Risk-limiting Unit Commitment in Smart Grid with Intelligent Periphery

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Abstract—This paper proposes the risk-limiting unit commitment (RLUC) as the operational method to address the uncertainties in the smart grid with intelligent periphery (GRIP). Three key requirements are identified for the RLUC in GRIP. The first one requires the RLUC to be modeled as a multi-stage multi-period unit commitment problem considering power trades, operational constraints, and operational risks. The second one requires the RLUC considering the conditional prediction to achieve a globally optimal solution. It is addressed by using conditional probability in a scenario-based form. The last one requires the risk index in the RLUC to be both valid and computationally friendly, and it is tackled by the utilization of a coherent risk index and the mathematical proof of a risk chain theorem. Finally, the comprehensive RLUC in GRIP satisfying all the three requirements is solved by an equivalent transformation into a mixed integer piecewise linear programming problem. Case studies on a 9-bus system, a realistic provincial power system, and a regional power grid in China demonstrate the advantages of the proposed RLUC in GRIP.

Index Terms—Cluster, renewables, risk-limiting, smart grid with intelligent periphery (GRIP), unit commitment

I. NOMENCLATURE

A. Acronyms

CVaR	Conditional value at risk
DS	Dispatch stage
DP	Delivery period
GRIP	Smart grid with intelligent periphery
LOL	Loss of load
LOLP	Loss of load probability
PDF	Probability density function
R1, R2, R3	Requirement 1, requirement 2, requirement 3
RLD	Risk limiting dispatch
RLUC	Risk limiting unit commitment
UC	Unit commitment

B. Indices

i	Index of dispatch stages, from 1 to I.
t	Index of delivery periods, from 1 to T
k	Index of samples, from 1 to K
l	Index of transmission line, from 1 to L
C	Index of the current stage, namely $i = C$

C. Parameters

$\mathbf{c}^b{}_{it}$, $\mathbf{c}^s{}_{it}$	Price of buy and sell at DS i for DP t
$\mathbf{c}^{sp}{}_t$	Price of wind spillage at DP t
$\mathbf{c}^l$	Price of load shedding at DP t
$\mathbf{c}^u{}_t$	Start-up price of the units at DP t
$Risk_t$	Risk bound at DP t
$\mathbf{P}_{min}$, $\mathbf{P}_{max}$	Unit lower and upper capacity
$\mathbf{f}_{max}$	Line thermal limit
$\mathbf{P}^D$, $\mathbf{P}^U$	Unit ramping down and up capacity
α	Confidential level

D. Decision variables

$\mathbf{p}^b{}_{it}$, $\mathbf{p}^s{}_{it}$	Power buying and selling at DS i for DP t
$\mathbf{p}^b{}_{itk}$, $\mathbf{p}^s{}_{itk}$	Power buying and selling at DS i for DP t in sample k
$\mathbf{w}^{sp}{}_t$	Power of wind spillage at DP t
$\mathbf{w}^{sp}{}_{tk}$	Power of wind spillage at DP t in sample k
$\mathbf{l}_t$	Power of load shedding DP t
$\mathbf{l}_{tk}$	Power of load shedding DP t in sample k
$\mathbf{ON}_t$	On/off state of the units at DP t
$\mathbf{ON}_{tk}$	On/off state of the units at DP t in sample k
$\boldsymbol{\theta}_{tk}$	Phase angle at DP t in sample k
β_{lt}	Ancillary variable for line l at DP t

E. Random variables

$\mathbf{L}_t$	Load demand at DP t
$\mathbf{L}_{tk}$	Load demand at DP t in sample k
$\mathbf{w}_t$	Wind power at DP t
$\mathbf{w}_{tk}$	Wind power at DP t in sample k
Y_i	Prediction information available at DS i

II. INTRODUCTION

THE Energy Internet is introduced to achieve a sustainable and green energy-oriented power energy system [1], in which the electric grid is envisioned as plug-in, energy shared, and distributed controlled [2], taking advantages of advanced

data communication technologies [3].

GRIP was proposed as a future architecture for the Energy Internet in [4]. There are three key functions in the GRIP architecture. The first function focuses on the system operation in order to maintain the instantaneous power balance. The second function is about the frequency fluctuation alleviation, and the last function is about the system resiliency.

This paper concentrates on the first function. In GRIP, the operational method should be in the spirit of "risk-limiting" [4], which means the operational risk should be mitigated through multiple dispatch stages before the real-time [5]. This is the *premise* of the operational method in GRIP [4]. Based on this premise, the risk-limiting operational method in GRIP should be extended to satisfy 3 requirements, in addition to the traditional requirements on system operation, towards the realistic implementation.

Requirement 1 (R1): The multi-stage multi-period UC in the spirit of "risk-limiting" should be modeled, considering the power trades in GRIP and operational constraints such as the transmission and ramping constraint.

