Cyclic Interference Alignment for Full-Duplex Multi-Antenna Cellular Networks

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***Abstract*— This paper studies full-duplex (FD) cellular networks in which a base station (BS) operated in FD mode with multiple antennas supports multiple uplink and downlink users simultaneously in the same wireless channel. Two typical FD cellular scenarios are considered, one with half-duplex (HD) users and the other with FD users along with the FD BS. For both the cases, a novel constructive method is developed for finding a closed-form interference alignment (IA) solution, named *cyclic IA*. The core idea behind this approach is to construct a set of loop-equations enabling IA in a *cyclic* manner, so that beamforming vectors are sequentially determined by solving an eigenvalue problem. It is shown analytically that the proposed cyclic IA can achieve the *optimal* sum degrees-of-freedom (DoF) when the number of user antennas is large enough to meet the derived conditions. In particular, it is shown that the proposed scheme achieves a twofold DoF gain compared with conventional HD cellular networks even in the presence of inter-link interference, provided the number of users becomes large enough compared with the ratio of the number of BSs and user antennas. Simulation results demonstrate that not only are the analytical DoF results valid, but under a practical multi-cell scenario, the proposed cyclic IA offers significant throughput gains depending on the cell radius.**

***Index Terms*— Interference alignment, full-duplex, degrees of freedom, eigenvalue problem.**

I. INTRODUCTION

IN-BAND full-duplex (FD) transmission has received considerable attention from both academia and industry due to its potential to double the capacity of existing wireless communication networks by enabling simultaneous transmission and reception on the same channel. The key challenge in achieving an FD wireless system is suppressing self-interference to a sufficiently low level, such as within a few dB of the noise floor. This unwanted self-interference generated from a device's own transmissions is billions of times ($>$100 dB) stronger than the signal of interest coming from a distance source. Such a large power differential between the self-interference and the signal of interest has been a fundamental bottleneck in the progress of FD radio technology. Fortunately, recent advances in active cancelation technologies using radio frequency (RF) and analog/digital baseband cancellation mechanisms are capable of canceling most of the self-interference [1]- [5]. This opens up a new possibility: applying FD radios to the base station (BS) in a cellular network [6] in order to allow the uplink (UL) and downlink (DL) users to communicate on the same channel. However, to fully capitalize on the benefits of FD radios in a typical cellular network in which multiple users are simultaneously supported by a single BS, a new challenge in managing interference, the so-called *inter-link interference* effect, must be further resolved. This inter-link interference arises from the fact that the uplink transmissions of UL users severely interfere with the downlink reception of DL users owing to the simultaneous operation of uplink and downlink in the same wireless channel.

To overcome this challenge, there have been several recent studies on efficiently managing inter-link interference in FD cellular networks. Most prior work has mainly focused on linear precoding techniques based on the concept of interference alignment (IA) [7]- [11], and iterative precoder designs via rank-constrained optimization [12], [13]. Under ergodic phase fading, reference [7] has provided a tight characterization of the degrees-of-freedom (DoF) through the concept of *ergodic* IA for the single-cell FD cellular network, in which a multi-antenna FD BS serves a set of single-antenna FD users. Following up on the concept of IA, under generic time-varying channels, the DoF region [8] and the sum-DoF [9] of the single-cell FD cellular network have been fully characterized by developing a new *asymptotic* IA as well as obtaining matching outer bounds. In particular, it has been shown in [7] and [9] that the sum-DoF can approach a *twofold* gain over the half-duplex (HD) system, in which the uplink and downlink transmissions occur in separate channels, when the number of users is sufficiently large compared to the number of antennas at the BS. The main limitation of the aforementioned two IA methods is that these approaches commonly rely on the ideal assumption of an arbitrarily large number of symbol

Manuscript received July 5, 2016; revised December 8, 2016 and February 20, 2017; accepted February 22, 2017. Date of publication March 3, 2017; date of current version June 14, 2017. This work of W. Shin and J. Lee was supported in part by the Basic Science Research Program (NRF-2015R1A2A1A15052493) through the National Research Foundation of Korea (NRF) funded by the Ministry of Science, ICT & Future Planning (MSIP), the Bio-Mimetic Robot Research Center funded by the Defense Acquisition Program Administration (UD130070ID), INMAC, and BK21-plus. The work of H. H. Choi was supported by Basic Science Research Program through the NRF funded by MSIP (NRF-2016R1C1B1016261). The work of H. V. Poor was supported in part by the U.S. National Science Foundation under Grants CNS-1456793 and ECCS-1647198. The associate editor coordinating the review of this paper and approving it for publication was M. Tao. *(Corresponding author: Hyun-Ho Choi.)*

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Digital Object Identifier 10.1109/TCOMM.2017.2677946

extensions over rapidly time-varying channels, which is quite challenging in practice.[1]

To resolve such practical restrictions, Kim *et al.* [11] have studied linear IA along *spatial* dimensions without symbol extension (i.e., one-shot linear IA), and also provided sufficient and necessary feasibility conditions of the linear IA from algebraic geometry for a single-cell FD multi-input multi-output (MIMO) cellular network. The limitation of the work in [11] is that although the feasibility condition of IA is satisfied for the existence of the solution, a nonlinear system of polynomial equations has to be solved to determine the solution because the transmit and receive beamforming vectors are coupled. A general closed-form solution therefore does not exist for all cases so that feasible IA solutions can be found *numerically* only in general cases, making it hard to implement in practice. Meanwhile, iterative algorithms [12], [13] have been suggested to find numerically the joint beamforming that maximizes the spectral efficiency via a rank-constrained optimization for a class of FD MIMO cellular networks. However, their computational complexity can be *prohibitively* high if the number of antennas at the nodes is large, so that such iterative approaches are unsuitable for practical implementation.

Motivated by the prior work [11]–[13] from a practical perspective, in this paper, we propose a novel constructive method for finding a closed-form IA solution along the spatial dimension in multi-antenna FD cellular networks. The core idea behind our approach is to construct a set of loop-equations enabling IA in a cyclic manner, so that beamforming vectors are sequentially determined by solving an eigenvalue problem; this is referred to as *cyclic* IA. We show analytically that the proposed IA can achieve the *optimal* sum-DoF when the number of user antennas is large enough to meet the derived conditions. Through this analysis, we also identify feasibility conditions under which the proposed IA offers a twofold DoF gain compared to conventional HD cellular networks. Simulation results demonstrate that not only are the analytical DoF results valid, but the proposed cyclic IA offers significant throughput gains under a practical multi-cell scenario.

The rest of the paper is organized as follows. In Section II, we present the FD cellular system models and the sum-DoF metrics considered in this paper. In Section III, we describe the proposed cyclic IA in the FD MIMO cellular network with a set of HD users for the single-beam case (i.e. each user sends or receives one data stream). Section IV further considers the multi-beam cases for the FD MIMO cellular network, and provides feasibility conditions for the proposed cyclic IA to achieve the optimal sum-DoF. In Section V, we turn our attention to another typical FD MIMO cellular network model with a set of FD users along with the FD BS, and establish a feasibility condition of cyclic IA for such a channel. In Section VI, we present our numerical results under different simulation settings in a multi-cell network environment. Finally, Section VII presents our conclusions.

[1]In contrast, it has recently been shown in [10] that the sum-DoF of an FD cellular network with no or partial channel state information at the transmitter side is characterized under an ideal assumption in which the coherence time of the channel is assumed to be long enough to span the coding block, but that may also rarely occur in realistic wireless scenarios.

Notation: We use $\mathbf{A}^T$ and $\mathbf{A}^\dagger$ to indicate the transpose and the conjugate transpose of a matrix $\mathbf{A}$, respectively. In addition, $\mathsf{span}(\mathbf{A})$ and $\mathsf{null}(\mathbf{A})$ represent the subspace spanned by the column vectors and an orthonormal basis for the null space of the matrix $\mathbf{A}$, respectively. $\mathbf{0}_x$ is used for a zero vector of size $x \times 1$. For two sets $\mathcal{A}$ and $\mathcal{B}$, the cardinality of the set $\mathcal{A}$ is denoted by $|\mathcal{A}|$, and $\mathcal{B} \setminus \mathcal{A}$ represents the relative complement of $\mathcal{A}$ in $\mathcal{B}$. In addition, $\mathbb{C}^{m\times n}$ and $\mathcal{CN}(0, \sigma^2)$ denote an $m \times n$ matrix whose components are complex values and a complex Gaussian random variable with zero mean and variance σ^2. Given two positive integers, n_1 and n_2, $n_1|n_2$ denotes the remainder of the division of n_1 by n_2.

II. System Model

In this section, we describe the system model of the considered FD MIMO cellular network with a single FD BS that communicates in the downlink and uplink simultaneously with a set of users. As for the user terminals, the capability of operating in FD mode depends on the cost or form factor of the devices. This motivates us to consider two types of network models: one consisting of a set of HD users, and the other consisting of a set of FD users along with an FD BS, as illustrated in Fig. 1.

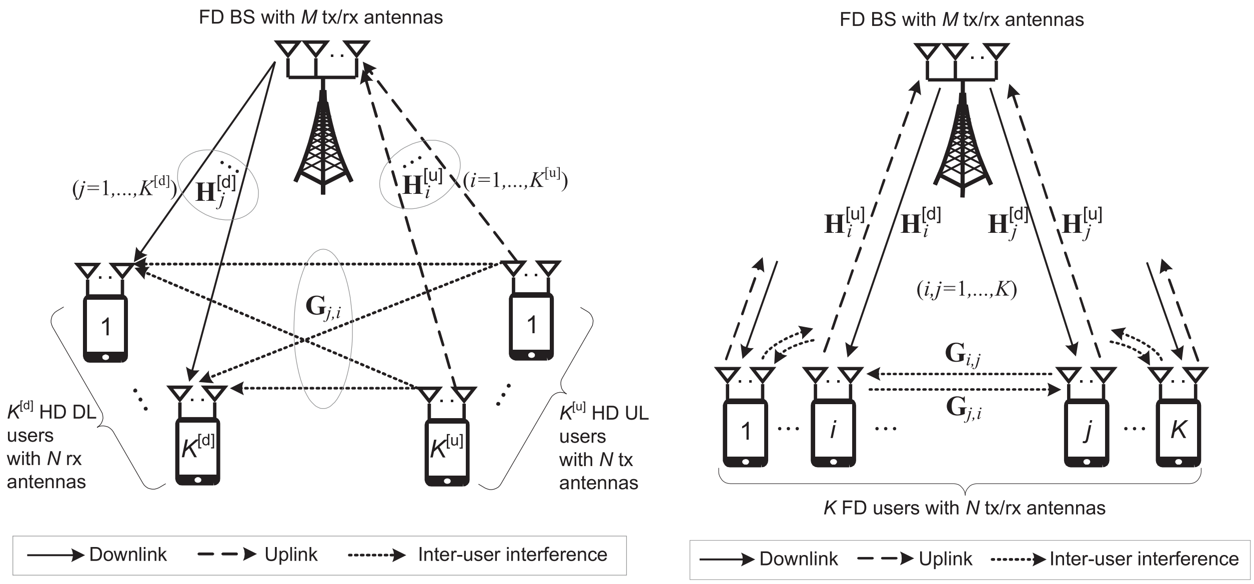
A. FD-BS-HD-User Cellular Networks

For the case of HD users, we consider an $(M, N, K^{[\mathsf{d}]}, K^{[\mathsf{u}]})$ FD-BS-HD-user cellular network, in which the FD BS is equipped with M antennas to serve both uplink and downlink users in the same time slot and frequency band. To be specific, the FD BS serves $K^{[\mathsf{d}]}$ DL users and $K^{[\mathsf{u}]}$ UL users, each of which is limited to HD operation. It is assumed that $M \geq \max\{K^{[\mathsf{d}]}, K^{[\mathsf{u}]}\}$ because the BS is not able to schedule more than M active users at the same time. We further assume that each user has N transmit/receive antennas. Suppose that the BS wants to send a set of independent messages $\mathcal{W}^{[\mathsf{d}]} = (W_1^{[\mathsf{d}]}, W_2^{[\mathsf{d}]}, \ldots, W_{K^{[\mathsf{d}]}}^{[\mathsf{d}]})$ to the DL users. At the same time, the BS also wants to receive a set of independent messages $\mathcal{W}^{[\mathsf{u}]} = (W_1^{[\mathsf{u}]}, W_2^{[\mathsf{u}]}, \ldots, W_{K^{[\mathsf{u}]}}^{[\mathsf{u}]})$ from the UL users.

