

Piecewise and Unified Phase Variables in the Control of a Powered Prosthetic Leg

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Abstract—Many control methods have been proposed for powered prosthetic legs, ranging from finite state machines that switch between discrete phases of gait to unified controllers that have a continuous sense of phase. In particular, recent work has shown that a mechanical phase variable can parameterize the entire gait cycle for controlling a prosthetic leg during steady rhythmic locomotion. However, the unified approach does not provide voluntary control over non-rhythmic motions like stepping forward and back. In this paper we present a phasing algorithm that uses the amputee’s hip angle to control both rhythmic and non-rhythmic motion through two modes: 1) a piecewise (PW) function that provides users voluntary control over stance and swing in a piecewise manner, and 2) a unified function that continuously synchronizes the motion of the prosthetic leg with the amputee user at different walking speeds. The two phase variable approaches are compared in experiments with a powered knee-ankle prosthesis used by an above-knee amputee subject.

I. INTRODUCTION

The methodology of analyzing the gait cycle as a sequence of discrete events (e.g., heel strike, toe off, etc.) [1], [2] often guides the design of control strategies used for powered prosthetic legs [3]–[8]. Under this control architecture, the joint motions of a powered prosthetic leg are represented as a sequence of finite states synchronized to the amputee motion by predefined switching rules. The prosthetic leg transitions from one state to another (e.g., from push-off to swing) based on switching rules measured from specific sensors located on the prosthetic or contralateral leg. A disadvantage of this methodology is that each finite state in the controller needs to be carefully tuned for each subject [3].

Recent efforts have unified the control of powered prosthetic legs over the gait cycle through the use of phase variables, i.e., monotonic signals that represent gait progression. The heel-to-toe movement of the Center Of Pressure (COP) controlled the progression of the prosthetic stance period in [9], whereas the swing period was controlled by two impedance-based finite states. The horizontal hip position

served as a phase variable in separate controllers for the stance and swing periods (i.e., two finite states for the gait cycle) in the simulations of [10]. Because the hip position is monotonic over the entire gait cycle, this “unified” phase variable enabled a single controller in the prosthetic leg simulations of [11]. Piecewise monotonic phase variables like the global hip angle can separately control the stance and swing periods [12] or be converted into unified phase variables during rhythmic motion [13], [14]. This allowed the experimental implementation of unified controllers in a powered ankle prosthesis used by a below-knee amputee [15] and more recently in a powered knee-ankle prosthesis used by an able-bodied subject wearing a leg-bypass adapter [16].

A unified phase variable provides many benefits during rhythmic locomotion but is incompatible with voluntary, non-rhythmic motions. Control methods based on a unified phase variable do not switch between finite states, resulting in smoother joint motion. Because there are fewer controllers and switching rules to tune, the unified approach provides “plug-and-play” functionality between users with minimal or no tuning [16]. However, unified phase variables based on phase oscillators require steady and rhythmic locomotion in order to have a well-defined sense of phase [13], [15]. In a recent implementation, the powered prosthetic leg remained rigid until rhythmic locomotion was recognized [16]. This weakness could be resolved by using a modified phase-based controller during these first few steps before transitioning to a unified phase-based controller. This modified phase-based controller should give the amputee subject voluntary control over the motion of the powered prosthetic leg during non-rhythmic motions (e.g., swinging the leg back and forth, stepping forward and back, and walking backwards).

Current controllers that give amputees voluntary control over robotic prostheses use a state-machine logic or direct EMG control dictated by residual antagonist muscle activation measurements [4], [17], [18]. These EMG-based approaches have been effective at controlling a single joint, but simultaneous volitional control of multiple joints remains a challenge. On the other hand, phase-based control methods are well suited for coordinating multiple joints through their mutual dependence on a single phase variable.

In this paper, we propose a piecewise variation of the phase-based controller [16] that uses the amputee’s hip angle to voluntarily control both a powered knee and ankle in a synchronized manner. Because the hip angle is piecewise monotonic through the gait cycle, this phase variable is divided between the stance and swing periods. This piecewise

This work was supported by the Eunice Kennedy Shriver National Institute of Child Health & Human Development of the National Institutes of Health under Award Number DP2HD080349. The content is solely the responsibility of the authors and does not necessarily represent the official views of the NIH. This work was also supported by NSF Award CMMI-1637704. Robert D. Gregg, IV, Ph.D., holds a Career Award at the Scientific Interface from the Burroughs Wellcome Fund. Dario J. Villarreal holds a Graduate Fellowship from the National Council of Science and Technology (CONACYT) from Mexico.