Requirement 2 (R2): The continuously updated prediction and prediction errors for the renewables should be considered.

Requirement 3 (R3): A valid risk index [1] in terms of risk-limiting should be adopted in the RLUC, being computationally friendly to any kind of random distributions.

The reasons for such requirements are as follows. First, the UC is a critical step in any operational methodology in power grids [6]. In addition, the UC should consider power trades, the multi-stage multi-period operational framework, and the operational constraints in GRIP. Power trade is a basic feature in GRIP [4]. Multi-period operational framework is the operational framework required by the premise, and the operational constraints should be incorporated in system operation. For example, transmission line congestion may alter the system economic status, or even lead to the system operational infeasibility [7]. Second, large percentage of renewables integration in GRIP brings about uncertainties [8]. The prediction errors of the uncertain renewable generation affects the operational decision, because it reflects the deviation from the prediction of renewables and thus critical for the recourse decision. Mathematically, the utilization of the information on the prediction and prediction error on uncertainties is a guarantee of a globally optimal operational decision [5]. Neglecting the prediction error will lead to a local optimal decision [9]. At last, not all risk indexes are valid in terms of risk-limiting operation in GRIP. Here a "valid" risk index means it should satisfy the property defined in [5]. Details of the valid risk index are illustrated in Section VI. In addition, there are numerous renewable prediction methods which vary in time scale [10], so it is arbitrary to assume the prediction as any specific distribution. Therefore, a valid risk index compatible to any kind of distribution is critical.

The state-of-the-art RLD approaches which satisfy the premise addressed the 3 requirements in different perspectives.

However, none of them fulfilled the 3 requirements simultaneously, and thus they were not qualified to be a realistic and comprehensive operational method in GRIP. For the R1, the multi-period power delivery framework was considered in [11], and the ramping constraint and energy storage were addressed in [12], [13]. However, real UC problems on multiple delivery periods were not considered. In addition, the transmission network constraint was tackled in [14], and the impact of both ramping and transmission capacity was analyzed in [15], but the power trade was not addressed. For the R2, the prediction error was interpreted as the conditional prediction and described by the conditional probability distribution in [5], [11]-[14]. For the R3, the LOLP was the risk index in [5], [11]-[14], but it is not capable to precisely describe the operational risk with low probability but high consequences, which is a major concern for operators, and it is not easy in computation. The energy expected not served was the risk index in [16], but the validity of this risk index was not proved.

Some other literatures studied the unit commitment problem for systems with high renewables penetration from the perspective of operational risk considering different risk measures. However, none of them meet the premise and the three requirements simultaneously. [17] proposed a two-stage unit commitment considering the LOLP and transmission line overloading probability (TLOP). In addition to the LOLP and the TLOP, the probability of wind curtailment was considered as the operational risk and integrated into the chance constrained UC [18]. [19] combined the probability and expectation as the risk index and formulated the UC model. Some other references adopted CVaR as the risk index, because of its merits in computation and in describing the risk of the tail loss. [20] proposed the two-stage CVaR based UC including the reserve requirements in isolated systems. The energy storage and demand response was integrated into the CVaR based UC in [21]. These works neither mitigate the operational risk in a multi-stage multi-period framework, nor consider the power trade, the conditional prediction and the validity of using CVaR as a risk index. Therefore, they are not the feasible operational methods in GRIP.

In sum, the research gap lies in the deficiency of a qualified operational method in GRIP, because all previous work did not satisfy the premise and all three requirements simultaneously, summarized in Table I.

TABLE I
PREVIOUS WORKS REGARDING THE PREMISE AND REQUIREMENTS

	[17]-[21]	[5]	[11]-[14]	[15]	[16]
Premise	No	Yes	Yes	Yes	Yes
R1	Partial	No	Partial	Partial	No
R2	No	Yes	Yes	No	Yes
R3	Partial	No	No	No	No

This paper fills this gap by proposing the RLUC which meets the premise and all three requirements concurrently, as the comprehensive operational method in GRIP. At first, an operational framework and a general model formulation of the RLUC for each cluster in GRIP are proposed so as to meet the R1. In order to satisfy the other two requirements, there are two

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technical challenges which lie in the integration of the conditional prediction information into the RLUC and the choice of a valid risk index in the RLUC. To address the first challenge, the prediction error is interpreted as the conditional prediction in a form of scenario based probability distribution. For the second one, CVaR is selected as the risk index for the operational risk. To our knowledge, the validity of using CVaR in the RLUC is mathematically proved for the first time. Therefore, the final RLUC model satisfies the premise and the three requirements because it is a multi-stage multi-period unit commitment problem encapsulating the power trades, the operational constraints and the operational risk, and armed with the conditional prediction and the valid risk index. In addition, two more theorems are applied to reformulate the RLUC as a mixed integer piecewise linear optimization problem which can be efficiently solved by existing solvers.