In the $(M, N, K^{[\mathsf{d}]}, K^{[\mathsf{u}]})$ FD-BS-HD-user model, the input-output relationship at the BS is given by

$$\mathbf{y}^{[\mathsf{u}]} = \sum_{i=1}^{K^{[\mathsf{u}]}} \mathbf{H}_i^{[\mathsf{u}]} \mathbf{x}_i^{[\mathsf{u}]} + \mathbf{z}^{[\mathsf{u}]}, \tag{1}$$

where $\mathbf{x}_i^{[\mathsf{u}]} \in \mathbb{C}^{N\times 1}$ is the transmit signal vector of the ith UL user, $\mathbf{H}_i^{[\mathsf{u}]} \in \mathbb{C}^{M\times N}$ is the channel matrix from the ith UL user to the BS, and $\mathbf{z}^{[\mathsf{u}]} \in \mathbb{C}^{M\times 1}$ is the additive noise vector at the BS. We assume that all entries of channel matrices are independent and identically distributed (i.i.d.) complex Gaussian random variables with zero means and unit variances, i.e., $\mathcal{CN}(0, 1)$. Furthermore, it is assumed that all entries of $\mathbf{z}^{[\mathsf{u}]}$ are i.i.d. $\mathcal{CN}(0, \sigma^2)$ where σ^2 represents the noise variance. The average power constraint of the signal transmitted by each UL user, $\mathbf{x}_i^{[\mathsf{u}]}$, is given by $\mathbb{E}\left[\mathbf{x}_i^{[\mathsf{u}]^\dagger}\mathbf{x}_i^{[\mathsf{u}]}\right] \leq P$, $\forall i$. We also assume for now that perfect self-interference suppression in the FD operation at the BS is achieved so that



(a) $(M, N, K^{[\mathsf{d}]}, K^{[\mathsf{u}]})$ FD-BS-HD-user cellular network

(b) (M, N, K) FD-BS-FD-user cellular network

Fig. 1. System models of FD MIMO cellular networks.

there is no self-interference term for the above input-output relationship as in [7]- [11].[2]

On the other hand, the received signal vector at the jth DL user is given by

$$\mathbf{y}_j^{[\mathsf{d}]} = \mathbf{H}_j^{[\mathsf{d}]}\mathbf{x}^{[\mathsf{d}]} + \sum_{i=1}^{K^{[\mathsf{u}]}} \mathbf{G}_{j,i}\mathbf{x}_i^{[\mathsf{u}]} + \mathbf{z}_j^{[\mathsf{d}]}, \tag{2}$$

where $\mathbf{x}^{[\mathsf{d}]} \in \mathbb{C}^{M\times 1}$ is the transmit signal vector of the BS; $\mathbf{H}_j^{[\mathsf{d}]}$ and $\mathbf{G}_{j,i}$ are the channel matrix from the BS to the jth DL user and the channel matrix from the ith UL user to the jth DL user, respectively; and $\mathbf{z}_j^{[\mathsf{d}]}$ is an additive noise vector at the jth DL user. Here, we assume that the instantaneous channel state information (CSI) of all the channels is available at all the nodes in the FD cellular network.

B. FD-BS-FD-User Cellular Networks

For the case of FD users, we consider that there are K FD users that intend to transmit uplink messages and receive downlink messages to/from the BS. That is, the FD BS wishes to send a set of independent messages $\mathcal{W}^{[\mathsf{d}]} = (W_1^{[\mathsf{d}]}, W_2^{[\mathsf{d}]}, \ldots, W_K^{[\mathsf{d}]})$ to the users. At the same time, the BS also wants to receive a set of independent messages $\mathcal{W}^{[\mathsf{u}]} = (W_1^{[\mathsf{u}]}, W_2^{[\mathsf{u}]}, \ldots, W_K^{[\mathsf{u}]})$ from the users. As before, we assume that the FD BS is equipped with M antennas while each FD user is equipped with N antennas. We refer to this network model as the (M, N, K) FD-BS-FD-user cellular network.

For the (M, N, K) FD-BS-FD-user cellular network, the received signal vectors at the BS and the jth FD user are, respectively, expressed as

$$\mathbf{y}^{[\mathsf{u}]} = \sum_{i=1}^{K} \mathbf{H}_i^{[\mathsf{u}]}\mathbf{x}_i^{[\mathsf{u}]} + \mathbf{z}^{[\mathsf{u}]}, \tag{3}$$

$$\mathbf{y}_j^{[\mathsf{d}]} = \mathbf{H}_j^{[\mathsf{d}]}\mathbf{x}^{[\mathsf{d}]} + \sum_{i=1, i\neq j}^{K} \mathbf{G}_{j,i}\mathbf{x}_i^{[\mathsf{u}]} + \mathbf{z}_j^{[\mathsf{d}]}. \tag{4}$$

Similarly, we assume that no self-interference remains for the FD users along with the FD BS. The rest of the assumptions regarding channel coefficients and noises are the same as that apply to FD-BS-HD-user cellular networks.

C. Degrees of Freedom

We define the sum-DoF of the uplink and downlink as

$$\mathsf{DoF}^{[\theta]} = \lim_{\mathsf{SNR}\to\infty} \frac{\sum_{i=1}^{K^{[\theta]}} R_i^{[\theta]}(\mathsf{SNR})}{\log \mathsf{SNR}} = \sum_{i=1}^{K^{[\theta]}} d_i^{[\theta]}, \quad \theta \in \{\mathsf{u}, \mathsf{d}\}, \tag{5}$$

where $R_i^{[\mathsf{u}]}(\mathsf{SNR})$ and $R_i^{[\mathsf{d}]}(\mathsf{SNR})$ denote the achievable rate for the ith DL user and the ith UL user at the corresponding SNR, respectively,[3] $\mathsf{SNR} = \frac{P}{\sigma^2}$, and $d_i^{[\theta]} = \lim_{\mathsf{SNR}\to\infty} \frac{R_i^{[\theta]}(\mathsf{SNR})}{\log \mathsf{SNR}}$. In particular, for codewords spanning n channel uses, a rate of message $W_k^{[\theta]}$, $R_i^{[\theta]}(\mathsf{SNR}) = \frac{\log_2 |W_i^{[\theta]}(\mathsf{SNR})|}{n}$, is achievable if the probability of error for the message $W_k^{[\theta]}$ approaches zero as $n \to \infty$. The symmetric sum-DoF is defined as the largest value of $\mathsf{DoF}^{[\theta]} = \sum_{i=1}^{K^{[\theta]}} d^{[\theta]}$ for which the DoF tuple $(d_1^{[\theta]}, \cdots, d_{K^{[\theta]}}^{[\theta]}) = (d^{[\theta]}, \cdots, d^{[\theta]})$ is achievable. As a fair and compact metric, we are interested in the symmetric sum-DoF as well as the sum-DoF.

[2]We will evaluate how imperfect self-interference suppression affects the throughput performance in Section VI.

[3]For the FD-BS-FD-user case, the ith FD user corresponds to both the ith DL user and ith UL user, so that we use them interchangeably.

III. Cyclic Interference Alignment

In this section, we propose a cyclic IA in FD-BS-HD-user cellular networks. The key idea behind the proposed algorithm is to formulate a set of loop-equations for aligning interference signals in a cyclic manner, which results in finding the closed-form IA solution. To facilitate a clear understanding of the proposed cyclic IA scheme, we first consider a symmetric network topology where $K^{[\mathrm{d}]} = K^{[\mathrm{u}]} = K \geq 2$. Thereafter, we will investigate an asymmetric network topology where M, N, $K^{[\mathrm{d}]}$, and $K^{[\mathrm{u}]}$ are given arbitrarily.

A. Symmetric FD MIMO Cellular Networks

In this subsection, we will show that the symmetric DoF tuple of $(d^{[\mathrm{d}]}, d^{[\mathrm{u}]}) = (1, 1)$ is achievable when $K^{[\mathrm{d}]} = K^{[\mathrm{u}]} = K$ and $M = N = K$ (i.e., symmetric topology). Let the information symbol of the ith UL user and the ith DL user be denoted by $s_i^{[\mathrm{u}]}$ and $s_i^{[\mathrm{d}]}$, respectively. The ith UL user sends its information $s_i^{[\mathrm{u}]}$ to the BS using the transmit beamforming vector $\mathbf{v}_i^{[\mathrm{u}]} \in \mathbb{C}^{K\times 1}$. At the same time, the BS sends its data $s_i^{[\mathrm{d}]}$ to the ith DL user using the transmit vector $\mathbf{v}_i^{[\mathrm{d}]} \in \mathbb{C}^{K\times 1}$, i.e., $\mathbf{x}^{[\mathrm{d}]} = \sum_{i=1}^{K} \mathbf{v}_i^{[\mathrm{d}]} s_i^{[\mathrm{d}]}$. At each DL user, in addition to the one desired symbol, there are $K-1$ DL users' interference symbols as well as K UL users' interference symbols (i.e., a total of $2K-1$ interference symbols). Because the total number of dimensions is only K, K of all the $2K-1$ interfering symbols must be aligned within the vector space spanned by the remaining $K-1$ interfering symbols, in order to leave one interference-free signal dimension. To accomplish this design goal in closed-form, two steps are required to construct the transmit beamforming vectors $\left\{\mathbf{v}_i^{[\mathrm{u}]}\right\}$ for UL users and $\left\{\mathbf{v}_i^{[\mathrm{d}]}\right\}$ for DL users where $i \in \{1, 2, \ldots, K\}$. The set of transmit beamformers of UL users is first determined such that interference alignment is achieved among inter-link interference signals. Subsequently, interference alignment is further achieved between inter-link and DL interference signals by carefully designing the transmit-and-receive beamforming vector pairs.

1) Design of the Beamforming Vector for UL Users: In the following, we will show how to make the UL user interference signals $\mathbf{G}_{i,j}\mathbf{v}_j^{[\mathrm{u}]}, \forall j$ lie in the subspaces of at most $K-1$ dimensions at each DL user. Toward this end, there has to exist a set of $\mu_{i,j}$'s where at least one of the $\mu_{i,j}$'s is nonzero, such that

$$\sum_{j=1}^{K} \mu_{i,j} \mathbf{G}_{i,j} \mathbf{v}_j^{[\mathrm{u}]} = \mathbf{0}_K. \tag{6}$$

If we directly solve (6), we have to handle an intricate nonlinear equation system because the variables $\mu_{i,j}$ and $\mathbf{v}_j^{[\mathrm{u}]}$ are coupled. To solve the equation in a closed-form, we introduce the *loop-equation* deduced from (6) as

$$\begin{aligned}
\mathbf{G}_{1,1}\mathbf{v}_1^{[\mathrm{u}]} &= \lambda_1 \mathbf{G}_{1,2}\mathbf{v}_2^{[\mathrm{u}]} \\
&\Rightarrow \mathbf{v}_2^{[\mathrm{u}]} = \frac{1}{\lambda_1}\mathbf{G}_{1,2}^{-1}\mathbf{G}_{1,1}\mathbf{v}_1^{[\mathrm{u}]}, \\
\mathbf{G}_{2,2}\mathbf{v}_2^{[\mathrm{u}]} &= \lambda_2 \mathbf{G}_{2,3}\mathbf{v}_3^{[\mathrm{u}]} \\
&\Rightarrow \mathbf{v}_3^{[\mathrm{u}]} = \frac{1}{\lambda_2}\mathbf{G}_{2,3}^{-1}\mathbf{G}_{2,2}\mathbf{v}_2^{[\mathrm{u}]}, \\
&\vdots \qquad\qquad \vdots \\
\mathbf{G}_{K-1,K-1}\mathbf{v}_{K-1}^{[\mathrm{u}]} &= \lambda_{K-1} \mathbf{G}_{K-1,K}\mathbf{v}_K^{[\mathrm{u}]} \\
&\Rightarrow \mathbf{v}_K^{[\mathrm{u}]} = \frac{1}{\lambda_{K-1}}\mathbf{G}_{K-1,K}^{-1}\mathbf{G}_{K-1,K-1}\mathbf{v}_{K-1}^{[\mathrm{u}]}, \\
\mathbf{G}_{K,K}\mathbf{v}_K^{[\mathrm{u}]} &= \lambda_K \mathbf{G}_{K,1}\mathbf{v}_1^{[\mathrm{u}]} \\
&\Rightarrow \mathbf{v}_1^{[\mathrm{u}]} = \frac{1}{\lambda_K}\mathbf{G}_{K,1}^{-1}\mathbf{G}_{K,K}\mathbf{v}_K^{[\mathrm{u}]},
\end{aligned} \tag{7}$$

where λ_i denotes the coefficient of the linear combination by putting $\mu_{i,i} = 1$, $\mu_{i,(i+1)|K} = -\lambda_i$ and $\mu_{i,\bar{j}} = 0$, $\forall \bar{j} \in \{1, 2, \ldots, K\}\setminus\{i, (i+1)\,|\,K\}$ in (6). This loop-equation shows that each IA condition at the ith DL user associates with $\mathbf{v}_i^{[\mathrm{u}]}$ and $\mathbf{v}_{i+1}^{[\mathrm{u}]}$ until $i < K$, whereas the last IA condition involves the last UL user's beamforming vector $\mathbf{v}_K^{[\mathrm{u}]}$ and the first UL user's beamforming vector $\mathbf{v}_1^{[\mathrm{u}]}$ back again, which we call *cyclic IA*. By arranging the *cyclic IA* conditions in (7), we can formulate the standard eigenvalue problem,

$$\left(\prod_{i=1}^{K}\lambda_i\right)\mathbf{v}_1^{[\mathrm{u}]} = \left\{\mathbf{G}_{K,1}^{-1}\mathbf{G}_{K,K}\left(\prod_{i=K-1}^{1}\mathbf{G}_{i,i+1}^{-1}\mathbf{G}_{i,i}\right)\right\}\mathbf{v}_1^{[\mathrm{u}]}. \tag{8}$$

Because all the channel matrices $\mathbf{G}_{i,j}$, $\forall i, j$ are of full rank almost surely owing to the assumption that the channel elements are drawn as i.i.d. Gaussian, the IA beamforming vectors that satisfy (8) exist as follows:

$$\mathbf{v}_k^{[\mathrm{u}]} = \begin{cases} \mathsf{eig}\left\{\mathbf{G}_{K,1}^{-1}\mathbf{G}_{K,K}\left(\prod_{i=K-1}^{1}\mathbf{G}_{i,i+1}^{-1}\mathbf{G}_{i,i}\right)\right\}, & k = 1, \\ \dfrac{\mathbf{G}_{k-1,k}^{-1}\mathbf{G}_{k-1,k-1}\mathbf{v}_{k-1}^{[\mathrm{u}]}}{\|\mathbf{G}_{k-1,k}^{-1}\mathbf{G}_{k-1,k-1}\mathbf{v}_{k-1}^{[\mathrm{u}]}\|_2}, & 2 \leq k \leq K, \end{cases} \tag{9}$$

