Proceedings of the ASME 2017 Dynamic Systems and Control Conference
DSCC 2017
October 11-13, 2016, Virginia, USA

DSCC2017-5153

DRAFT: DYNAMICS OF A CIRCULAR CYLINDER WITH A PASSIVE DEGREE OF FREEDOM INTERACTING WITH AN INVISCID FLUID CONTAINING A POINT VORTEX

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ABSTRACT

In the recent past the design of many aquatic robots has been inspired by the motion of fish. Actuated internal rotors or moving masses have been frequently used either for propulsion and or the control of such robots. However the effect of internal passive degrees of freedom or passive appendages on the motion of such robots is poorly understood. In this paper we present a minimal model that demonstrates the influence of passive degrees of freedom on an aquatic robot. The model is of a circular cylinder with a passive internal rotor, immersed in an inviscid fluid interacting with point vortices. We show through numerics that the motion of the cylinder containing a passive degree of freedom is significantly different than one without. These results show that the mechanical feedback via passive degrees of freedom could be a useful way to control the motion of aquatic robots.

1 INTRODUCTION

Research into aquatic robots has grown significantly in the past few decades due to their potential uses in applications such as surveying and exploration, environmental monitoring, search and rescue, as well as numerous military applications. It is advantageous for such robots to have certain characteristics such as high manuverability, energy efficiency and the capability to harvest energy from the currents. This has led researchers to turn to natural swimmers for inspiration to mimick specific features of natural swimmers, see for instance [1–4]. One such feature is the possibility that many fish use passive dynamics of the interaction of the water with the body to both improve their propulsive efficiency as well maneuverability. In a well known series of experiments, it was demonstrated that a euthanized fish could swim upstream in a water tunnel, [5]. This propulsion was made possible due to the wake generated by an obstruction in the water tunnel, suggesting that fish can swim passively by taking advantage of the vorticity field in the fluid. Even more interestingly, it has been suggested that the deformations of the body of a fish or their appendages could partly be the result of fluid-structure interaction and that such passive mechanics improve the maneuverability of the fish, [6].

The passive motion of appendages of segments on the body of a fish can influence its motion through two means, one of the which is by modifying the interaction of the body with fluid. Passive body deformations or the motion of passive segments of a fish also change the inertia tensor of the body and can thus effect its motion. The relative importance of these two factors is worthy of investigation. In this paper we present a minimal model

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that incorporates the role of a passive degree of freedom in the interaction of a rigid body and an inviscid fluid with singular distributions of vorticity. The model consists of a rigid cylinder with a passive rotor that is entirely internal to the cylinder. The momentum that is absorbed by the passive rotor modulates the motion of the rigid body. We derive the equations of motion that describe the dynamics of the cylinder-fluid-passive rotor system using Lagrange-D'Alembert method. We show that the motion of a cylinder with a passive rotor is significantly different from one without and discuss ways in which this could be taken advantage of to improve the maneuverability of aquatic robots.

2 Problem setup

Consider a circular cylinder of radius, r_c, mass m_c immersed in an inviscid fluid. We will assume that the fluid domain is $\mathbb{R}^2 - \mathscr{B}$, where $\mathscr{B}$ denotes the region occupied by the cylinder. A passive rotor of mass m_r and centroidal moment of inertia I_r is attached to the center of the circular cylinder. The distance of the center of mass of the rotor from the cylinder's center is denoted by L. The rotor is contained entirely within the cylinder and does not directly interact with the fluid. The combined mass of the cylinder and the rotor is such that $m_c + m_r = \rho \pi r_c^2$. Without loss of generality we will assume that the fluid is of unit density and the cylinder is of unit radius. While the fluid is inviscid, it is assumed to contain N singular distributions of vorticity, the so called point vortices. While the formulation we derive in this paper is valid for the interaction of a circular cylinder with many point vortices, we will focus in our computations on the specific case of a single point vortex.