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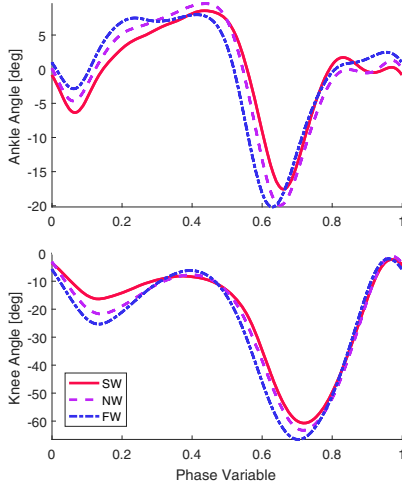


Fig. 1. The reference ankle (top) and knee (bottom) trajectories as functions of the phase variable for different walking speeds based on able-bodied data from [2]. These phase-based trajectories are the virtual constraints enforced by the prosthetic control system.

(PW) phase variable allows the amputee user to voluntarily take steps forward or back and accelerate into rhythmic walking, after which the prosthesis transitions to a unified phase-based controller. An improved version of the unified phase variable from [16] is implemented to adapt to different cadences, enabling the first experiments of the unified control approach with an above-knee amputee subject.

A description of the phase variable algorithm for controlling a powered prosthetic leg during both rhythmic and non-rhythmic motion will be given in Section II-A. The experimental protocol and the results of an amputee subject walking at different speeds using this algorithm are presented in Section II-B and III, respectively. Finally, a discussion about the advantages and disadvantages of the two phase variable methods is presented in Section IV.

II. METHODS

The powered knee and ankle prosthetic leg was controlled using a phase-based virtual constraint controller. A virtual constraint is a desired kinematic trajectory as a function of the phase variable. In the case of the powered prosthetic leg, the virtual constraints were designed from average able-bodied data for different speeds (Fig. 1) [2]. As the phase variable progresses through the gait cycle, the virtual constraints were enforced through a torque PD controller as described in [16].

Details of the virtual constraints, torque control law, and hardware used in the UT Dallas powered knee-ankle prosthesis have already been discussed in [16]. In the present paper, we focus on a new phase variable algorithm and its implementation for experiments with an above-knee amputee subject (Fig. 2).

A. Phase Variable Algorithm

The algorithm structure is mainly composed of two subsystems: 1) a PW phase variable algorithm and 2) a unified phase variable algorithm ($\hat{\phi}$). The default output of the phase

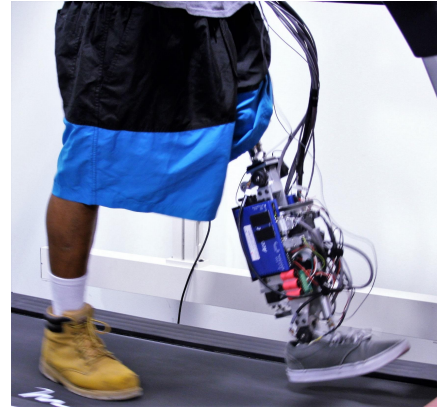


Fig. 2. The UT Dallas powered knee and ankle prosthetic leg being worn by a transfemoral amputee subject during the experiment.

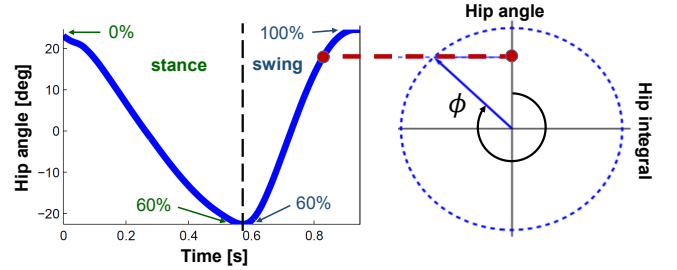


Fig. 3. Converting a sinusoidal hip trajectory (left) into a phase portrait (right) to determine phase variable $\phi \in [0, 2\pi)$. During steady walking the hysteresis in the left plot can be resolved by the signal's integral (right). The phase portrait is not circular during non-rhythmic motion, so ground contact sensing can be used to resolve this hysteresis.