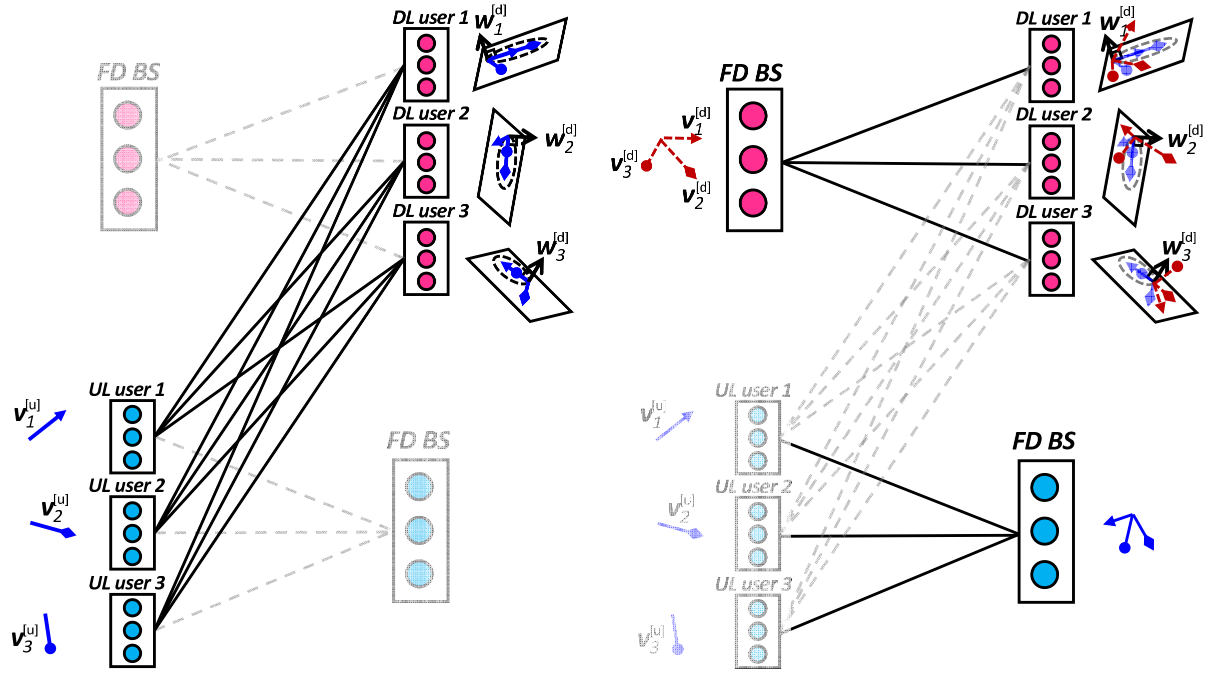
where $\mathsf{eig}(\cdot)$ denotes an eigenvector of a matrix with unit norm.

Note that in the special case of $M = N = K = 2$, another closed-form expression of the transmit beamforming vector for UL users can be derived by using the quadratic formula as follows:

$$\mathbf{v}_1^{[\mathrm{u}]} = \begin{bmatrix} \frac{1}{\sqrt{1+\nu^2}} \\ \frac{\nu}{\sqrt{1+\nu^2}} \end{bmatrix} \quad \text{and} \quad \mathbf{v}_2^{[\mathrm{u}]} = \frac{\mathbf{G}_{1,2}^{-1}\mathbf{G}_{1,1}\mathbf{v}_1^{[\mathrm{u}]}}{\|\mathbf{G}_{1,2}^{-1}\mathbf{G}_{1,1}\mathbf{v}_1^{[\mathrm{u}]}\|_2}, \tag{10}$$

where $\nu = \frac{\lambda-\overline{\mathbf{G}}(1,1)}{\overline{\mathbf{G}}(1,2)}$, $\lambda = \frac{\overline{\mathbf{G}}(1,1)+\overline{\mathbf{G}}(2,2)\pm\sqrt{\left(\overline{\mathbf{G}}(1,1)+\overline{\mathbf{G}}(2,2)\right)^2-4\left(\overline{\mathbf{G}}(1,1)\overline{\mathbf{G}}(2,2)-\overline{\mathbf{G}}(1,2)\overline{\mathbf{G}}(2,1)\right)}}{2}$, and $\overline{\mathbf{G}} = \mathbf{G}_{2,1}^{-1}\mathbf{G}_{2,2}\mathbf{G}_{1,2}^{-1}\mathbf{G}_{1,1} = \begin{bmatrix} \overline{\mathbf{G}}(1,1) & \overline{\mathbf{G}}(1,2) \\ \overline{\mathbf{G}}(2,1) & \overline{\mathbf{G}}(2,2) \end{bmatrix}$. We omit the detailed derivations here due to space limitations.

When the IA solution derived in (9) is applied to the transmit beamforming of UL users, the interference signals occupy only a $(K-1)$-dimensional subspace. Therefore, the dimension of the orthogonal subspace of this interference subspace becomes one almost surely due to the *rank-nullity* theorem. Thus, each DL user should select its received beamforming vector, $\mathbf{w}_k^{[\mathrm{d}]}$, orthogonal to the subspace spanned by



(a) Step 1: Beamforming vector design for UL users (b) Step 2: Beamforming vector design for DL users

Fig. 2. A diagram illustrating *cyclic IA* for $K^{[\mathrm{u}]} = K^{[\mathrm{d}]} = 3$ and $M = N = 3$.

the aligned interference signals as follows:

$$\mathbf{w}_k^{[\mathrm{d}]} \in \mathrm{null}\left(\left[\mathbf{G}_{k,1}\mathbf{v}_1^{[\mathrm{u}]}\mathbf{G}_{k,2}\mathbf{v}_2^{[\mathrm{u}]}\cdots\mathbf{G}_{k,K}\mathbf{v}_K^{[\mathrm{u}]}\right]^\dagger\right),\quad k \in \{1,2,\ldots,K\}, \tag{11}$$

$$\overset{(a)}{=} \mathrm{null}\left(\underbrace{\left[\mathbf{G}_{k,1}\mathbf{v}_1^{[\mathrm{u}]} \quad \mathbf{G}_{k,2}\mathbf{v}_2^{[\mathrm{u}]} \quad \cdots \quad \mathbf{G}_{k,k}\mathbf{v}_k^{[\mathrm{u}]}, \; \tfrac{1}{\lambda_k}\mathbf{G}_{k,k}\mathbf{v}_k^{[\mathrm{u}]} \quad \cdots \; \mathbf{G}_{k,K}\mathbf{v}_K^{[\mathrm{u}]}\right]^\dagger}_{\text{rank-}(K-1)}\right). \tag{12}$$

where (a) follows from the IA condition in (7), i.e., $\mathbf{G}_{k,k}\mathbf{v}_k^{[\mathrm{u}]} = \lambda_k \mathbf{G}_{k,(k+1)|K}\mathbf{v}_{(k+1)|K}^{[\mathrm{u}]}$.

2) Design of the Beamforming Vector for DL Users: Given the receive beamforming vector $\mathbf{w}_k^{[\mathrm{d}]}$ for each DL user, we can also construct the transmit beamforming vector for each DL user in order to guarantee a one-dimensional interference-free subspace at each DL user:

$$\mathbf{w}_j^{[\mathrm{d}]^\dagger}\mathbf{H}_j^{[\mathrm{d}]}\mathbf{v}_k^{[\mathrm{d}]} = 0, \quad \forall j \neq k, \; k \in \{1,2,\ldots,K\}. \tag{13}$$

Without loss of generality, we can rewrite (13) as

$$\mathbf{v}_k^{[\mathrm{d}]} \in \mathrm{null}\left(\underbrace{\left[\mathbf{H}_1^{[\mathrm{d}]^\dagger}\mathbf{w}_1^{[\mathrm{d}]} \quad \cdots \quad \mathbf{H}_{k-1}^{[\mathrm{d}]^\dagger}\mathbf{w}_{k-1}^{[\mathrm{d}]}, \; \mathbf{H}_{k+1}^{[\mathrm{d}]^\dagger}\mathbf{w}_{k+1}^{[\mathrm{d}]} \quad \cdots \quad \mathbf{H}_K^{[\mathrm{d}]^\dagger}\mathbf{w}_K^{[\mathrm{d}]}\right]^\dagger}_{\text{rank-}(K-1)}\right). \tag{14}$$

Using the fact that the size of the matrix, $[\mathbf{H}_1^{[\mathrm{d}]^\dagger}\mathbf{w}_1^{[\mathrm{d}]}\cdots\mathbf{H}_{k-1}^{[\mathrm{d}]^\dagger}\mathbf{w}_{k-1}^{[\mathrm{d}]}, \mathbf{H}_{k+1}^{[\mathrm{d}]^\dagger}\mathbf{w}_{k+1}^{[\mathrm{d}]}\cdots\mathbf{H}_K^{[\mathrm{d}]^\dagger}\mathbf{w}_K^{[\mathrm{d}]}]$ is $K \times (K-1)$ and the computation of $\mathbf{w}_k^{[\mathrm{d}]}$ does not depend on the downlink channel matrices $\mathbf{H}_i^{[\mathrm{d}]}$, $\forall i$, we find the *unique* one-dimensional null space of the matrix almost surely by the *rank-nullity* theorem.

An important observation here is the following. We can see that the transmit beamforming vector for DL users from (14) achieves IA as illustrated in Fig. 2 (e.g., the operational example of cyclic IA when $K^{[\mathrm{u}]} = K^{[\mathrm{d}]} = 3$). To be more specific, it is seen that $2K-1$ interference signal vectors (i.e., K UL users' interferences and $K-1$ DL users' interferences) are aligned into only a $(K-1)$-dimensional linear subspace. In order to keep the interference contained in a $(K-1)$-dimensional subspace at each DL user, all the $K-1$ DL users' interfering symbols must be aligned within the vector space spanned by the interference symbols sent from UL users. From the set of conditions (11) and (14), our transmit beamforming design for DL users guarantees that the kth DL user's transmit signal $\mathbf{H}_j^{[\mathrm{d}]}\mathbf{v}_k^{[\mathrm{d}]}, k \neq j$ lies in the interference subspace spanned by the UL users' signals, which is expressed as

$$\mathbf{w}_j^{[\mathrm{d}]} \perp \mathbf{H}_j^{[\mathrm{d}]}\mathbf{v}_k^{[\mathrm{d}]} \quad \text{and}$$

$$\mathbf{w}_j^{[\mathrm{d}]} \perp \mathrm{span}\left(\underbrace{\left[\mathbf{G}_{j,1}\mathbf{v}_1^{[\mathrm{u}]}\cdots\mathbf{G}_{j,K}\mathbf{v}_K^{[\mathrm{u}]}\right]}_{\text{rank-}(K-1)}\right), \quad \forall j \neq k,$$

$$\Rightarrow \dim\left(\mathbf{H}_j^{[\mathrm{d}]}\mathbf{v}_k^{[\mathrm{d}]} \cap \mathrm{span}\left(\left[\mathbf{G}_{j,1}\mathbf{v}_1^{[\mathrm{u}]}\cdots\mathbf{G}_{j,K}\mathbf{v}_K^{[\mathrm{u}]}\right]\right)\right) > 0$$

$$\overset{(a)}{\Rightarrow} \mathbf{H}_j^{[\mathrm{d}]}\mathbf{v}_k^{[\mathrm{d}]} \in \mathrm{span}\left(\left[\mathbf{G}_{j,1}\mathbf{v}_1^{[\mathrm{u}]}\cdots\mathbf{G}_{j,K}\mathbf{v}_K^{[\mathrm{u}]}\right]\right), \tag{15}$$

where $\mathbf{v} \perp \mathsf{span}(\mathbf{V})$ denotes that the vector of $\mathbf{v}$ is perpendicular to all the vectors in $\mathsf{span}(\mathbf{V})$, and (a) follows from the fact that $\mathbf{H}_j^{[\mathsf{d}]}\mathbf{v}_k^{[\mathsf{d}]}$ is a column vector.

Next, we consider decodability, which leads to achieving one-DoF for each DL user. We need to show that each DL user is able to decode its desired symbol sent by the BS. To prove this, it is sufficient to check whether the effective channel coefficient, $\mathbf{w}_k^{[\mathsf{d}]^\dagger}\mathbf{H}_k^{[\mathsf{d}]}\mathbf{v}_k^{[\mathsf{d}]}$, carrying the desired symbol $s_k^{[\mathsf{d}]}$, has at least non-zero values where $k \in \{1, 2, \dots, K\}$. Because $\mathbf{w}_k^{[\mathsf{d}]}$ is independently constructed with respect to $\mathbf{H}_k^{[\mathsf{d}]}\mathbf{v}_k^{[\mathsf{d}]}$, the element of $\mathbf{w}_k^{[\mathsf{d}]^\dagger}\mathbf{H}_k^{[\mathsf{d}]}\mathbf{v}_k^{[\mathsf{d}]}, \forall k$ is non-zero almost surely. Thus, each DL user can decode the desired symbol $s_k^{[\mathsf{d}]}$ for $k \in \{1, 2, \dots, K\}$. Without loss of generality, we focus on the decoding process at DL user 1. With the help of cyclic IA, i.e., (7) and (15), the rank of the effective interference channel matrix for DL user 1 is given as

$$\begin{aligned}&\mathsf{rank}\left(\left[\mathbf{H}_1^{[\mathsf{d}]}\mathbf{v}_2^{[\mathsf{d}]}\cdots\mathbf{H}_1^{[\mathsf{d}]}\mathbf{v}_K^{[\mathsf{d}]}\ \mathbf{G}_{1,1}\mathbf{v}_1^{[\mathsf{u}]}\cdots\mathbf{G}_{1,K}\mathbf{v}_K^{[\mathsf{u}]}\right]\right)\\&\quad=\mathsf{rank}\left(\left[\mathbf{G}_{1,1}\mathbf{v}_1^{[\mathsf{u}]}\cdots\mathbf{G}_{1,K}\mathbf{v}_K^{[\mathsf{u}]}\right]\right)=K-1.\end{aligned} \tag{16}$$

With the receive beamforming vector $\mathbf{w}_1^{[\mathsf{d}]}$ computed in (11), DL user 1 can cancel the aligned interference signals, which enables DL user 1 to recover the desired symbol $s_1^{[\mathsf{d}]}$ without interference. Similarly, we can set the all interference subspace spanned by $2K-1$ interference signal vectors to be aligned within a $(K-1)$-dimensional subspace at the other DL users, which enables the $d^{[\mathsf{d}]}\,(=1)$ desired symbol to be decoded at each DL user.