Two reference frames, shown in 1, are useful for the analysis of the dynamics of the cylinder. The first is an inertial frame of reference fixed at $(0,0)$. The second is a body fixed frame of reference, $X_b - Y_b$, that is attached to the center of the cylinder. The body frame moves and rotates with the cylinder. The location of the center of the cylinder with the respect to the inertial frame is $\mathbf{b} = (x_c, y_c)$. The locations of the point vortices in the body fixed frame are $\mathbf{l}_\alpha = (x_\alpha, y_\alpha)$ where $\alpha = 1, \ldots, N$. The circulations of each of the point vortices are denoted by Γ_α respectively. Position vectors, $\mathbf{l}$ in the body fixed frame are transformed to position vectors in the inertial frame, $\mathbf{r}$ by the equation

$$\mathbf{r} = \mathbf{b} + \mathbf{R}\mathbf{l} \tag{1}$$

where $\mathbf{R} \in SO2$ gives the orientation of the body fixed frame with respect to the inertial frame. The velocity of the cylinder in the inertial frame, $\mathbf{U} = (\dot{x}_c, \dot{y}_c)$ is related to its velocity in the body frame through the equation

$$\mathbf{U} = \mathbf{R}\mathbf{V} \tag{2}$$

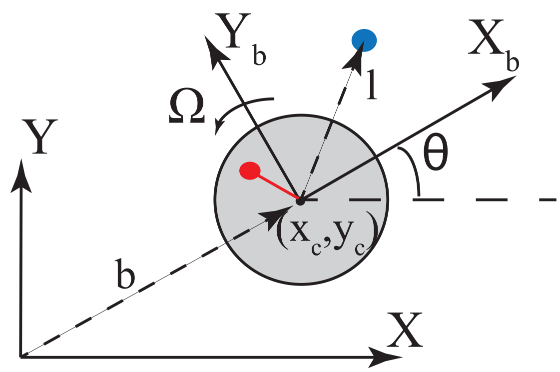

FIGURE 1: A schematic of the cylinder with an internal passive rotor (shown in red) interacting with a single point vortex (shown in blue). the body frame is $X_b - Y_b$ and the fixed inertial frame is $X - Y$.

We verified that the results shown in fig. 2 are the same either through the use of the method of the conservation of linear and angular impulse or the Lagrange-D'Alembert approach. To verify this the initial position of the cylinder and the point vortex and the initial velocity of the cylinder are chosen such that the initial impulse of the entire system to $\mathbf{P} = 0$. The impulse due to the point vortex at $t = 0$ is obtained from (15), $\mathbf{R}\mathbf{P}_v(0) = \Gamma(2 - \frac{1}{2})\hat{j}$. The initial velocity of the cylinder is therefore chosen to be

$$\mathbf{U}(0) = \frac{\mathbf{R}\mathbf{P}_v(0)}{(m_c + m_a)}. \tag{3}$$

The initial angular impulse is chosen to zero as well. Therefore the initial angular velocity of the cylinder is

$$\Omega(0) = \frac{1}{2I_c}\Gamma||\mathbf{l}_\alpha||^2. \tag{4}$$

2.1 Velocity of the fluid

The velocity field of the fluid is denoted by $\mathbf{u} = (u_x, u_y)$. The boundary conditions for the fluid flow are that there be no normal velocity relative to the cylinder on its surface, $(\mathbf{u}-\mathbf{U}).\mathbf{n} = 0$ where $\mathbf{n}$ is the unit normal vector at any point on the surface of the cylinder. The velocity of the fluid is a linear superposition of two components, one due to the motion of the cylinder, $\mathbf{u}_b$ and the other due to the presence of the point vortices, $\mathbf{u}_V$. Each of these components will satisfy the boundary conditions separately. The first component is $\mathbf{u}_b(x,y) = -\nabla\Phi_b$ where Φ_b is the Kirchoff's potential function,

$$\Phi_b = V_x r_c^2 \frac{x-x_c}{(x-x_c)^2+y^2} + V_y r_c^2 \frac{y-y_c}{(x-x_c)^2+(y-y_c)^2} \quad (5)$$