variable algorithm is the PW phase variable. This variable is used when the subject is at rest or during non-rhythmic motions. When the algorithm detects the subject is rhythmically walking, then the output of the algorithm transitions to the unified phase variable (Fig. 3). The algorithm detects a steady walking condition by measuring rhythmic patterns of the hip angular position and velocity measurements. The transition between phase variables (i.e., PW to unified) does not happen until the measurement of both phase variables agree. This condition avoids undesired jumps in the phase variable value. In other words, once the algorithm detects the person is walking, the transition happens only when the unified phase variable crosses the PW phase variable. This generally takes a few strides. When the subject stops walking, the algorithm switches back to the PW phase variable.

1) *Piecewise Phase Variable Algorithm (PW)*: A diagram of the PW phase variable algorithm is presented in Fig. 4. The PW phase variable is a function of the global hip angle, denoted as $q_H(t)$, that is measured using an Inertial Measurement Unit (IMU - LORD MicroStrain Sensing Systems, Vermont, USA) attached to the top of the robotic leg's knee joint. The PW phase variable is divided into stance and swing periods depending on the measurement of a force sensitive resistor sensor (FSR - FlexiForce A401, Tekscan Inc., Massachusetts, USA) located inside the pyramid adapter of the prosthetic foot. A high value on the sensor measurement denotes stance (i.e., the subject's foot is in contact with the ground) whereas a low value denotes swing. In

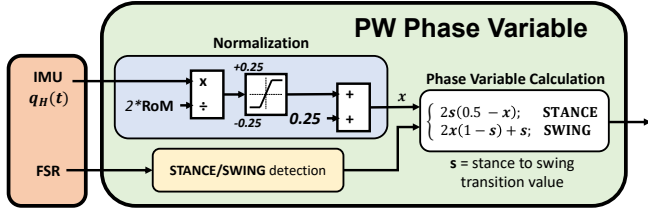


Fig. 4. Diagram of the piecewise phase variable algorithm. The variable s denotes the transition between stance and swing during locomotion (at a normal walking speed $s = 0.57$ according to [1]). The variable x represents the global hip angle $q_H(t)$ after normalization.

order to calculate a piecewise phase variable that is between the values of 0 and 1 (i.e., corresponding to 0 and 100% of the gait cycle), the hip angle needs to be normalized to a predefined range of motion. In particular, stance is normalized between $[0, s]$ and swing between $(s, 1)$, where $s \in (0, 1)$ denotes the desired phase transition value between stance and swing. This implies that we need to normalize the hip angle to twice a predefined range of motion ($2 \cdot \text{RoM}$). The normalized hip motion is saturated if its value is greater than 0.25 or smaller than -0.25 (corresponding to a hip angle outside the predefined normalized RoM). A constant offset value of 0.25 is added to the normalized hip angle in order to compute a PW phase variable that starts at zero. The PW phase variable is then calculated by

$$\begin{cases} 2s(0.5 - x), & \text{stance} \\ 2x(1 - s) + s, & \text{swing,} \end{cases} \quad (1)$$

where x is the hip angle after normalization and saturation. The stance-to-swing phase transition value was chosen as $s = 0.57$ based on able-bodied walking [1], Fig. 3.

Normalizing the PW phase variable using the RoM of the hip angle introduces a new challenge to the algorithm as humans typically walk with a varying RoM. The RoM value for normalization was selected to be fairly small (corresponding to short steps) in order to avoid instantaneous jumps of the phase variable value between stance and swing. However, a small RoM results in phase variable saturation whenever a subject increases his/her RoM (corresponding to longer steps). It was decided that, for the safety of the hardware and its user, saturation in the phase variable was preferred over instantaneous jumps, since the latter can introduce high frequency accelerations to the joint actuators. Moreover, longer steps are more typical during steady walking, which will be performed by the unified phase variable algorithm.