On the other hand, in order to show the achievability of one-DoF for each UL user, we should check whether the BS is able to decode its K desired UL users' symbols. The received signal vector at the BS from UL users is given by

$$\mathbf{y}^{[\mathsf{u}]} = \underbrace{\left[\mathbf{H}_1\mathbf{v}_1^{[\mathsf{u}]}\ \mathbf{H}_2^{[\mathsf{u}]}\mathbf{v}_2^{[\mathsf{u}]}\ \cdots\ \mathbf{H}_K^{[\mathsf{u}]}\mathbf{v}_K^{[\mathsf{u}]}\right]}_{\mathsf{rank}\text{-}K}\begin{bmatrix} s_1^{[\mathsf{u}]}\\ s_2^{[\mathsf{u}]}\\ \vdots\\ s_K^{[\mathsf{u}]}\end{bmatrix} + \mathbf{z}^{[\mathsf{u}]}. \tag{17}$$

Since the transmit beamforming vectors of UL users, $\mathbf{v}_i^{[\mathsf{u}]}$, are designed independently of all the uplink channel matrices $\mathbf{H}_j^{[\mathsf{u}]}, \forall j$, and all the channel coefficients are drawn from a continuous random distribution, the effective channel matrix $\left[\mathbf{H}_1\mathbf{v}_1^{[\mathsf{u}]}\ \mathbf{H}_2^{[\mathsf{u}]}\mathbf{v}_2^{[\mathsf{u}]}\ \cdots\ \mathbf{H}_K^{[\mathsf{u}]}\mathbf{v}_K^{[\mathsf{u}]}\right]$ has the full rank of K almost surely. This implies that the BS can individually decode the K UL users' signals by applying the following zero-forcing decoder that eliminates the effect of UL user interference:

$$\begin{aligned}&\mathbf{W}^{[\mathsf{u}]^\dagger}\\&=\left[\mathbf{w}_1^{[\mathsf{u}]}\ \mathbf{w}_2^{[\mathsf{u}]}\ \cdots\ \mathbf{w}_K^{[\mathsf{u}]}\right]^\dagger\\&=\left[\mathbf{H}_1\mathbf{v}_1^{[\mathsf{u}]}\ \mathbf{H}_2^{[\mathsf{u}]}\mathbf{v}_2^{[\mathsf{u}]}\ \cdots\ \mathbf{H}_K^{[\mathsf{u}]}\mathbf{v}_K^{[\mathsf{u}]}\right]^{-1}\begin{bmatrix}\gamma_1 & 0 & \cdots & 0\\ 0 & \gamma_2 & \cdots & 0\\ \vdots & & \ddots & \vdots\\ 0 & \cdots & 0 & \gamma_K\end{bmatrix},\end{aligned} \tag{18}$$

where γ_k is a normalization factor introduced to satisfy the receive power constraint.

TABLE I

ACHIEVABILITY OF CYCLIC IA FOR $(d^{[\mathsf{d}]}, d^{[\mathsf{u}]}) = (1, 1)$ WHEN $M = K$

N \ K	2	3	4	5	$\cdots$
2	$\boxed{\star}$				
3	$\circledast$	$\boxed{\star}$	$\star$		
4	$\circledast$	$\circledast$	$\boxed{\star}$	$\star$	
$\vdots$				$\ddots$	

In this way, the proposed cyclic IA enables each DL user to achieve one-DoF while the UL user also achieves one-DoF. This leads to achieving the optimal $2K$ sum-DoF by leveraging the following converse argument:

$$\sum_{k=1}^{K} d^{[\mathsf{u}]} + \sum_{k=1}^{K} d^{[\mathsf{d}]} \overset{(a)}{\leq} 2\min\{M, KN\} \overset{(b)}{=} 2K. \tag{19}$$

Note that the bound (a) follows from the DoF result of the MIMO two-way channel in [19], which corresponds to the FD-BS-HD-user cellular network with full cooperation among the $2K$ DL/UL users, and (b) comes from the assumption that $M = N = K$ and $K \geq 2$ made in this subsection.

Remark 1 (On Feasibility Conditions of Cyclic IA): The closed-form solution of cyclic IA achieves the DoF tuple $(d^{[\mathsf{d}]}, d^{[\mathsf{u}]}) = (1, 1)$ for the (K, K, K, K) FD-BS-HD-user cellular network, which turns out to be *optimal* in a sum-DoF sense. However, this does not necessarily imply that the cyclic IA method requires the minimum number of transmit/receive antenna configurations to achieve the optimal sum-DoF. Recently, the feasibility condition has been fully characterized for the symmetric case $K^{[\mathsf{u}]} = K^{[\mathsf{d}]} = K$, while the *exact* feasibility condition for $(M, N, K^{[\mathsf{u}]}, K^{[\mathsf{d}]})$ FD-BS-HD-user cellular networks has not been found so far in the general case. To be specific, for the (M, N, K, K) FD-BS-HD-user cellular network, the DoF tuple $(d^{[\mathsf{d}]}, d^{[\mathsf{u}]}) = (1, 1)$ is feasible almost surely if and only if $\min\left(\frac{M}{K}, \frac{2N}{K+2}\right) \geq 1$ [11]. According to the feasibility condition, the DoF tuple $(d^{[\mathsf{d}]}, d^{[\mathsf{u}]}) = (1, 1)$ is not feasible in the $(K-1, N, K, K)$ FD-BS-HD-user cellular network for any K. In addition to this, when $K \leq 3$, the DoF tuple is not feasible in the $(M, K-1, K, K)$ FD-BS-HD-user cellular network regardless of the value of M. Consequently, we conclude that cylic IC can achieve the *optimal* sum-DoF with the *minimum* transmit/receive antenna configurations of $M = N = K$ by simply solving a set of linear system equations as long as $K \leq 3$. This assumption is favorable from the practical standpoint when considering a realistic situation in which natural attenuation effects cause loss of connectivity between certain receivers and the interfering transmitter as K increases. In particular, we summarize the feasibility from [11] and achievability by cyclic IA in Table I when $M = K$. Note that $\star$ and $\square$ represent the feasibility from [11] and achievability by cyclic IA, respectively, and $\bigcirc$ denotes the achievability by a simple variant of cyclic IA, in which the $N-K$ extra antennas at each user are not exploited in achieving the DoF tuple $(d^{[\mathsf{d}]}, d^{[\mathsf{u}]}) = (1, 1)$.

Remark 2 (Feedback Mechanism): We have assumed that instantaneous CSI of all the channels is available at all the nodes in the FD cellular network. As a matter of fact, under mild assumptions on CSI feedback, the proposed method can be implemented in practice such as 3GPP LTE-A [14]. Each DL user first feeds back its incoming channels associated with UL users and the serving BS, and then the BS forms the beamforming vectors of DL/UL users via the cyclic IA solution. The BS sends the transmit beamforming vectors as data to its corresponding UL user via the downlink feed-forward channel. Note that, on the other hand, each DL user can determine its own receive beamforming vector by using of a demodulation reference signal without feed-forward processing. That is, the receive beamforming vectors for DL users are chosen as null space vectors of effective interference channels after applying transmit beamformings. Therefore, the cyclic IA can be implemented with little change to an existing cellular system supporting multi-user MIMO in 3GPP LTE-A.

B. Asymmetric FD MIMO Cellular Networks

In this subsection, we establish the minimum required antenna configuration to achieve the optimal DoF by exploiting cyclic IA for $(M, N, K^{[\mathsf{d}]}, K^{[\mathsf{u}]})$ FD-BS-HD-user cellular networks under the realistic assumption $M \geq \max\{K^{[\mathsf{d}]}, K^{[\mathsf{u}]}\}$.[4]

Theorem 1: To support $K^{[\mathsf{u}]}$ UL users and $K^{[\mathsf{d}]}$ DL users simultaneously via the FD BS with M transmit/receive antennas, the required minimum number of user antennas to achieve the symmetric DoF of $(d^{[\mathsf{d}]}, d^{[\mathsf{u}]}) = (1, 1)$ by using the cyclic IA is given by[5]

$$N^{\star} = \begin{cases} \max\left\{K^{[\mathsf{u}]}\left(1 - \frac{1}{K^{[\mathsf{u}]}}\left\lfloor\frac{K^{[\mathsf{u}]}-K^{[\mathsf{d}]}}{K^{[\mathsf{d}]}}\right\rfloor\right), 2\right\}, & K^{[\mathsf{u}]} \geq K^{[\mathsf{d}]}, \\ K^{[\mathsf{u}]} + 1, & K^{[\mathsf{u}]} < K^{[\mathsf{d}]}. \end{cases} \tag{20}$$

That is, if this condition is satisfied, the cyclic IA achieves a symmetric sum-DoF of $K^{[\mathsf{u}]} + K^{[\mathsf{d}]}$.

Proof: The proof of this theorem is very similar to one presented in Section III-A. Hence, we only highlight the difference in this proof.

1) Achievability when $K^{[\mathsf{u}]} \geq K^{[\mathsf{d}]}$: There are $K^{[\mathsf{d}]} - 1$ DL users' interference symbols as well as $K^{[\mathsf{u}]}$ UL users' interference symbols at each DL user. Similar to the case in Section III-A, the main goal for designing the transmit beamforming vectors of UL users is to align $K^{[\mathsf{u}]}$ UL users' interfering symbols into the $\max\left\{K^{[\mathsf{u}]} - \left\lfloor\frac{K^{[\mathsf{u}]}}{K^{[\mathsf{d}]}}\right\rfloor, 1\right\}$-dimensional subspace. To accomplish this goal, we form the *cyclic* IA conditions with a set of loop-equations as follows:

$$\mathbf{G}_{j|K^{[\mathsf{d}]},j}\mathbf{v}_j^{[\mathsf{u}]} = \lambda_j \mathbf{G}_{j|K^{[\mathsf{d}]},(j+1)|K^{[\mathsf{u}]}}\mathbf{v}_{(j+1)|K^{[\mathsf{u}]}}^{[\mathsf{u}]} \tag{21}$$

$$\Rightarrow \mathbf{v}_{(j+1)|K^{[\mathsf{u}]}}^{[\mathsf{u}]} = \frac{1}{\lambda_j}\mathbf{G}_{j|K^{[\mathsf{d}]},(j+1)|K^{[\mathsf{u}]}}^{-1}\mathbf{G}_{j|K^{[\mathsf{d}]},j}\mathbf{v}_j^{[\mathsf{u}]}, \quad \forall j \in \{1, 2, \ldots, K^{[\mathsf{u}]}\}. \tag{22}$$

Without loss of generality, we focus on the IA conditions associated with the j^{th} DL user, which are given as follows.