where (V_x, V_y) is the velocity of the cylinder in the body frame. The velocity field $-\nabla\Phi_b$ satisfies the required boundary conditions. To satisfy the boundary conditions due to the presence of a point vortex, the Milne-Thomson circle theorem is used, [7]. Accordingly, for a point vortex at $\mathbf{l}_\alpha$ an image point vortex of circulation $\Gamma_{\alpha+N} = \Gamma_\alpha$ is placed inside the circular cylinder at the location $\mathbf{l}_{\alpha+N} = r_c^2 \frac{\mathbf{l}_\alpha}{|\mathbf{l}_\alpha|^2}$. The velocity field of the fluid at any point that does not coincide with a point vortex is obtained by superposition,

$$\mathbf{u}_V(\mathbf{r}) = \sum_{\alpha=1}^{N} \Gamma_\alpha \left(\frac{\mathbf{r}-\mathbf{r}_\alpha}{||\mathbf{r}-\mathbf{r}_\alpha||^2} - \frac{\mathbf{r}-\mathbf{r}_{\alpha+N}}{||\mathbf{r}-\mathbf{r}_{\alpha+N}||^2} \right) \times \hat{k}. \quad (6)$$

The total velocity field at any point $\mathbf{r}$ is given by Hodge decomposition,

$$\mathbf{u}(\mathbf{r}) = \mathbf{u}_b + \mathbf{r}_V \quad (7)$$

The velocity of a point vortex is the velocity of a hypothetical fluid particle at the same location less the self induced velocity of the point vortex, i.e.

$$\dot{\mathbf{r}}_\alpha = \mathbf{u}_V - \Gamma_\alpha \frac{\mathbf{r}-\mathbf{r}_\alpha}{||\mathbf{r}-\mathbf{r}_\alpha||} \times \hat{k}. \quad (8)$$

2.2 Lagrange-D'Alembert equations

The equations of motion of the cylinder and that of the motion of the point vortex are calculated using the Lagrange D'Alembert approach. The fluid exerts a net force, (F_x, F_y) and a moment, τ on the cylinder. The cylinder's motion exerts a force (F_x, F_y) and a moment, τ on the fluid. These forces and moment on the fluid can be found by calculating the rate at which the linear and angular momentum of the fluid change. Technically the linear and angular momentum of the fluid are unbounded, but we are interested in the non singular component of these momenta, called the linear and angular impulse, [8]. The calculations of the linear and angular impulse of the fluid can be found in a number of recent papers, notably, [9–11] and the reader is referred to these for details. The dynamics of the cylinder are governed by the Lagrange D'Alembert equations where the external forces acting on the cylinder are $(-F_x, -F_y)$ and the external moment is, $-\tau$.

The configuration space of the cylinder and rotor is parametrized by the variables, (x_c, y_c), the angle θ that the cylinder's body frame makes with the inertial frame and the angle ϕ made by the rotor with the inertial frame. The Lagrangian for the cylinder-rotor system is

$$\mathscr{L} = \frac{1}{2}m_c(\dot{x}^2+\dot{y}^2) + \frac{1}{2}I_c\dot{\theta}^2 + \frac{1}{2}m_r v_r^2 + \frac{1}{2}I_r\dot{\phi}^2 \quad (9)$$

where $v_r = (\dot{x_c} - L\dot{\phi}\sin\phi)\hat{\imath} + (\dot{y} + L\dot{\phi}\cos\phi)\hat{\jmath}$ is the velocity of the center of mass of the rotor. The Lagrange D'Alembert equations are

$$m\ddot{x}_c - m_r L\ddot{\phi}\sin\phi - m_r L\dot{\phi}^2\cos\phi = -F_x \quad (10)$$

$$m\ddot{y}_c + m_r L\ddot{\phi}\cos\phi - m_r L\dot{\phi}^2\sin\phi = -F_y \quad (11)$$

$$I_c\ddot{\theta} = -\tau \quad (12)$$

$$I_{rc}\ddot{\phi} + m_r L\dot{\phi}(\dot{y}\sin\phi - \dot{x}\cos\phi) - m_r L(\ddot{x}\sin\phi + \ddot{y}\cos\phi) = 0 \quad (13)$$

where $I_{rc} = I_r + m_r L^2$ is the moment of inertia of the passive rod about the center of the cylinder.