2) *Unified Phase Variable Algorithm* ($\hat{\phi}$): The unified phase variable was introduced in [14] and [16], where we defined the variable $\tilde{q}_H(t)$ as the integral of $q_H(t)$ over a gait cycle (i.e., $\tilde{q}_H(t) = \int_0^T q_H(\tau) d\tau$, where $t \in [0, T]$ represents the time duration of a gait cycle). It was shown in [13] that the global hip angle has a high correlation coefficient to a cosine function during a stride, thus it is expected that its integral traces a sinusoidal trajectory over a gait cycle [14]. The unified phase variable $\phi(t)$ is calculated as

$$\phi(t) = \frac{\text{atan2}(k\tilde{q}_H(t), q_H(t)) + \pi}{2\pi}, \quad (2)$$

where atan2 is the four-quadrant inverse tangent function. The variable k is a scaling factor that increases the linearity of the phase variable [13], Fig. 3. It is calculated every gait cycle as

$$k = \frac{|\max(q_H) - \min(q_H)|}{|\max(\tilde{q}_H) - \min(\tilde{q}_H)|}. \quad (3)$$

The global maximum and minimum hip angle and its integral need to be known for the phase variable computation in (3), so values from the previous gait cycle are used. It should be noted that in contrast to the PW phase variable, the unified phase variable does not have problems with normalization since by construction it is always in the range of $[0, 2\pi)$ due to the atan2 function.

The integral of the hip angle gets reset at heel strike every gait cycle in order to avoid drift. The heel strike condition is measured by the FSR inside the pyramid adapter. Furthermore, at every heel strike event the algorithm uses the information from previous strides to compute a correction term for the integral. For linearity of the unified phase variable, the sinusoidal trajectory from the integral of the global hip angle should start and end at zero for each gait cycle (given the cyclic nature of the global hip angle [13]). However, due to the variability of human locomotion the integral might not end exactly at zero, so values from the previous gait cycle are used. If the value of the integral at the end of the gait cycle is different than zero, then the positive area under the curve of the hip angle will not equal the negative area. This issue could potentially introduce a phase shift to the phase variable. One solution to this problem would be to shift the hip angle by subtracting the residual of its integral from the previous stride. Ideally this adjustment term in the hip angle would allow us to reach a perfect phase estimation over time. Nonetheless, during experiments the best adaptation was to gradually adjust the signal of the hip angle every gait cycle. The adapted unified phase variable is finally computed as

$$\hat{\phi}(t) = \frac{\text{atan2}(\hat{k}\hat{\theta}(t), \theta(t)) + \pi}{2\pi}. \quad (4)$$

where $\theta(t) = q_H(t) - x_0(T_N)$ is the adjusted hip angle for the N stride and $\hat{\theta}(t)$ is its integral. The term $x_0(T_N)$ is the gradual adjustment per stride defined by

$$x_0(T_N) = \sum_{n=1}^N \text{sgn}(\tilde{q}_H(T_n)/T_n), \quad (5)$$

where N represents the number of strides the subject has taken. The variable $\hat{k}$ is calculated as in (3) but taking into account the adjusted hip angle and its integral.

Computing a phase variable that is linear with respect to time enables smooth control of a powered prosthetic leg. The virtual constraints define joint angles as polynomial functions of a perfectly linear phase variable [16]. Thus, nonlinear regions in the phase trajectory will cause excessively slow or fast progression through the joint patterns [11].

B. Experimental Protocol

The experimental protocol was reviewed and approved by the Institutional Review Board (IRB) at the University of Texas at Dallas. The phase variable algorithm was implemented in the controller of the powered knee-ankle prosthesis from [16] and tested by a transfemoral amputee subject. These experiments used the same control gains determined during previous trials with an able-bodied subject wearing a bypass adapter [16]. The gains were tuned to a point where the system was more compliant, as it was noticed that stronger gains were uncomfortable during locomotion.

The experimental setup began by attaching an IMU to the top of the robotic leg's knee joint and aligning the sensor along the subject's sagittal plane. A rotation matrix $R \in SO(2)$ [19] was applied to the pitch and roll Euler angles in order to improve the accuracy of the hip angle reading with respect to the sagittal plane. The objective of this rotation matrix was to decouple the readings of the hip angle between the frontal and sagittal planes. This rotation matrix was computed by using a principal component analysis (PCA) [20] on the recorded signals of the IMU across several strides. Aligning the IMU to the sagittal plane was imperative to achieving reliable calculations of the phase variable. This calibration process took only a couple of minutes. Once the IMU was mounted and signals realigned, the amputee subject was given at least 10 minutes to get acclimated to the powered prosthetic leg for level ground walking between handrails. During this time a certified prosthetist made necessary adjustments to the pyramid adapter below the socket to ensure proper alignment of the prosthetic leg.