- **Case I-a**: $K^{[\mathsf{d}]} \geq 2$ and $j \leq K^{[\mathsf{u}]}|K^{[\mathsf{d}]}$

$$\begin{aligned} &\mathbf{G}_{j,\bar{j}}\mathbf{v}_{\bar{j}}^{[\mathsf{u}]} \\ &= \lambda_{\bar{j}}\mathbf{G}_{j,(\bar{j}+1)|K^{[\mathsf{u}]}}\mathbf{v}_{(\bar{j}+1)|K^{[\mathsf{u}]}}^{[\mathsf{u}]}, \\ &\quad \bar{j} \in \left\{j, j+K^{[\mathsf{d}]}, \ldots, j+\bar{r}K^{[\mathsf{d}]}\right\} \\ &\Rightarrow \begin{cases} \mathbf{G}_{1,1}\mathbf{v}_1^{[\mathsf{u}]} = \lambda_1\mathbf{G}_{1,2}\mathbf{v}_2^{[\mathsf{u}]} = \frac{1}{\lambda_{1+\bar{r}K^{[\mathsf{d}]}}}\mathbf{G}_{1,K^{[\mathsf{d}]}}\mathbf{v}_{K^{[\mathsf{u}]}}^{[\mathsf{u}]}, \\ \quad \text{if } j = 1 \text{ and } (K^{[\mathsf{u}]}-1)|K^{[\mathsf{d}]} = 0, \\ \mathbf{G}_{j,\bar{j}}\mathbf{v}_{\bar{j}}^{[\mathsf{u}]} = \lambda_{\bar{j}}\mathbf{G}_{j,(\bar{j}+1)|K^{[\mathsf{u}]}}\mathbf{v}_{(\bar{j}+1)|K^{[\mathsf{u}]}}^{[\mathsf{u}]}, \\ \quad \text{otherwise,} \end{cases} \end{aligned} \tag{23}$$

  where $\bar{r} = \left\lceil\frac{K^{[\mathsf{u}]}}{K^{[\mathsf{d}]}}\right\rceil - 1$. Thus, the number of dimensions of the interference signals sent by UL users at the j^{th} DL user becomes $K^{[\mathsf{u}]} - \left\lceil\frac{K^{[\mathsf{u}]}}{K^{[\mathsf{d}]}}\right\rceil - 1$ if $j = 1$ and $(K^{[\mathsf{u}]}-1)|K^{[\mathsf{d}]} = 0$, and $K^{[\mathsf{u}]} - \left\lceil\frac{K^{[\mathsf{u}]}}{K^{[\mathsf{d}]}}\right\rceil$ otherwise.
- **Case I-b**: $K^{[\mathsf{d}]} \geq 2$ and $j > K^{[\mathsf{u}]}|K^{[\mathsf{d}]}$

$$\begin{aligned} \mathbf{G}_{j,\bar{j}}\mathbf{v}_{\bar{j}}^{[\mathsf{u}]} &= \lambda_{\bar{j}}\mathbf{G}_{j,(\bar{j}+1)|K^{[\mathsf{u}]}}\mathbf{v}_{(\bar{j}+1)|K^{[\mathsf{u}]}}^{[\mathsf{u}]}, \\ &\quad \bar{j} \in \left\{j, j+K^{[\mathsf{d}]}, \ldots, j+\underline{r}K^{[\mathsf{d}]}\right\} \end{aligned} \tag{24}$$

  where $\underline{r} = \left\lfloor\frac{K^{[\mathsf{u}]}}{K^{[\mathsf{d}]}}\right\rfloor - 1$. Note that the UL users' interference signals always occupy the $K^{[\mathsf{u}]} - \left\lfloor\frac{K^{[\mathsf{u}]}}{K^{[\mathsf{d}]}}\right\rfloor$ dimension at the j^{th} DL user as illustrated in Fig. 3 (e.g. $\forall j > (K^{[\mathsf{u}]}|K^{[\mathsf{d}]}) = 0$ and $K^{[\mathsf{d}]} = 2$).
- **Case II**: $K^{[\mathsf{d}]} = 1$

$$\begin{aligned} \mathbf{G}_{1,\bar{j}}\mathbf{v}_{\bar{j}}^{[\mathsf{u}]} &= \lambda_{\bar{j}}\mathbf{G}_{1,(\bar{j}+1)|K^{[\mathsf{u}]}}\mathbf{v}_{(\bar{j}+1)|K^{[\mathsf{u}]}}^{[\mathsf{u}]}, \\ &\quad \bar{j} \in \left\{1, 2, \ldots, K^{[\mathsf{u}]}\right\} \end{aligned} \tag{25}$$

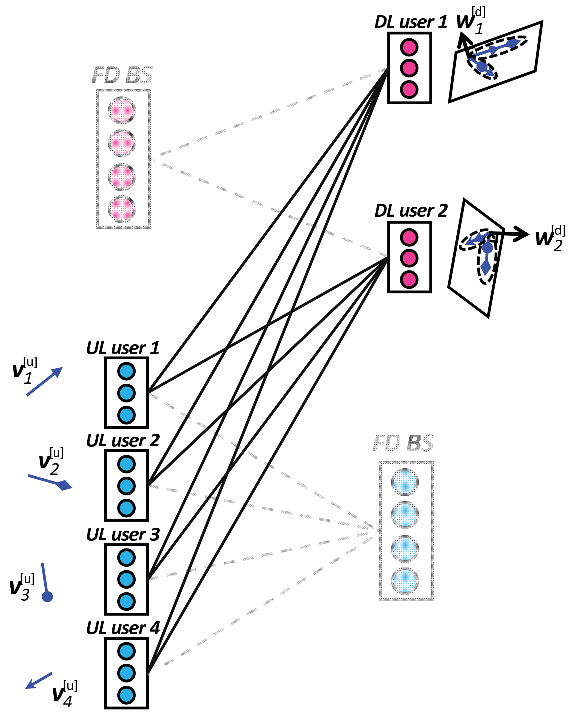
$$\begin{aligned} \Rightarrow \mathbf{G}_{1,1}\mathbf{v}_1^{[\mathsf{u}]} &= \lambda_1\mathbf{G}_{1,2}\mathbf{v}_2^{[\mathsf{u}]} = \left(\prod_{i=1}^{2}\lambda_i\right)\mathbf{G}_{1,3}\mathbf{v}_3^{[\mathsf{u}]} = \cdots \\ \cdots &= \left(\prod_{i=1}^{K^{[\mathsf{u}]}-1}\lambda_i\right)\mathbf{G}_{1,K^{[\mathsf{u}]}}\mathbf{v}_{K^{[\mathsf{u}]}}^{[\mathsf{u}]} \\ &= \left(\prod_{i=1}^{K^{[\mathsf{u}]}}\lambda_i\right)\mathbf{G}_{1,1}\mathbf{v}_1^{[\mathsf{u}]}, \end{aligned} \tag{26}$$

  where the product of all the coefficients λ_i becomes one, i.e., $\left(\prod_{i=1}^{K^{[\mathsf{u}]}}\lambda_i\right) = 1$. As a special case, the *cyclic IA* conditions imply that all the interference symbols sent by UL users are aligned into a one dimensional linear subspace.

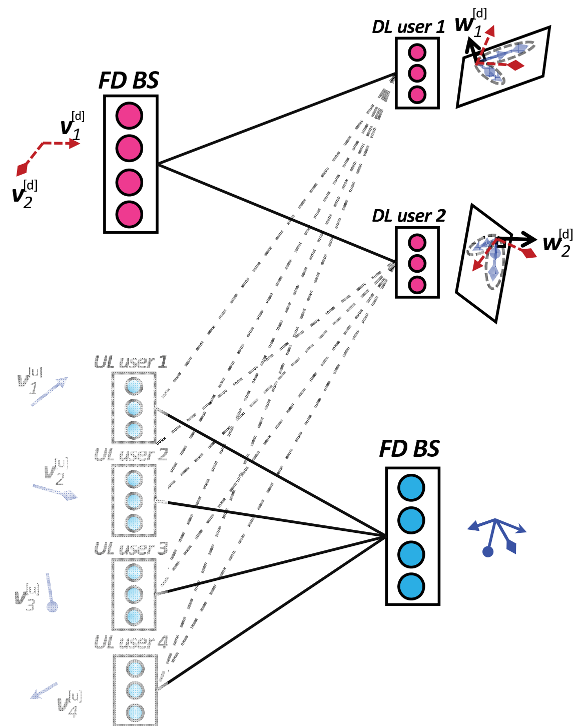
To sum up, the *cyclic IA* enables us to constrain the interference subspace of each DL user within at most $\max\{K^{[\mathsf{u}]} - \left\lfloor\frac{K^{[\mathsf{u}]}}{K^{[\mathsf{d}]}}\right\rfloor, 1\}$-dimensions by considering all the cases. Furthermore, similar to the symmetric topology case, the transmit beamforming vectors for each UL user are explicitly given by

[4]As mentioned in Section II, it is assumed that the FD BS does not schedule more than M DL users and/or M UL users at the same time, i.e., $M \geq \max\{K^{[\mathsf{d}]}, K^{[\mathsf{u}]}\}$.

[5]Note that the feasibility condition directly reduces to K as in Section III-A if the symmetric scenario is taken into account, i.e., $K^{[\mathsf{u}]} = K^{[\mathsf{d}]} = K$.



(a) Step 1: Beamforming vector design for UL users



(b) Step 2: Beamforming vector design for DL users

Fig. 3. A diagram illustrating *cyclic IA* for $K^{[\mathrm{u}]}=4$, $K^{[\mathrm{d}]}=2$, $M=4$, and $N=3$.

solving an eigenvalue problem, which is

$$\mathbf{v}_j^{[\mathrm{u}]}=\begin{cases}\mathsf{eig}\Big\{\mathbf{G}^{-1}_{K^{[\mathrm{u}]}|K^{[\mathrm{d}]},1}\mathbf{G}_{K^{[\mathrm{u}]}|K^{[\mathrm{d}]},K^{[\mathrm{u}]}}\\ \quad\times\Big(\prod_{i=K^{[\mathrm{u}]}-1}^{1}\mathbf{G}^{-1}_{i|K^{[\mathrm{d}]},i+1}\mathbf{G}_{i|K^{[\mathrm{d}]},i}\Big)\Big\},\ j=1,\\ \dfrac{\mathbf{G}^{-1}_{(j-1)|K^{[\mathrm{d}]},j}\mathbf{G}_{(j-1)|K^{[\mathrm{d}]},(j-1)|K^{[\mathrm{d}]}}\mathbf{v}^{[\mathrm{u}]}_{(j-1)|K^{[\mathrm{d}]}}}{\|\mathbf{G}^{-1}_{(j-1)|K^{[\mathrm{d}]},j}\mathbf{G}_{(j-1)|K^{[\mathrm{d}]},(j-1)|K^{[\mathrm{d}]}}\mathbf{v}^{[\mathrm{u}]}_{(j-1)|K^{[\mathrm{d}]}}\|_2},\\ \qquad\qquad 2\le j\le K^{[\mathrm{u}]}.\end{cases}\tag{27}$$

To guarantee a one-dimensional interference-free space at each DL user, the number of transmit/receive antennas at UL/DL users for the design of the cyclic IA is given by

$$N\ge\max\left\{K^{[\mathrm{u}]}-\left\lfloor\frac{K^{[\mathrm{u}]}}{K^{[\mathrm{d}]}}\right\rfloor,1\right\}+1\tag{28}$$

$$=\max\left\{K^{[\mathrm{u}]}\left(1-\underbrace{\frac{1}{K^{[\mathrm{u}]}}\left\lfloor\frac{K^{[\mathrm{u}]}-K^{[\mathrm{d}]}}{K^{[\mathrm{d}]}}\right\rfloor}_{\text{antenna-reduction factor}}\right),2\right\},\tag{29}$$

where $\frac{1}{K^{[\mathrm{u}]}}\left\lfloor\frac{K^{[\mathrm{u}]}-K^{[\mathrm{d}]}}{K^{[\mathrm{d}]}}\right\rfloor$ represents the antenna-reduction factor obtained by reducing the number of serving DL users $K^{[\mathrm{d}]}$ compared to that of UL users $K^{[\mathrm{u}]}$. It is observed that the factor reduces to zero when $K^{[\mathrm{u}]}=K^{[\mathrm{d}]}$, whereas it approaches one as $K^{[\mathrm{d}]}\to 1$. This result backs up the intuition that as the number of receivers increases in an interference-limited network, each transmitter needs to carefully control transmit signals with more antennas so that all of the unwanted receivers mitigate the interference signals.

Similar to (11) in the symmetric topology case, each DL user determines its received beamforming vector orthogonal to the subspace spanned by the aligned interference signals such that $\mathbf{w}_k^{[\mathrm{d}]^\dagger}\mathbf{G}_{k,j}\mathbf{v}_j^{[\mathrm{u}]}=0,\forall k\in\{1,2,\dots,K^{[\mathrm{d}]}\},\forall j\in\{1,2,\dots,K^{[\mathrm{u}]}\}$. In addition, given the receive beamforming vectors of DL users, the BS can design transmit beamforming vectors of DL users such that they are aligned into at most $\max\{K^{[\mathrm{u}]}-\lfloor\frac{K^{[\mathrm{u}]}}{K^{[\mathrm{d}]}}\rfloor,1\}$- dimensions already occupied by UL users' interference at each undesired DL user as long as $M\ge K^{[\mathrm{d}]}$. At the same time, similar to the symmetric topology case, the BS can also separately decode each UL user's symbol from the combined received signals by simple zero-forcing decoding when $M\ge K^{[\mathrm{u}]}$.