The rate at which the momentum of the fluid, $\mathbf{P}_f$ in the domain $A_R = \mathbb{R}^2 - B$ changes is

$$\mathbf{F} = \frac{d}{dt}\mathbf{P}_f = \frac{d}{dt}\int_{A_R} (\nabla\Phi_b + \mathbf{u}_V)dA = \frac{d}{dt}\mathbf{R}\left(\mathbf{M}_a\mathbf{V} + \mathbf{P}_v\right). \quad (14)$$

The first term on the right hand side of (14) is the change in momentum of the fluid due to the rigid body motion of the cylinder.

This is equivalent to increasing the inertia of the cylinder by $\mathbf{M}_a$ the added mass tensor. For a circular cylinder immersed in a fluid of unit density this is just $\mathbf{M}_a = 0.5\pi r_c^2$. The term $\mathbf{P}_v$ is the non singular component of the momentum of the fluid due to the vortices in the body fixed frame,

$$\mathbf{P}_v = \sum_{\alpha=1}^{N} \Gamma_\alpha (\mathbf{l}_\alpha - \mathbf{l}_{\alpha+N}). \tag{15}$$

The force $\mathbf{F}$ is

$$\mathbf{F} = \mathbf{R}\left(\frac{d\mathbf{V}}{dt} + \frac{d\mathbf{P}_v}{dt}\right) + \mathbf{R}\Omega \times (\mathbf{M}_a\mathbf{V} + \mathbf{P}_v) \tag{16}$$

The moment exerted by the fluid on the cylinder is obtained by differentiating the angular momentum of the fluid about the cylinder's center,

$$-\tau = \frac{d}{dt}\int_{A_R} \mathbf{l} \times (\nabla\Phi_b + \mathbf{u}_V)dA = \frac{d}{dt}\left(-\frac{1}{2}\Gamma_\alpha ||\mathbf{l}_\alpha||^2\right). \tag{17}$$

If the cylinder did not contain a passive rotor, the equations of motion of such a cylinder could be obtained by setting $m_r = 0$ and $I_r = 0$ in equations (10)-(13). Alternatively it should be recognized that the linear impulse of the cylinder and the fluid together would be conserved, [9] and that the angular impulse of the fluid-cylinder system about the origin of the inertial frame would be conserved. Knowing the initial linear and angular impulse of the system together with (8) gives $2N+3$ equations in $2N+3$ unknowns. The velocity of the cylinder and the velocity of the point vortices can thus be computed. Such an approach to model the dynamics of circular cylinders and Joukowski foils can be found in [9, 12–14] and extensions to swimming in three dimensions in [15, 16].

When the cylinder contains a passive rotor, the combined linear impulse of the cylinder, the rotor and the fluid would still be conserved. Similarly the angular impulse of the combined system about the origin of the inertial frame would be conserved. With the additional degree of freedom that the passive rotor represents, the number of unknowns is now $2N+4$ while the number of equations that can be obtained from the conservation laws is still $2N+3$. The increase in the number of degrees of freedom necessitates the Lagrange-D'Alembert formulation of the equations of motion that we adopt in this paper.

3 SIMULATION RESULTS

To identify the effect of the passive rotor on the motion of the cylinder, we first discuss simulations of the motion of a cylinder without a passive rotor. We restrict our attention to the case where the number of vortices present is $N = 1$. Figure 2(a) shows the trajectory of the center of the cylinder (red graph) and the trajectory of a point vortex (blue graph) of circulation $\Gamma = 1$. The cylinder's initial position is at $(x_c, y_c) = (0,0)$ and the initial position of the vortex is at $\mathbf{r}_\alpha = (2,0)$ in the inertial frame of reference.

