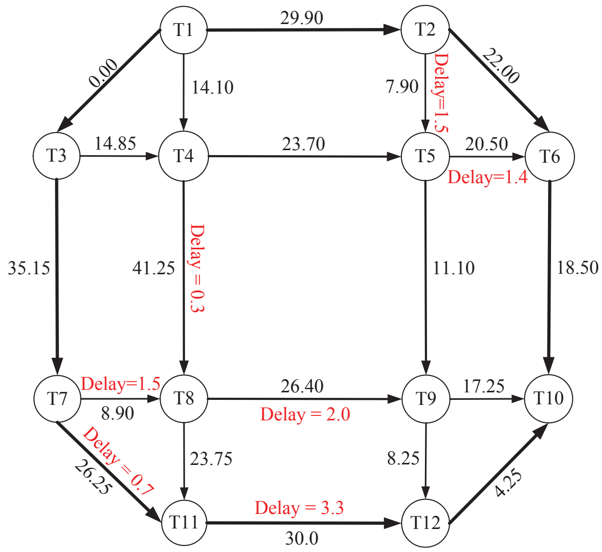

Fig. 5. Distribution of traffic flows and delay at the UE.

TABLE IV
ROAD EXPANSION STRATEGIES

Link	Demand increase			
	25%	50%	75%	100%
T1-T3	0	0	0	0
T1-T2	0	1	1	1
T2-T6	0	0	1	1
T1-T4	0	0	0	1
T2-T5	0	0	0	0
T3-T4	1	1	1	2
T4-T5	1	2	2	2
T5-T6	1	1	3	3
T3-T7	1	1	2	2
T4-T8	0	1	2	4
T5-T9	0	1	0	0
T6-T10	0	0	0	0
T7-T8	0	1	0	0
T8-T9	0	0	0	2
T9-T10	0	2	0	2
T7-T11	1	0	2	1
T8-T11	0	2	2	3
T9-T12	0	0	0	0
T12-T10	0	0	0	0
T11-T12	0	0	1	1

facilities in this particular test. Certainly, the proposed method can be applied to more general cases with growing traditional power demands and EV charging loads. In such circumstance, the investment in the PDN will have a larger share of the total investment cost.

A few more observations can be made about upgrading the generators and distribution lines. The ultimate goal of ETN capacity expansion is to meet the charging demand without violating operating constraints. There are two options for acquiring electricity: producing it from local generators, or purchasing it from the main grid. If the marginal cost of energy production is lower than the price in the market, and the generators can be arbitrarily placed where they are needed, then there is no need to build additional distribution lines. On the other hand, when producing electricity is expensive and

TABLE V
CHARGING FACILITY EXPANSION STRATEGIES

Link	Demand increase			
	25%	50%	75%	100%
T1-T3	1	0	0	0
T1-T2	0	1	2	2
T2-T6	0	1	1	2
T1-T4	0	1	1	2
T2-T5	0	0	0	0
T3-T4	0	1	1	1
T4-T5	1	3	3	2
T5-T6	1	1	3	3
T3-T7	2	2	3	4
T4-T8	0	2	3	6
T5-T9	0	2	0	0
T6-T10	1	1	2	2
T7-T8	0	1	0	1
T8-T9	0	1	1	4
T9-T10	0	2	0	2
T7-T11	1	0	2	2
T8-T11	0	2	2	3
T9-T12	0	0	0	0
T12-T10	0	0	0	0
T11-T12	0	0	2	2

TABLE VI
GENERATOR EXPANSION STRATEGIES

Link	Demand increase			
	25%	50%	75%	100%
E7	0	0	0	0
E10	0	0	1	1
E11	0	0	0	0
E14	0	0	0	0

TABLE VII
DISTRIBUTION LINE EXPANSION STRATEGIES

Link	Demand increase			
	25%	50%	75%	100%
E10-E13	0	0	1	1
E13-E16	1	0	0	0
E11-E8	0	0	0	1
E8-E3	0	0	1	0

TABLE VIII
OTHER QUANTITIES

Quantities	Demand increase			
	25%	50%	75%	100%
Composite total cost (p.u.)	3482.9	4536.9	5814.3	7030.0
Average travel time (min)	27.3	26.8	26.98	27.42
Computation time (s)	2.34	15.9	126.1	101.5

more energy is delivered from the slack bus, or the available sites for generators are restricted, additional distribution lines may need upgrading to circumvent potential voltage or power flow violation.

The composite total cost (defined as $F_{TN} + F_{PDN}$), the average vehicle travel time of TN (defined as $\sum_a x_a(t_a^0 + \lambda_a)/\sum_{rs} q_{rs}^r$), and the computation times in different scenarios are provided in Table VIII. Table VIII shows that when the traffic demand grows, the total cost increases, but the average travel time is maintained at about 27 minutes per vehicle, which indicates that the road expansion planning strategy is reasonable. The congestion pattern shown in Fig. 5 associating with the highest traffic demand scenario demonstrates that the delay on each congested road is moderate and acceptable. In general, the computation time increases as the traffic demand grows. However, this will not become a critical limitation for practical usage. On the one hand, the time scale of planning problems is long, hence it is acceptable and desired to spend a few hours or even a few days to seek a better solution which may cut down the expense notably in the long run. On the other hand, the infrastructures of the ETN should be upgraded every few years; in this regard, the traffic demand is unlikely to grow remarkably (usually less than 100%). A multi-period model can also be used to tackle long term planning problems. Nevertheless, solving mixed integer programs for realistic large-scale instances is still very challenging. In such circumstances, one can accept the best solution found in a pre-specified time limit.

VI. Conclusions

As one of the first few attempts in studying the interdependent urban electrified transportation infrastructure, this paper proposes a comprehensive model for the optimal expansion planning of roads, charging facilities, generators, and distribution lines, which captures the interaction between the TN and PDN. The proposed model minimizes the total composite cost subject to the traffic equilibrium condition and the power flow constraint. The traffic equilibrium is modeled by Nesterov's TAP, which does not depend on the particular road latency function and defines the delay through Lagrangian dual multipliers. It is suitable for the planning problem because it incorporates explicit capacity limit on road traffic flow. The linearized BPF model directly models bus voltage magnitudes and line power flows, and is suitable for PDN with a tree topology. An exact MICP for the proposed ETN expansion planning model is developed without exploiting approximation. This approach is able to find the global optimal solution for moderately sized ETNs with reasonable computation effort.

Future research will be focused on developing more sophisticated models that incorporate multiple traffic patterns (representing several critical scenarios, such as morning rush hours and evening rush hours, or potential uncertainties), as well as suitable decomposition methods for solving this challenging problem, such as the progressive hedging approach [47] and the Benders decomposition algorithm [48]. More dedicated description of the charging demand using link vehicular flow is also worth studying. This may require developing distance and battery status constrained UE model. Dynamic UE model [49], [50], which allows the modeling of global positioning system enabled dynamic rerouting behavior, is also one of our on-going research. In summary, we believe the ETN in line with the emerging trend of transportation electrification will be a very promising research direction in the near future.

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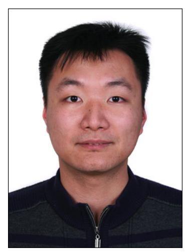

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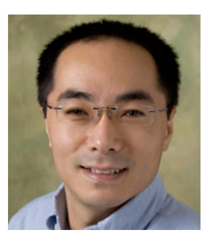

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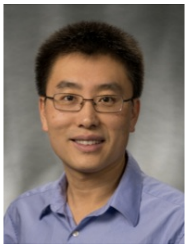

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