2) Achievability when $K^{[\mathrm{u}]}<K^{[\mathrm{d}]}$: Recall that the total number of *cyclic IA* conditions is always $K^{[\mathrm{u}]}$ for the single-beam case regardless of how many DL users there are in the network (i.e., regardless of $K^{[\mathrm{d}]}$). In other words, the symbols sent from any of $K^{[\mathrm{u}]}$ UL users cannot be aligned with each other at any of the last $K^{[\mathrm{d}]}-K^{[\mathrm{u}]}$ DL users while the aligned interference occupies only $(K^{[\mathrm{u}]}-1)$-dimensions at all the first $K^{[\mathrm{u}]}$ DL users if $K^{[\mathrm{u}]}<K^{[\mathrm{d}]}$. As in the symmetric topology case, the BS then sends the message for the j^{th} DL user in the direction that lies in the (aligned) UL users' interference subspace at all the other DL users, so that the received signal can be projected along the null space of the aligned interference signal subspace. To ensure a one-dimensional interference-free signal subspace at all the DL users, each DL/UL user has at least $K^{[\mathrm{u}]}+1$ antennas, thus making the null space of the aligned interference subspace

available at each DL user. In addition, the BS makes use of zero-forcing receive beamforming vectors to recover all the $K^{[\mathsf{u}]}$ messages with no interference. This completes the proof. ■

Remark 3 (Comparison to Other Feasibility Condition): Note that the problem of characterizing the *minimum* number of user antenna configurations by developing a new tighter necessary condition is still open and this will be left as future work. On the other hand, we can provide a comparison to the existing best-known feasibility result in the literature [11], which is given by

$$d^{[\mathsf{d}]} \leq M/K^{[\mathsf{d}]} \quad \text{and} \quad d^{[\mathsf{u}]} \leq M/K^{[\mathsf{u}]}, \tag{30}$$

$$\begin{aligned} |\mathcal{J}_{\mathsf{d}}||\mathcal{J}_{\mathsf{u}}|d^{[\mathsf{d}]}d^{[\mathsf{u}]} \leq & \sum_{k\in\mathcal{J}_{\mathsf{d}}} d^{[\mathsf{d}]}(N-d^{[\mathsf{d}]}) \\ & + \sum_{k\in\mathcal{J}_{\mathsf{d}}} d^{[\mathsf{d}]}(N-d^{[\mathsf{d}]}), \quad \forall \mathcal{J}_{\mathsf{d}}, \mathcal{J}_{\mathsf{u}}, \end{aligned} \tag{31}$$

where $\mathcal{J}_{\mathsf{d}} \subseteq \left[1 : K^{[\mathsf{d}]}\right]$ and $\mathcal{J}_{\mathsf{u}} \subseteq \left[1 : K^{[\mathsf{u}]}\right]$. Considering the underlying assumptions in Theorem 1, the two conditions in (30) always hold, and the condition in (31) is reduced to $N \geq \frac{|\mathcal{J}_{\mathsf{d}}||\mathcal{J}_{\mathsf{u}}|}{|\mathcal{J}_{\mathsf{d}}|+|\mathcal{J}_{\mathsf{d}}|} + 1$, $\forall \mathcal{J}_{\mathsf{d}}$, $\mathcal{J}_{\mathsf{u}}$. To sum up, the best-known feasibility condition can be written as $N \geq \frac{K^{[\mathsf{d}]}K^{[\mathsf{u}]}}{K^{[\mathsf{d}]}+K^{[\mathsf{u}]}} + 1$. This is because $\triangledown f(\mathbf{x}) \geq 0$ where $f(\mathbf{x}) = \frac{x_1 x_2}{x_1+x_2} = \frac{1}{1/x_1+1/x_2}$ for $\mathbf{x} = [x_1, x_2]^T$, which enables us to maximize $f(\mathbf{x})$ at the largest possible values of x_1 and x_2, respectively. The difference between the feasibility condition with integer constraints and the derived condition for cyclic IA (in Theorem 1) is given by

$$\Delta = \begin{cases} \max\left\{\left\lfloor \frac{K^{[\mathsf{u}]}}{K^{[\mathsf{d}]}/K^{[\mathsf{u}]}+1} \right\rfloor - \left\lfloor \frac{K^{[\mathsf{u}]}}{K^{[\mathsf{d}]}} \right\rfloor, 0\right\}, & K^{[\mathsf{u}]} \geq K^{[\mathsf{d}]}, \\ \left\lfloor \frac{K^{[\mathsf{u}]}}{K^{[\mathsf{d}]}/K^{[\mathsf{u}]}+1} \right\rfloor, & K^{[\mathsf{u}]} < K^{[\mathsf{d}]}. \end{cases} \tag{32}$$

The gap can be interpreted as the cost of cyclic IA with much less computational complexity in designing the transmit-and-receive beamforming vector pairs. Note that the gap tends to approach 0 as $K^{[\mathsf{d}]}$ decreases (e.g. $\Delta = 0$ when $K^{[\mathsf{d}]} = 1$).

IV. Extension to Multi-Beam Cases

So far, we have focussed our attention on the single beam case (i.e., each UL/DL user wishes to send/receive one data stream) for the $(M, N, K^{[\mathsf{d}]}, K^{[\mathsf{u}]})$ FD-BS-HD-user cellular network. We now move on to another typical scenario in which the throughput required in the downlink is higher than that required in the uplink owing to the asymmetry of the uplink and downlink traffic. Accordingly, we focus on an asymmetric DoF tuple allowing different numbers of $d_i^{[\mathsf{d}]}(\geq 1)$ for the downlink, while maintaining the equality constraint on $d_i^{[\mathsf{u}]}(= 1)$ for the uplink. In this setup, we derive a feasibility condition for antenna configurations to achieve the optimal DoF for the $(M, N, K^{[\mathsf{d}]}, K^{[\mathsf{u}]})$ FD-BS-HD-user cellular network in the following.

Theorem 2: To support $K^{[\mathsf{u}]}$ UL users and $K^{[\mathsf{d}]}$ DL users simultaneously via the FD BS with M transmit/receive antennas, the required minimum number of user antennas to achieve the optimal sum-DoF of $2M$ (i.e., $\mathsf{DoF}^{[\mathsf{u}]} = \mathsf{DoF}^{[\mathsf{d}]} = M$) is given by

$$N^\star = \begin{cases} \left(K^{[\mathsf{u}]} - \left\lfloor \frac{K^{[\mathsf{u}]}}{K^{[\mathsf{d}]}} \right\rfloor\right) + \frac{M - K^{[\mathsf{u}]}|K^{[\mathsf{d}]}\left(\left\lceil \frac{K^{[\mathsf{u}]}}{K^{[\mathsf{d}]}} \right\rceil - \left\lfloor \frac{K^{[\mathsf{u}]}}{K^{[\mathsf{d}]}} \right\rfloor\right)}{K^{[\mathsf{d}]}}, \\ \qquad \text{if } \left(K^{[\mathsf{u}]} - 1\right)|K^{[\mathsf{d}]} \neq 0, \\ \left(K^{[\mathsf{u}]} - \left\lfloor \frac{K^{[\mathsf{u}]}}{K^{[\mathsf{d}]}} \right\rfloor\right) + \frac{M - K^{[\mathsf{u}]}|K^{[\mathsf{d}]} - 1}{K^{[\mathsf{d}]}}, \\ \qquad \text{if } K^{[\mathsf{u}]}|K^{[\mathsf{d}]} = 1, K^{[\mathsf{d}]} \neq 1, \\ M+1, \qquad \text{if } K^{[\mathsf{u}]}|K^{[\mathsf{d}]} = 1, K^{[\mathsf{d}]} = 1, \end{cases} \tag{33}$$

for $K^{[\mathsf{u}]} = M$ and $\frac{M}{N} \leq K^{[\mathsf{d}]} \leq M$.[6]

Proof: The converse argument follows from the same line of reasoning applied to (19). Recall that the DoF region of the two-user $(M, K^{[\mathsf{d}]}N, K^{[\mathsf{u}]}N, M)$ MIMO Z-channel with output feedback for encoding and message side information for decoding is not in general smaller than the DoF region of the $(M, N, K^{[\mathsf{d}]}, K^{[\mathsf{u}]})$ FD-BS-HD-user cellular network [9], [15]. Denote $\mathsf{DoF}^{[\mathsf{u}]} = \sum_{i=1}^{K^{[\mathsf{u}]}} d_i^{[\mathsf{u}]}$ and $\mathsf{DoF}^{[\mathsf{d}]} = \sum_{i=1}^{K^{[\mathsf{d}]}} d_i^{[\mathsf{d}]}$. From the cut-set bound of the MIMO Z-channel, the outer bounds on the sum-DoF of the $(M, N, K^{[\mathsf{d}]}, K^{[\mathsf{u}]})$ FD-BS-HD-user cellular network can be derived as $\mathsf{DoF}^{[\mathsf{u}]} \leq M$ and $\mathsf{DoF}^{[\mathsf{d}]} \leq M$, which trivially yields $\mathsf{DoF}^{[\mathsf{u}]} + \mathsf{DoF}^{[\mathsf{d}]} \leq 2M$.

We now move on to the achievability part to prove this theorem. In particular, each UL user sends one symbol as above, so the FD BS can recover its desired M symbols using a zero-forcing method with M received antennas (i.e., $\mathsf{DoF}^{[\mathsf{u}]} = K^{[\mathsf{u}]} \cdot 1 = M$). Hence, we need to focus on the achievability part for the DL users ($\mathsf{DoF}^{[\mathsf{d}]} = \sum_{i=1}^{K^{[\mathsf{d}]}} d_i^{[\mathsf{d}]} = M$) to prove this theorem. Similar to the proof of *Theorem 1*, we consider two cases:

- **Case I**: $\left(K^{[\mathsf{u}]} - 1\right)|K^{[\mathsf{d}]} \neq 0$
  When the cyclic IA condition is satisfied as formulated in (21), at DL user j, $\forall j \leq K^{[\mathsf{u}]}|K^{[\mathsf{d}]}$, the total $K^{[\mathsf{u}]}$ UL users' interference signals are aligned into a $\left(K^{[\mathsf{u}]} - \left\lceil \frac{K^{[\mathsf{u}]}}{K^{[\mathsf{d}]}} \right\rceil\right)$-dimensional subspace, yet at all the rest of the DL users (DL user j for $j > K^{[\mathsf{u}]}|K^{[\mathsf{d}]}$) those interference signals are aligned into a $\left(K^{[\mathsf{u}]} - \left\lfloor \frac{K^{[\mathsf{u}]}}{K^{[\mathsf{d}]}} \right\rfloor\right)$-dimensional subspace. Here, if the FD BS can design the transmit beamforming vectors of DL users such that they are aligned into the vector subspace already occupied by UL users' interferences, it is possible for DL user j to resolve up to $N - \left(K^{[\mathsf{u}]} + \left\lceil \frac{K^{[\mathsf{u}]}}{K^{[\mathsf{d}]}} \right\rceil\right)$ independent messages if $j \leq K^{[\mathsf{u}]}|K^{[\mathsf{d}]}$, and $N - \left(K^{[\mathsf{u}]} + \left\lfloor \frac{K^{[\mathsf{u}]}}{K^{[\mathsf{d}]}} \right\rfloor\right)$ independent messages otherwise by cancelling the corresponding aligned interference. To achieve the M sum-DoF of DL users through the IA, the feasibility condition on the number of antennas for the FD BS and DL/UL users

[6] We here assume that the number of DL or UL users is large enough to maintain the cut-set DoF outer bound of M for each DL or UL channel as well as to guarantee at least a single beam per user, i.e., $K^{[\mathsf{u}]} \leq M$, $K^{[\mathsf{d}]} \leq M$, $K^{[\mathsf{u}]}N \geq M$ and $K^{[\mathsf{d}]}N \geq M$. Further, when the number of antennas, M, exceeds $K^{[\mathsf{u}]}$, the extra $M - K^{[\mathsf{u}]}$ antennas are ignored for simplicity when applying the proposed cyclic IA, resulting in $K^{[\mathsf{u}]} = M$.

should be satisfied as follows:

$$\begin{aligned} M &\le \left(N - K^{[u]} + \left\lceil \frac{K^{[u]}}{K^{[d]}} \right\rceil\right) K^{[u]}|K^{[d]} \\ &\quad + \left(N - K^{[u]} + \left\lfloor \frac{K^{[u]}}{K^{[d]}} \right\rfloor\right) \left(K^{[d]} - K^{[u]}|K^{[d]}\right), \\ \Longrightarrow N &\ge \left(K^{[u]} - \left\lfloor \frac{K^{[u]}}{K^{[d]}} \right\rfloor\right) \\ &\quad + \frac{M - K^{[u]}|K^{[d]}\left(\left\lceil \frac{K^{[u]}}{K^{[d]}} \right\rceil - \left\lfloor \frac{K^{[u]}}{K^{[d]}} \right\rfloor\right)}{K^{[d]}}. \end{aligned} \tag{34}$$

We note that $\left\lceil \frac{K^{[u]}}{K^{[d]}} \right\rceil - \left\lfloor \frac{K^{[u]}}{K^{[d]}} \right\rfloor$ becomes 0 or 1 according to whether or not the value of $\frac{K^{[u]}}{K^{[d]}}$ is an integer. Accordingly, it can be seen that the minimum requirement of N is simply reduced to $K^{[u]}$, provided that $K^{[u]}$ is a multiple of $K^{[d]}$ (i.e., $K^{[u]} = \alpha K^{[d]}$ for any positive integer α).

- **Case II**: $\left(K^{[u]} - 1\right)|K^{[d]} = 0$

The only difference between Case I and Case II is that in Case II the cyclic IA as formulated in (21) provides an opportunity for the first DL user (i.e., DL user 1) to align interference signals from UL users within an even smaller dimension than that in Case I due to the intrinsic nature of the cyclic IA method, leaving the extra dimensions available to the desired signals free of interference. To be more specific, it can be seen that the number of dimensions of the interference signals sent by UL users at the DL user 1 becomes $\max\left\{K^{[u]} - \left\lceil \frac{K^{[u]}}{K^{[d]}} \right\rceil - 1, 1\right\}$ from the foregoing conditions (23) and (25). Thus, in this case, the feasibility condition to support the M sum-DoF of DL users can be slightly changed from (34) as follows:

$$\begin{aligned} M &\le \left(N - \max\left\{K^{[u]} - \left\lceil \frac{K^{[u]}}{K^{[d]}} \right\rceil - 1, 1\right\}\right) \\ &\quad + \left(N - K^{[u]} + \left\lceil \frac{K^{[u]}}{K^{[d]}} \right\rceil\right) \cdot \left(K^{[u]} - 1\right)|K^{[d]} \\ &\quad + \left(N - K^{[u]} + \left\lfloor \frac{K^{[u]}}{K^{[d]}} \right\rfloor\right) \cdot \left(K^{[d]} - K^{[u]}|K^{[d]}\right), \\ \Rightarrow N &\ge \begin{cases} \frac{M - K^{[u]}|K^{[d]}\left(\left\lceil \frac{K^{[u]}}{K^{[d]}} \right\rceil - \left\lfloor \frac{K^{[u]}}{K^{[d]}} \right\rfloor\right) - 1}{K^{[d]}} \\ + \left(K^{[u]} - \left\lfloor \frac{K^{[u]}}{K^{[d]}} \right\rfloor\right), & \text{if } K^{[d]} \ne 1 \\ M + 1, & \text{if } K^{[d]} = 1. \end{cases} \end{aligned} \tag{35}$$

As for the case $K^{[d]} \ne 1$, the minimum required number of user antennas in Case II is further reduced by $\frac{1}{K^{[d]}}$ (i.e. a decrease of at most one antenna under the integer constraint) as compared to that in Case I. From the fact that $\left(K^{[u]} - 1\right)|K^{[d]} = 0$ implies that $\left\lceil \frac{K^{[u]}}{K^{[d]}} \right\rceil - \left\lfloor \frac{K^{[u]}}{K^{[d]}} \right\rfloor = 1$, the feasibility conditions can be further simplified as (33).