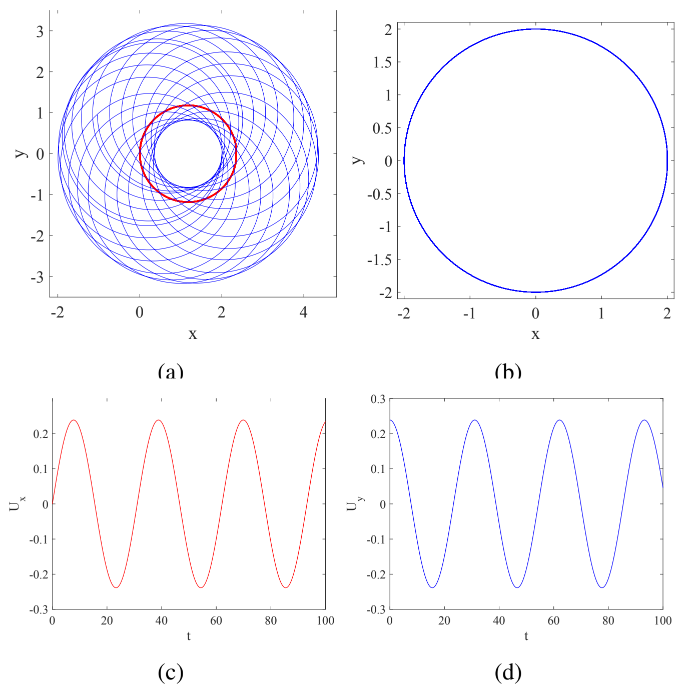

FIGURE 2: Dynamics of a cylinder and a point vortex - (a) Trajectory of the center of the cylinder (red) and of the point vortex (blue). (b) Trajectory of the point vortex in the body frame of reference. (c)-(d) Velocity of the center of the cylinder, U_x (c) and U_y (d).

We verified that the results shown in fig. 2 are the same either through the use of the method of the conservation of linear and angular impulse or the Lagrange-D'Alembert approach. To verify this the initial position of the cylinder and the point vortex and the initial velocity of the cylinder are chosen such that the initial impulse of the entire system to $\mathbf{P} = 0$. The impulse due to the point vortex at $t = 0$ is obtained from (15), $\mathbf{RP}_v(0) = \Gamma(2-$

$\frac{1}{2})\hat{j}$. The initial velocity of the cylinder is therefore chosen to be

$$\mathbf{U}(0) = \frac{\mathbf{RP}_v(0)}{(m_c + m_a)}. \tag{18}$$

The initial angular impulse is chosen to zero as well. Therefore the initial angular velocity of the cylinder is

$$\Omega(0) = \frac{1}{2I_c}\Gamma||\mathbf{l}_\alpha||^2. \tag{19}$$

Figure 2(a) shows that the cylinder moves along a circular trajectory with point vortex orbiting around it. Figure 2(b) shows the trajectory of the point vortex in the body fixed frame of reference. This trajectory is a circle, which implies that the distance of the point vortex from the center of the cylinder remains the same. It can in fact be shown that the $||\mathbf{l}_\alpha||^2$ is an invariant of the system, [11]. It follows from (19) that the angular velocity of the cylinder remains constant. The velocity of the cylinder has a constant magnitude, with its individual components, U_x and U_y being periodic, as shown in 2(c) and 2(d).

In the case of the cylinder with the passive rotor, we once again choose $\mathbf{r}_c(0) = (0,0)$, $\mathbf{r}_1 = (2,0)$ and set $\phi = 0$ and $\dot{\phi} = 0$. The initial velocity and angular velocity of the cylinder are chosen to be the same values as given by (18) and (19). The velocity of the center of mass of the rotor is given by $\mathbf{u}_r = \mathbf{U} + L\dot{\phi}\hat{k} \times (\cos\phi\hat{i} + \sin\phi\hat{j})$. At the initial time, since $\dot{\phi} = 0$, the velocity of the center of the rotor is the same as that of the center of the cylinder. With these initial conditions, both the linear and angular impulse of the system are zero. We choose $\frac{m_r}{m_c} = \frac{0.1}{0.9}$, i.e. the rotor is light compared to the cylinder. The sum of the masses of the rotor and the cylinder is chosen such that the over all body is neutrally buoyant.

The trajectory of the center of the cylinder over a period of $t = 1000$ is shown in 3(a). The trajectory is no longer a circle, but instead is aperiodic. Similarly the trajectory of the point vortex in the body fixed frame departs from being a circle. Instead as shown in 3(b) it fills up a circular annulus. The trajectory of the point vortex and the cylinder are nevertheless bounded in the plane. The space filling trajectory of the point vortex suggests that its motion and that of the cylinder are at least quasi periodic.