After the setup and acclimation period, the amputee subject performed an overground walking trial. During this trial the subject was asked to walk the length of the handrails, stop, turn around, and start again for a total of 60 seconds. This trial primarily tested voluntary control of non-steady walking motions, and consequently the phasing algorithm primarily output the PW phase variable.

After the overground trial was completed, the subject was then asked to step onto a treadmill for the next trials. The subject was provided a safety harness and began walking at a comfortable treadmill speed until acclimated. The subject was asked to select their preferred slow (SW), normal (NW), and fast (FW) walking speeds for level ground. The subject then performed a 60 second treadmill walking trial for each of the three speed conditions. Note that across all trials the control gains and the phase variable algorithm were not modified. However, for each trial the prosthetic leg enforced a predefined virtual constraint representing the average kinematic trajectories of an able-bodied subject for that particular walking speed [11].

Outlier gait cycles were removed for analyzing each trial. An outlier was defined as a gait cycle where the phase variable trajectory had a value outside three standard deviations from the average trajectory for that particular trial. Note that the average and standard deviation of the phase variable had to be computed for each trial prior to running

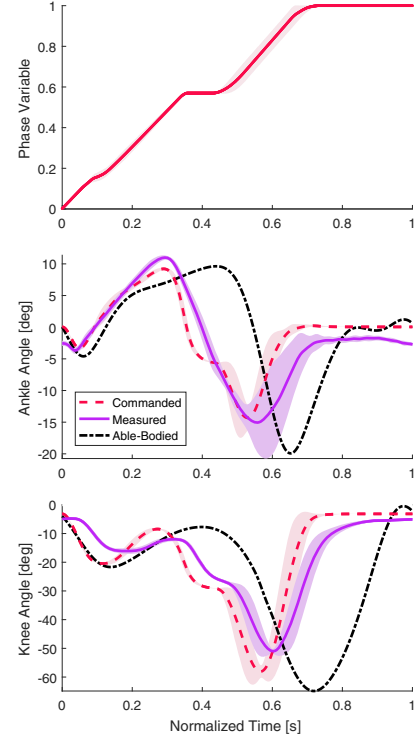


Fig. 5. The mean and ± 1 std ($N = 9$) of the PW phase variable algorithm is shown for the overground trial (top). The resulting commanded and measured joint trajectories (mean and ± 1 std) of the ankle (middle) and knee (bottom) are shown as a function of normalized time. The black dashed line represents the average joint angle trajectory of an able-bodied subject during rhythmic walking [2]. These able-bodied kinematics were enforced over phase.

the outlier detection algorithm. If an outlier gait cycle was detected, then that particular gait was discarded from the results. Outliers were removed because the amputee subject was an inexperienced user of powered prosthetic legs and had only a few minutes of training time. Thus, the hip angle trajectories of the amputee subject had more variability than the experienced able-bodied subject in the experiment of [16]. From the controls perspective, these outlier gait cycles yielded a phase variable calculation with a premature stance to swing transition.

III. RESULTS

Fig. 5 shows the results of the overground trial, which predominantly used the PW phase variable due to the short length of the parallel bars. The commanded and measured knee and ankle trajectories show that the controller produced fairly normative joint kinematics but with some differences in timing from rhythmic able-bodied data. These differences can be attributed to the non-steady nature of the overground trial and the saturation of the phase variable before the stance-to-swing transition (Fig. 5, top). However, the PW phase variable enabled the subject to comfortably control the powered leg during non-steady walking and voluntary motions such as starting and stopping. The supplemental video also shows the ability of the subject to walk backwards with the PW phase variable.