In conclusion, we have shown the feasibility conditions for achieving the optimal $2M$ sum-DoF (i.e., $\mathsf{DoF}^{[u]} = \mathsf{DoF}^{[d]} = M$) for all the cases we considered, which completes the proof. ∎

Remark 4 (DoF Gain from FD BS): To shed further light on the impact of the FD BS, it is instructive to compare the sum-DoF results compared to the conventional HD BS results. From the DoF results for the MIMO broadcast channel (BC) and multiple-access channel (MAC) in [18], the sum-DoF of the HD BS cellular network is upper bounded as $\mathsf{DoF}_{\mathsf{HD}} \le \max\{\mathsf{DoF}_{\mathsf{HD}}^{[u]}, \mathsf{DoF}_{\mathsf{HD}}^{[d]}\} = M$, where $\mathsf{DoF}_{\mathsf{HD}}^{[u]} = \min\{M, K^{[u]}N\} = M$ and $\mathsf{DoF}_{\mathsf{HD}}^{[d]} = \min\{M, K^{[d]}N\} = M$ are the sum-DoF of DL users and UL users for the HD BS, respectively, under the assumption used in Theorem 2. Moreover, for achievability, zero-forcing is sufficient to utilize the sum-DoF of the HD BS cellular network. Thus, we conclude that even when inter-link interference from UL users to DL users exists, the FD operation at the BS can double the sum-DoF over the conventional HD BS cellular networks, provided that the feasibility conditions of (33) are satisfied.

V. (M, N, K) FD-BS-FD-User Cellular Networks

In this section, we consider (M, N, K) FD-BS-FD-user cellular networks in which all users support FD operation along with the FD BS in the network. As before, we assume that the FD BS is equipped with M transmit/receive antennas, and wishes to send a set of independent messages to K FD users ($M \ge K$) each of which is equipped with N transmit/receive antennas. From the system model in Section II, it can be shown that the FD BS receives the same signal, regardless of the HD/FD operation at the user terminals. For the received signal at each (DL) user, unlike the (M, N, K, K) FD-BS-HD-user cellular network, there are only $K - 1$ inter-link interference signals from (UL) users in that the self-interference has been completely removed for the FD user case. In the network, one interesting question would be how much gain the system can provide if the FD capability is further implemented at the user terminals in the FD BS case. An answer to the aforementioned question will be addressed in terms of the required number of antennas at each user to achieve the symmetric DoF tuple $(d^{[d]}, d^{[u]}) = (1, 1)$, i.e., $2K$ sum-DoF.

Recall that the original cyclic IA conditions in (7) involve the inter-link interference from the UL user j to the DL user j, which is not supposed to be used in the FD user case. Thus, we need to reformulate the cyclic IA condition to fit into the new interference links so that the interference data symbols sent by user $i|K+1$ and user $(i+1)|K+1$ for the uplink are aligned at user i as follows:

$$\begin{aligned} \mathbf{G}_{K,1}\mathbf{v}_1^{[u]} &= \lambda_1 \mathbf{G}_{K,2}\mathbf{v}_2^{[u]} \\ \Rightarrow \mathbf{v}_2^{[u]} &= \frac{1}{\lambda_1}\mathbf{G}_{K,2}^{-1}\mathbf{G}_{K,1}\mathbf{v}_1^{[u]}, \\ \mathbf{G}_{1,2}\mathbf{v}_2^{[u]} &= \lambda_2 \mathbf{G}_{1,3}\mathbf{v}_3^{[u]} \\ \mathbf{v}_3^{[u]} &= \frac{1}{\lambda_2}\mathbf{G}_{1,3}^{-1}\mathbf{G}_{1,2}\mathbf{v}_2^{[u]}, \\ &\ \ \vdots \qquad\qquad \vdots \\ \mathbf{G}_{K-2,K-1}\mathbf{v}_{K-1}^{[u]} &= \lambda_{K-1}\mathbf{G}_{K-2,K}\mathbf{v}_K^{[u]} \\ \Rightarrow \mathbf{v}_K^{[u]} &= \frac{1}{\lambda_{K-1}}\mathbf{G}_{K-2,K}^{-1}\mathbf{G}_{K-1,K-1}\mathbf{v}_{K-1}^{[u]}, \\ \mathbf{G}_{K-1,K}\mathbf{v}_K^{[u]} &= \lambda_K \mathbf{G}_{K-1,1}\mathbf{v}_1^{[u]} \\ \Rightarrow \mathbf{v}_1^{[u]} &= \frac{1}{\lambda_K}\mathbf{G}_{K-1,1}^{-1}\mathbf{G}_{K-1,K}\mathbf{v}_K^{[u]}. \end{aligned} \tag{36}$$

By exploiting the special structure of the revised loop-equations such as (8), we can identify transmit beamforming vectors for users by solving the eigenvalue problem as follows:

$$\left(\prod_{i=1}^{K}\lambda_i\right)\mathbf{v}_1^{[\mathrm{u}]} = \left\{\mathbf{G}_{K-1,1}^{-1}\mathbf{G}_{K-1,K}\left(\prod_{i=K-2}^{1}\mathbf{G}_{i,i+2}^{-1}\mathbf{G}_{i,i+1}\right)\mathbf{G}_{K,2}^{-1}\mathbf{G}_{K,1}\right\}\mathbf{v}_1^{[\mathrm{u}]}, \tag{37}$$

and $\mathbf{v}_2^{[\mathrm{u}]} = \frac{1}{\lambda_1}\mathbf{G}_{K,2}^{-1}\mathbf{G}_{K,1}\mathbf{v}_1^{[\mathrm{u}]}$ and $\mathbf{v}_{j+1}^{[\mathrm{u}]} = \frac{1}{\lambda_{j+1}}\mathbf{G}_{j,j+2}^{-1}\mathbf{G}_{j,j+1}\mathbf{v}_j^{[\mathrm{u}]}$ for $\forall j > 2$.

This construction guarantees that inter-link interference from the UL signals at each user is restricted within $\max\{K-2,1\}$ dimensions, allowing each user to recover $N - \max\{K-2,1\}$ desired symbols, provided that the DL interference signals from the FD BS do not span more than $\max\{K-2,1\}$ dimensions. In order to achieve one DoF for the DL message of each user, the number of antennas at the user should be greater than or equal to $\max\{K-1,2\}$, i.e., $N \geq \max\{K-1,2\}$. At the same time, the FD BS can recover all the K UL messages under the assumption $M \geq K$, which leads to achieving K sum-DoF for UL messages. As a result, all symbols (for K DL messages and K UL messages) is resolved by cyclic IA, so that $2K$ sum-DoF (i.e., $(d^{[\mathrm{d}]}, d^{[\mathrm{u}]}) = (1,1)$) is achieved on the (M,N,K) FD-BS-FD-user cellular networks if $N \geq \max\{K-1,2\}$.

To show the optimality of the proposed cyclic IA in the FD user case, we investigate a sum-DoF outer bound in $(M, \max\{K-1,2\}, K)$ FD-BS-FD-user cellular networks. By allowing full cooperation among the K users in the $(M, \max\{K-1,2\}, K)$ FD-BS-FD-user cellular network, we obtain a MIMO two-way network. Thus, from the result in [19] we have the following upper bound:

$$\sum_{k=1}^{K} d^{[\mathrm{u}]} + \sum_{k=1}^{K} d^{[\mathrm{d}]} \leq 2\min\left\{M, K\max\{K-1,2\}\right\} \tag{38}$$

$$\leq 2\min\left\{M, K(K-1), 2K\right\} \tag{39}$$

$$= 2K. \tag{40}$$

From this converse argument, the proposed cyclic IA achieves the optimal sum-DoF of $2K$ if the derived feasibility condition is satisfied, i.e., $N \geq \max\{K-1,2\}$.

Remark 5 (Extension to the case of $N > K-1$): If the number of antennas at FD users is assumed to be large enough to cancel all of the $K-1$ inter-link interference without aligning interference, i.e., $N > K-1$, the optimal DoF can be achieved by a rather simple method. That is, no specific transmit beamforming vectors for UL users are required, but other beamforming designs are exactly the same as those of cyclic IA. Specifically, transmit beamforming vectors for DL users can be designed such that all of the DL users' interference must be aligned within the vector space spanned by the interference symbols sent from UL users, regardless of the transmit beamforming vectors for UL users. As for the receive beamforming design for UL users, a zero-forcing decoder can be applied as in cyclic IA to achieve the optimal DoF.

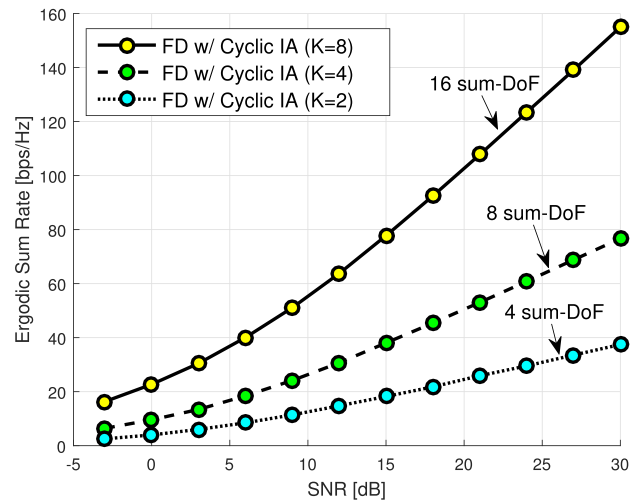


Fig. 4. Ergodic sum rate versus SNR when K is 2, 4, and 8.

VI. Numerical Results And Discussion

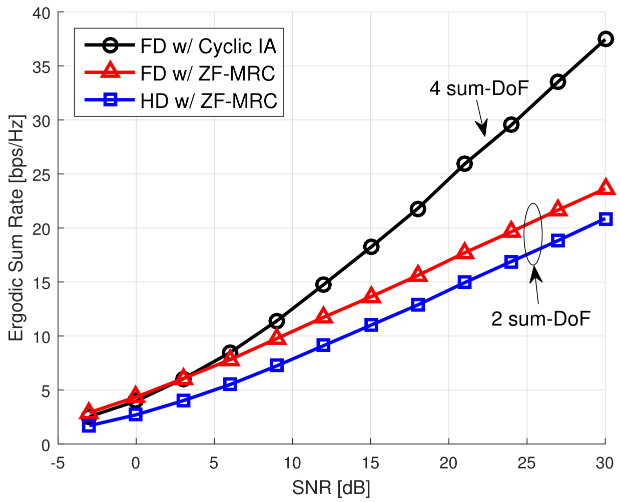
To evaluate the performance of the proposed cyclic IA scheme, we first present several numerical results in the single-cell FD cellular network obtained by means of Monte-Carlo simulations. Thereafter, we assess its performance in outdoor picocell networks in order to identify the effectiveness of cyclic IA in a practical multi-cell environment. In this evaluation, we focus on the scenario of a symmetric topology with HD users, in order to facilitate an understanding of the performance of the proposed scheme.

A. Achievable Ergodic Sum Rate Performance

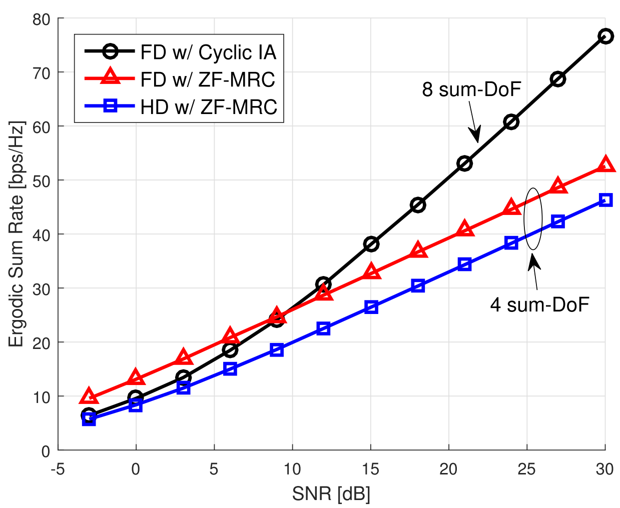
Fig. 4 shows the ergodic sum rate of the proposed cyclic IA versus SNR when $K = 2$, 4, and 8 in a symmetric (K,K,K,K) FD-BS-HD-user cellular network. Here, the sum rate indicates the sum of the achievable rates of all downlink and uplink transmissions. It is observed that the sum rate of the proposed cyclic IA increases linearly with the slope of 4, 8, and 16 when K is 2, 4, and 8, respectively. This numerically verifies that the proposed cyclic IA achieves the optimal $2K$ sum-DoF as proven earlier in Theorem 2.

Fig. 5 shows the ergodic sum rate of the cyclic IA and two conventional schemes as a function of the SNR when K is 2 and 4. For the conventional methods, each user is assumed to use the maximal ratio combining (MRC) for DL and the maximal ratio transmission (MRT) for UL to maximize the desired channel link. At the same time, the BS uses a zero-forcing precoder/combiner to nullify the inter-user interference in DL/UL, referred to as *ZF-MRC* for simplicity. This is a reasonable choice to maximize the achievable sum rate while guaranteeing zero inter-user interference, by assuming no inter-link interference.[7] We apply it to two different scenarios for performance comparison

[7]The method does not take into account the inter-link interference even in the FD BS scenario.