The velocity of the center of the cylinder is shown in the two panels in 3(c), U_x in top panel and U_y in bottom panel. The angular velocity of the cylinder is shown in the top panel of 3(d) while the angular velocity of the internal rotor is shown in the bottom panel of 3(d).

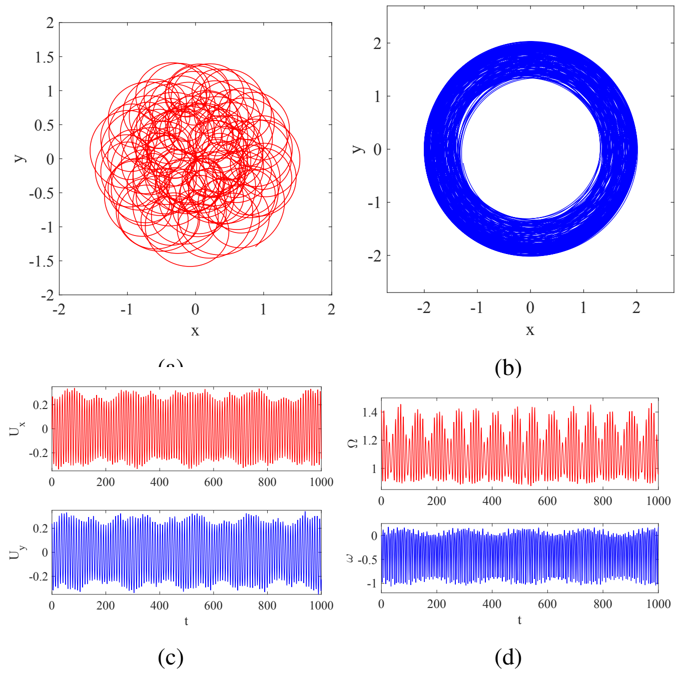

FIGURE 3: Dynamics of a cylinder with a passive rotor and a point vortex - (a) Trajectory of the center of the cylinder. (b) Trajectory of the point vortex in the body frame of reference. (c) Velocity of the center of the cylinder, U_x (top panel) and U_y (bottom panel). (d) Angular velocity of the cylinder, Ω (top panel) and angular velocity of the passive rotor, ω (bottom panel).

4 CONCLUSION

We showed through numerical simulations that even a 'light' passive rotor can significantly modify the motion of a circular cylinder moving in an inviscid fluid interacting with a single point vortex. The presence of the passive rotor changes the trajectory of the cylinder from a circle to a complicated irregular trajectory. The passive rotor does not significantly decrease the speed of the cylinder, as the graphs of the velocity of the cylinder in 2 and 3 show. The passive rotor influences the motion of the cylinder by changing the center of mass of the cylinder and hence its linear and angular momentum. The irregular trajectories of the cylinder and the point vortex can also be attributed to the loss of integrability. The system of the cylinder without a passive rotor and a point vortex is in integrable system, [9–11] with four constants of motion, the two components of the linear impulse, the angular impulse and the invariant $||\mathbf{l}_1||^2$. The last invariant can be shown to be due to existence of a Hamiltonian function. For the system of the cylinder with the passive rotor, the distance of the vortex from the center of the cylinder is no longer constant. While it is likely that a non canonical Hamiltonian exists for this system, the number of degrees of freedom for the system has increased by one due to the presence of the rotor. This leads to a loss of integrability and the resulting irregular trajectories.

A cylinder without a passive rotor is confined to a simple orbit and will need some form of actuation to change its trajectory to sample even a nearby region that does not lie on its circular orbit. The irregular trajectory of the cylinder with a passive rotor allows it to sample a larger region. The minimal model in this paper suggests that combinations of passive and active internal rotors or moving masses can be a viable means of propelling and maneuvering a swimming robot. Here it is useful to point out that it is possible for fish like robot to swims solely due to the actuation provided by a balanced internal rotor, [4, 12–14]. A minimally actuated robot that contains a passive rotor could take advantage of such passive dynamics to move easily from one region in the fluid to another.

5 ACKNOWLEDGMENTS

This paper is based upon work supported by the National Science Foundation under grant number CMMI 1563315.

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