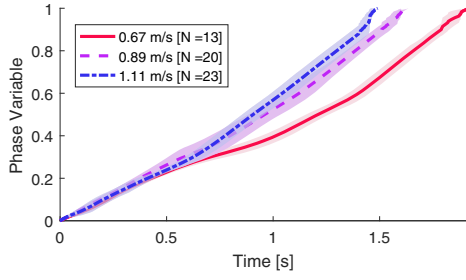


Fig. 6. The output of the phase variable algorithm for the amputee subject across three different self-selected speeds. The amputee subject self-selected the walking speeds to be: SW = 0.67 m/s, NW = 0.89 m/s, and FW = 1.11 m/s. The variable N represents the number of strides used in the calculation of the mean and ± 1 std for each walking speed condition.

Fig. 6 shows the average output of the unified phase variable for each walking speed trial on the treadmill. Observe that the slope of the phase variable with respect to time changed for each walking speed. This shows that the phase variable captures the speed of the amputee's gait. The variability of the phase variable is a direct consequence of variability in the user's hip motion, which subsequently causes variation in the prosthetic joint kinematics.

Fig. 7 shows the commanded and measured knee and ankle trajectories for the level ground NW treadmill trial over normalized time and phase. Over phase, the measured trajectories have a slight phase delay with respect to the commanded trajectories. This was a consequence of tuning the control gains to achieve more compliant behavior on the robotic leg, Section II. The tradeoff for increased user comfort was increased tracking error.

It can be seen from Fig. 7 (top) that over phase, the commanded trajectory has no variance and that the controller produced small variations in the measured joint kinematics. This is due to the virtual constraints of the controller being parameterized over the phase variable. On the other hand, there is more variability in the commanded and measured joint kinematics over normalized time (Fig. 7, bottom). This variability comes from the variance in the user's hip motion, which in turned affected the phase variable over time (Fig. 6). It is important to note that over time, the measured joint trajectories do resemble the kinematics of an able-bodied subject during locomotion. In conclusion, the prosthetic leg exhibited able-bodied behavior as the amputee subject walked at different speeds.

IV. DISCUSSION

The PW phase variable (Section II-A.1) parameterizes stance and swing independently and switches based on prosthetic ground contact. Fig. 5 shows that this phase variable is not entirely monotonic around a value of 0.6, which is a consequence of using the prior stride's hip RoM to normalize the phase variable. However, directly enslaving the prosthetic joint motion to the amputee's hip angle provides volitional control for starting, stopping, and non-steady walking. For example, the PW phase variable allowed the subject to walk both forward and backward at will (see supplemental video). On the other hand, once the subject achieved steady and

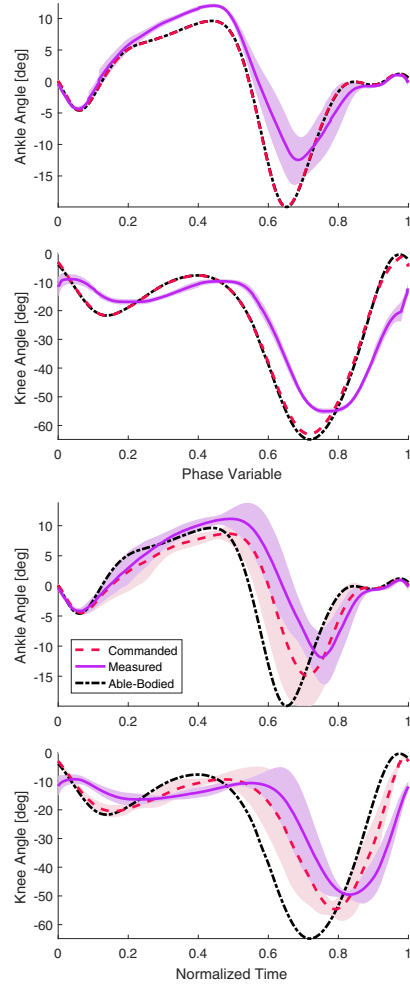


Fig. 7. The ankle and knee commanded and measured joint kinematics (mean and ± 1 std) as a function of unified phase variable (top) and normalized time (bottom) for the NW trial (N = 20). The black dashed line represents the average joint angle kinematics of an able-bodied subject.

rhythmic walking, the unified phase variable provided a smoother and more linear phase trajectory throughout the gait cycle as seen in Fig. 6.