(a) K=2



(b) K=4

Fig. 5. Comparison of ergodic sum rate when K is 2 and 4.

purposes. One is operated with the FD BS causing inter-link interference, and the other is operated with the HD BS without causing inter-link interference. It is observed that the proposed cyclic IA exhibits double sum-DoF compared to the conventional schemes. Thus, the proposed scheme significantly outperforms the conventional schemes in a high SNR regime. However, in a low SNR regime, the proposed scheme even provides poor sum rate performance compared to ZF-MRC in FD mode according to the value of K. This is mainly because when the proposed scheme is applied, the transmitters essentially sacrifice the desired signal power in the low SNR regime owing to alignment of interference signals over the spatial dimension, whereas ZF-MRC aims to maximize it. From these numerical results, we realize that the proposed cyclic IA is much more effective in a higher SNR regime with a severe inter-link interference effect.

B. Performance in Multi-Cell Networks

To evaluate the performance of the proposed cyclic IA in practical multi-cell networks, we consider an outdoor picocell

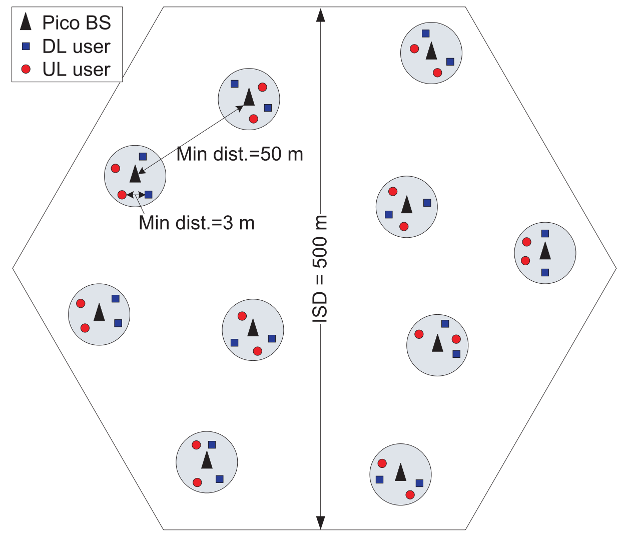


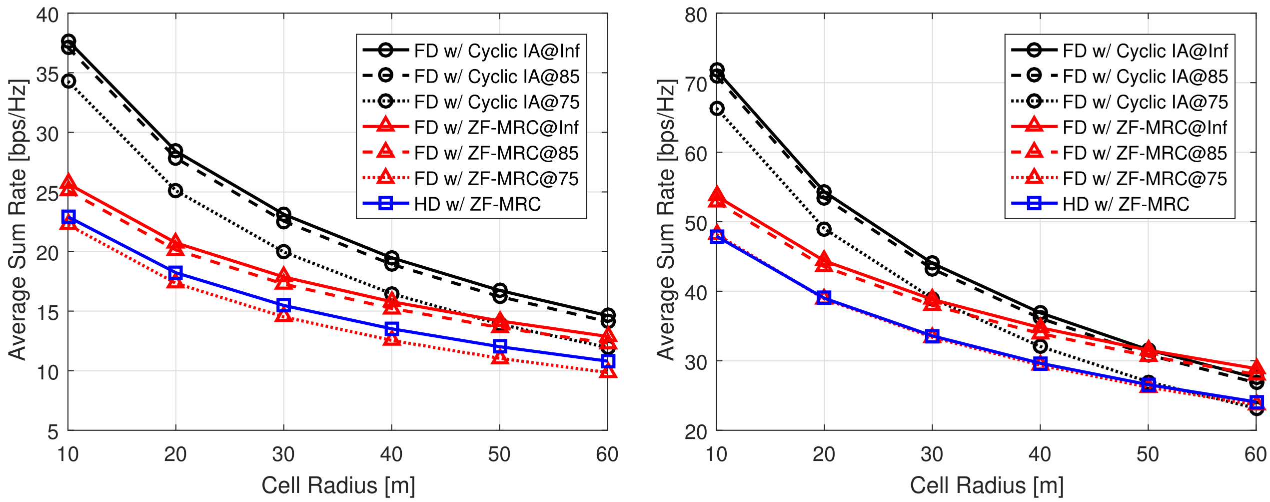
Fig. 6. Outdoor picocell network layout.

network in which the FD function is embedded in the picocell BS. The picocell is one of the candidates suitable for the application of the FD system because i) the picocell has a small coverage, and so it can operate in a high SNR regime; ii) the inter-cell interference among picocells is not so dominant because they are usually deployed sparsely in hotspots; and iii) the transmission power in the pico BS is smaller than that in the macro BS so that the remaining self-interference power due to the imperfect self-interference cancellation (SIC) at the pico BS is weak [20]. Fig. 6 illustrates the considered outdoor picocell network. We consider ten picocells deployed randomly in a macrocell area and K DL/UL users dropped randomly in each picocell. Therefore, there is inter-link interference from UL users to DL users within each picocell and inter-cell interference from the BS and the UL users from other picocells, but there is no interference from the macrocell assuming that the picocells use a dedicated frequency band different from that of the macrocell. In addition, we do not consider any specific user scheduling considering these interference properties since this is beyond the scope of this paper. This scenario reflects a commercial picocell deployment, which covers local traffic hotspots with low cost and complexity. The related parameters are based on the 3GPP simulation methodology [14] and are described in Table II. For the pathloss model, we assume that all the links within the picocell (i.e., DL/UL signal and inter-link interference links) have the line-of-sight (LOS) condition, and all the links among picocells (i.e., inter-cell interference links) have the non-line-of-sight (NLOS) condition, considering that the number of obstacles increases as the physical distance increases in outdoor environments [14]. Further, we consider a realistic SIC capability of 85 and 75 dB as well as an ideal SIC capability in order to investigate the effect of imperfect SIC in the FD BS, based on the state-of-the-art of SIC that achieves a cancellation range from 70 to 100 [21]. Here, FD@x denotes the FD system with an SIC of x dB and FD@Inf denotes that there is no self-interference.

Fig. 7 shows the average sum rate versus the cell radius when K is 2 and 4. We examine the proposed cyclic IA

TABLE II
SIMULATION PARAMETERS

Parameter	Value
Inter-site distance (ISD) of the macrocell	500 m
Number of picocells in a macrocell	10
Radius of a picocell	variable (10∼60 m)
Minimum distance between pico BSs	50 m
Minimum distance between users	3 m
Minimum distance between pico BS and users	picocell radius×0.12 m
Maximum BS transmission power	24 dBm
Maximum user transmission power	23 dBm
Channel bandwidth	10 MHz
Thermal noise density	-174 dBm/Hz
Noise figure	9 dB
Shadowing standard deviation	8 dB
Pathloss within a picocell	LOS: $PL(R) = 103.8 + 20.9log_{10}(R)$ dB (R in km)
Pathloss among picocells	NLOS: $PL(R) = 145.4 + 37.5log_{10}(R)$ dB (R in km)
SIC capability	Inf / 85 / 75 dB

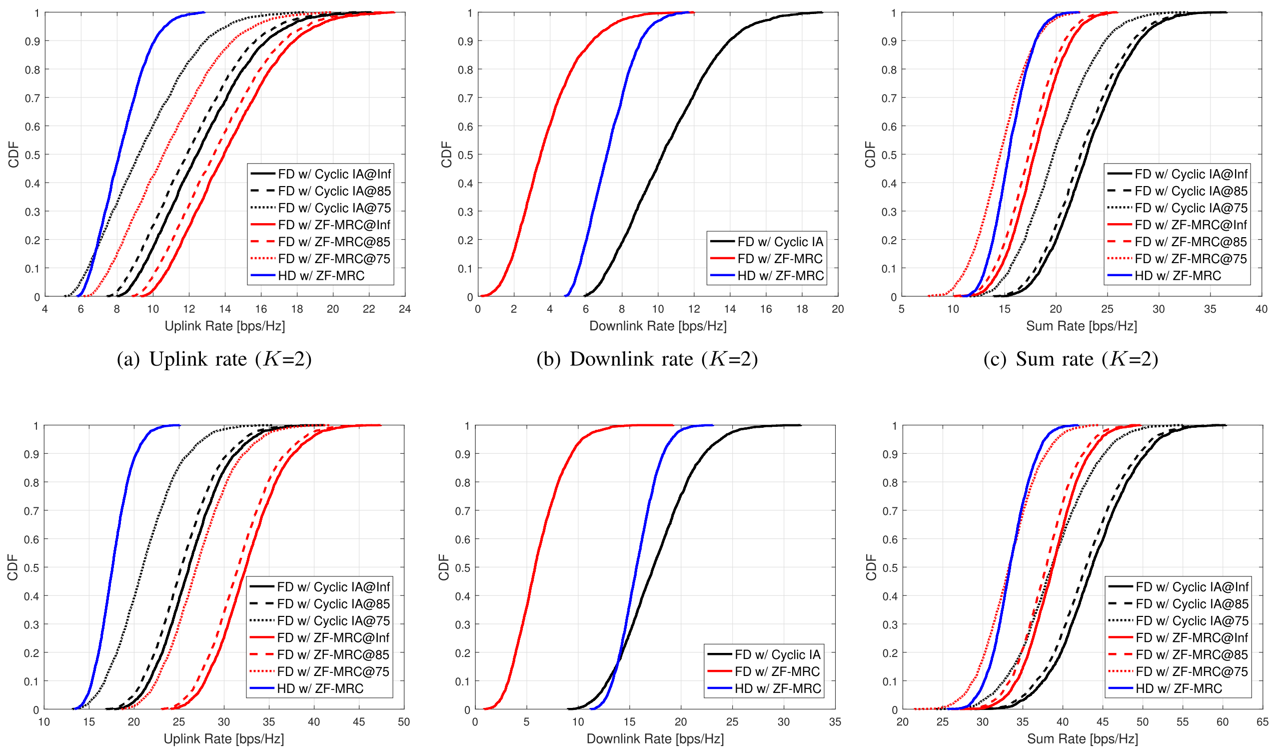


(a) K=2 (b) K=4

Fig. 7. Average sum rate versus cell size.

scheme according to the variation of the cell radius. The size of the picocell is an important operational parameter because the system has a trade-off performance that depends on the cell size. For example, the off-loading effect increases but the inter-cell interference increases as the picocell size increases. Moreover, the received signal strength increases but the inter-link interference increases in the FD system as the picocell size decreases. Therefore, it is meaningful to investigate over which range of cell radius the proposed IA scheme has a performance gain. As the cell radius increases, the sum rate decreases in all schemes because all the SNRs at the BS and users decrease. The performance difference between the proposed cyclic IA and the FD with ZF-MRC is reduced as the cell radius increases. When $K = 4$, the performance of the FD with ZF-MRC is better than that of the cyclic IA at a cell radius of 50 m. This is because the SNR decreases and the inter-link interference becomes less dominant as the cell radius increases. Moreover, as the SIC capability decreases, the sum rate decreases because the UL rate decreases due to the remaining self-interference. This imperfect SIC effect makes the performance of the FD with ZF-MRC worse than that of the HD with ZF-MRC. Overall, the proposed cyclic IA scheme outperforms the conventional FD and HD schemes in the common range of the picocell radius (which is less than 50 m) and the practical range of SIC capability.

Fig. 8 shows the cumulative distribution functions (CDFs) of the uplink rate, downlink rate, and sum rate when the cell radius is 30 m with $K = 2, 4$. In the case of the uplink rate, the FD with ZF-MRC is slightly better than that with cyclic IA because ZF-MRC aims to maximize the sum rate performance assuming no inter-link interference, which naturally fits into such an FD uplink case. However, in the case of the downlink rate, the proposed cyclic IA significantly outperforms the FD with ZF-MRC because the proposed scheme cancels the inter-link interference effectively but the FD with ZF-MRC suffers from the severe inter-link interference. In the downlink, the HD with ZF-MRC is better than the FD with ZF-MRC because it does not experience any inter-link interference, but it is worse than the proposed cyclic IA because it uses only half the bandwidth. Combining both the uplink and downlink rates, the proposed cyclic IA shows better sum-rate performance than the other conventional schemes according to the SIC



(a) Uplink rate (K=2) (b) Downlink rate (K=2) (c) Sum rate (K=2)

(d) Uplink rate (K=4) (e) Downlink rate (K=4) (f) Sum rate (K=4)

Fig. 8. Cumulative distribution functions of the uplink rate, downlink rate, and sum rate when the cell radius is 30 m.

capability because it uses the full resources for uplink and downlink transmissions simultaneously without the inter-link interference.

VII. Conclusion

In this paper, we have introduced a novel constructive method to find the closed-form IA solution for a class of FD multi-antenna cellular networks, in which the BS is capable of FD operation while users commonly support either FD or HD operation according to their hardware constraints. It has been shown that the proposed cyclic IA can achieve the optimal sum-DoF when the numbers of user antennas are sufficient to meet the derived feasibility condition. We have further demonstrated that the FD BS can double the sum-DoF even in the presence of inter-link interference compared to the conventional HD network, provided that the number of users is sufficiently large compared to the ratio of the number of BSs and user antennas. We have also confirmed through numerical results that the proposed cyclic IA significantly outperforms the existing algorithms for single-cell and multi-cell environments. In future work, we plan to consider the impact of local or incomplete CSI at the transmitters in FD multi-antenna cellular networks from a DoF perspective.

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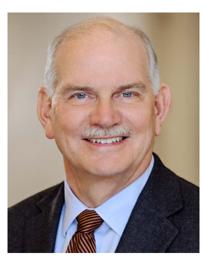

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