The fact that the unified phase variable works better in steady gait (i.e., dynamic walking) could be analogous to human walking when reflex pathways with the spinal cord are in control of the walking motion without involvement of the brain [21]. Therefore, it could be said that each of the phase variables (i.e., PW and unified) could control the powered prosthetic leg at different cognitive states of the subject's locomotion. The PW phase variable controls volitional movements whereas the unified phase variable takes over when the subject is walking using his/her reflex pathways. This specific behavior is only possible due to the flexibility and adaptability of the phase variable algorithm. This adaptability also allows the phase variable algorithm to seamlessly parameterize joint patterns across different walking speeds.

During experimentation it was noticed that the unified phase variable algorithm had some difficulties adapting to non-symmetric gaits as it was designed based on symmetric

and consistent able-bodied gaits. Section II-A.2 mentioned that the unified phase variable depends on the hip RoM from the previous stride. A different RoM in the current stride compromises the linearity of the phase variable trajectory, which was observed during occasional non-symmetric and non-consistent steps. During the acclimation period, it was essential for the subject to relearn how to walk symmetrically and trust the powered prosthetic leg. Once the amputee became accustomed to walking symmetrically, the phase variable algorithm adapted and computed monotonic and linear phase trajectories across the different walking speeds.

Even though the phase variable algorithm was able to automatically adapt to different walking speeds, the joint trajectories enforced by the controller (i.e., the virtual constraints) were manually changed across different speed conditions. It was important to select each virtual constraint according to each walking speed as there are clear differences in able-bodied joint kinematics between locomotion tasks (Fig. 1). A simple classifier based on cadence could automatically transition between the virtual constraints for each speed. It is also possible to model the desired joint kinematics as continuous functions of walking speed and slope so the controller does not have to rely on state-machine logic [22]. The phase variable algorithm allows the amputee subject to intuitively control progression through whatever virtual constraints are used.

This experiment also suggests that the phase-based control architecture could allow powered prosthetic legs to be used as plug-and-play devices. While it has been reported that the calibration process in other controllers used in powered knee-ankle prostheses can take several hours across multiple sessions [3], the phase-based control architecture has a relatively short calibration process (<10 min). The amputee subject was able to walk comfortably using the same control parameters obtained in previous experiments with an able-bodied subject wearing the prosthetic leg through a bypass adapter [16].

V. CONCLUSIONS

An algorithm capable of measuring the phase of the gait cycle was used to synchronize a powered prosthetic leg with the motion of its amputee user. This algorithm consists of 1) a piecewise phase variable that allowed the subject to control voluntary, non-steady leg motions, and 2) a unified phase variable that allows the amputee to smoothly control steady walking motions. Furthermore, the phase variable algorithm allowed the prosthetic leg to adapt to changes in the walking speed of the amputee. The control architecture only uses on-board sensors to measure phase and requires no major tuning across different walking speeds, which makes it viable for clinical applications. Future amputee experiments will let us validate the behavior of the phase variable algorithm and controller across other locomotion tasks (e.g., walking over various slopes).

ACKNOWLEDGMENT

The authors would like to thank Susan Kapp, CPO, Siavash Rezazadeh, Ph.D., and Christopher Nesler for their

help during the amputee experiment.

REFERENCES

- [1] J. Perry, J. M. Burnfield, and L. M. Cabico, *Gait analysis : normal and pathological function*. Thorofare, NJ: SLACK, 2010.
- [2] D. Winter, *Biomechanics and Motor Control of Human Movement*. Hoboken, New Jersey: J. Wiley and S, 2009.
- [3] A. M. Simon, K. a. Ingraham, N. P. Fey, S. B. Finucane, R. D. Lipschutz, A. J. Young, and L. J. Hargrove, "Configuring a powered knee and ankle prosthesis for transfemoral amputees within five specific ambulation modes," *PLoS ONE*, vol. 9, no. 6, 2014.
- [4] L. J. Hargrove, A. M. Simon, R. D. Lipschutz, S. B. Finucane, and T. A. Kuiken, "Real-time myoelectric control of knee and ankle motions for transfemoral amputees," *JAMA : J AM MED ASSOC*, vol. 305, no. 15, pp. 1542–1544, 2011.
- [5] F. Sup, A. Bohara, and M. Goldfarb, "Design and Control of a Powered Transfemoral Prosthesis," *INT J ROBOT RES*, vol. 27, no. 2, pp. 263–273, 2008.
- [6] K. H. Ha, H. A. Varol, and M. Goldfarb, "Volitional control of a prosthetic knee using surface electromyography," *IEEE Trans. Biomed. Eng.*, vol. 58, no. 1, pp. 144–151, 2011.
- [7] S. A. Murray, K. H. Ha, and M. Goldfarb, "An assistive controller for a lower-limb exoskeleton for rehabilitation after stroke, and preliminary assessment thereof," in *2014 36th Annual Int. Conf. of the IEEE EMBS*. IEEE, aug 2014, pp. 4083–4086.
- [8] F. Sup, H. A. Varol, and M. Goldfarb, "Upslope walking with a powered knee and ankle prosthesis: Initial results with an amputee subject," *IEEE Trans. Neural Syst. Rehabil. Eng.*, vol. 19, no. 1, pp. 71–78, 2011.
- [9] R. D. Gregg, T. Lenzi, L. J. Hargrove, and J. W. Sensinger, "Virtual Constraint Control of a Powered Prosthetic Leg : From Simulation to Experiments with Transfemoral Amputees," *IEEE Trans. Robot.*, vol. 30, no. 6, pp. 1455–1471, 2014.
- [10] A. E. Martin and R. D. Gregg, "Stable, robust hybrid zero dynamics control of powered lower-limb prostheses," *IEEE Trans. Autom. Control*, vol. PP, no. 99, pp. 1–1, 2017.
- [11] D. Quintero, A. E. Martin, and R. D. Gregg, "Toward unified control of a powered prosthetic leg: A simulation study," *IEEE Trans. Control Syst. Technol.*, vol. PP, no. 99, pp. 1–8, 2017.
- [12] D. J. Villarreal and R. D. Gregg, "A survey of phase variable candidates of human locomotion," in *2014 36th Annual Int. Conf. of the IEEE EMBS*. IEEE, aug 2014, pp. 4017–4021.
- [13] D. J. Villarreal, H. A. Poonawala, and R. D. Gregg, "A Robust Parameterization of Human Gait Patterns Across Phase-Shifting Perturbations," *IEEE Trans. Neural Syst. Rehabil. Eng.*, vol. 25, no. 3, pp. 265–278, 2017.
- [14] D. J. Villarreal and R. D. Gregg, "Unified phase variables of relative degree two for human locomotion," in *2016 38th Annual Int. Conf. of the IEEE EMBS*. IEEE, aug 2016, pp. 6262–6267.
- [15] M. a. Holgate, T. G. Sugar, and A. W. Bohler, "A novel control algorithm for wearable robotics using phase plane invariants," *Proceedings - IEEE Int. Conf. on Robotics and Automation*, pp. 3845–3850, 2009.
- [16] D. Quintero, D. J. Villarreal, and R. D. Gregg, "Preliminary experiments with a unified controller for a powered knee-ankle prosthetic leg across walking speeds," pp. 5427–5433, Oct 2016.
- [17] L. J. Hargrove, A. M. Simon, A. J. Young, R. D. Lipschutz, S. B. Finucane, D. G. Smith, and T. A. Kuiken, "Robotic leg control with EMG decoding in an amputee with nerve transfers," *NEW ENGL J MED*, vol. 369, no. 13, pp. 1237–42, 2013.
- [18] S. K. Au, P. Bonato, and H. Herr, "An EMG-position controlled system for an active ankle-foot prosthesis: An initial experimental study," *Proceedings of the 2005 IEEE 9th Int. Conf. on Rehabilitation Robotics*, vol. 2005, pp. 375–379, 2005.
- [19] R. Murray, Z. Li, and S. S. Sastry, *A Mathematical Introduction to Robotic Manipulation*. CRC Press Inc., 2013.
- [20] I. Jolliffe, *Principal Component Analysis (Springer Series in Statistics)*. Springer, 2002.
- [21] E. R. Kandel, *Principles of neural science*. New York: McGraw-Hill, 2013.
- [22] K. R. Embry, D. J. Villarreal, and R. D. Gregg, "A unified parameterization of human gait across ambulation modes," in *2016 38th Annual Int. Conf. of the IEEE EMBC*, Aug 2016, pp. 2179–2183.