The contributions of this paper are summarized as follows: (1) The multi-stage multi-period RLUC considering power trades and the operational constraints is formulated as the comprehensive operational method in GRIP. (2) The conditional prediction is used to model the random variables, e.g. wind power injection and load demand, and is incorporated in the RLUC, so the operational decisions are globally optimal. (3) The risk chain theorem is mathematically proved for the first time, justifying the validity of using CVaR in the RLUC in GRIP.

The remainder of the paper is organized as follows. Section III presents the architecture of GRIP and justifies the necessity of the RLUC. Section IV shows the operational framework and general model meeting the R1, and discusses the two technical challenges to meet the R2 and R3, which are tackled in Section V and VI respectively. The final model of the RLUC is solved in Section VII. Section VIII applies the proposed models to three cases, and conclusions are drawn in Section IX.

III. GRIP ARCHITECTURE

Bulk electric grids are facing some fundamental shifts from the aspect of system operation, due to the integration of renewables and mature energy storage technologies [4]. A promising vision addressing these shifts is the Energy Internet, in which the green energy can be generated and shared by millions of individual homes and buildings [22].

GRIP was proposed as a paradigm of the Energy Internet in [4] to realize the energy generating and sharing, built upon three pillars. First, peripheries of the core grid, e.g. micro-grids, smart buildings, are empowered so that the uncertainty of the renewables and loads can be mitigated closed to the uncertainty sources. Second, differences between the core grid and its peripheries are disappearing, and this yields a more universal operational paradigm. Third, a layered architecture is preferred in GRIP, so that the legacy grid can be seamlessly transformed to the future periphery empowered grid.

Combining the three pillars, the basic element of GRIP is called a cluster. Each cluster encapsulates its own generation (traditional and renewable units), load, control scheme and communication, and performs three basic functionalities.

(1) Risk-limiting operation. This function is performed in each cluster, aiming to maintain the internal power balance of each cluster, and keep the external power trade schedule by dispatching the generation/load inside each cluster. The operational risk due to the uncertainty of renewable generation is mitigated by multiple dispatch stages.

(2) Frequency regulation: This function smooths the frequency fluctuation by the local feedback control for the periphery clusters.

(3) Failure mitigation: This function mitigates the system failures by sophisticated failure detection and generation/load shedding.

The structure of GRIP is shown in Fig. 1. Cluster 1, 2 and 3 can be interpreted as the transmission, distribution, and micro grid. It is also possible to plug lower clusters in cluster 3, such as smart buildings and smart homes.
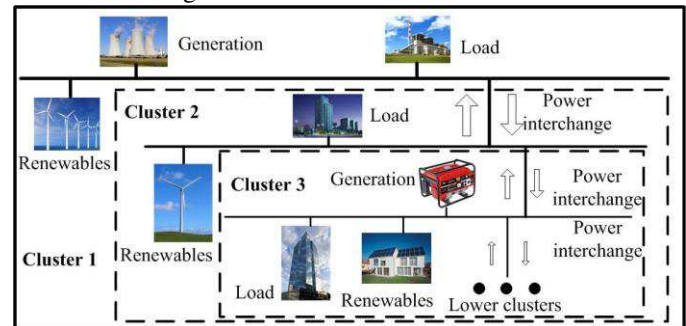

Fig. 1. The structure of GRIP

This paper focuses on the risk-limiting operation in each cluster. The cluster empowered to fulfil the risk-limiting operation function is called a *target cluster*, in which the internal power balance and external power trade schedule should be maintained by the risk-limiting operation. On this basis, there are many subordinate clusters of each target cluster. For example, in Fig. 1, if cluster 1 is the target cluster on which we make an operational plan by the risk-limiting operation, and the generators, loads and the distribution grid (cluster 2) are treated as its subordinate clusters. However, if cluster 2 is the target cluster on which we make an operational plan by the risk-limiting operation, then the generators, loads, cluster 1 and 3 are its subordinate clusters. In addition, the power flow between clusters are bi-directional, which means each cluster can sell and buy power from the other clusters.

This cluster-based architecture of GRIP yields a more universal risk-limiting operational paradigm in GRIP. Many kinds of operational problems, such as demand response, energy storage, etc., can be integrated in the generic risk-limiting operation.

Because the risk-limiting operation is used in each target cluster in GRIP, the comprehensive form of the "risk-limiting" operation needs to incorporate UC and the operational constraints, and to allow power trades between clusters (R1).

IV. RISK-LIMITING UNIT COMMITMENT FOR GRIP

In subsection A, a general operational framework of the RLUC in GRIP is constructed. Subsection B presents the model formulation of the RLUC which is a multi-stage multi-period unit commitment problem considering power trades and the operational constraints in the spirit of "risk-limiting", satisfying

the premise and R1.

A. Operational framework of the RLUC in GRIP

In each target cluster, the operational framework of the RLUC with I DSs and T DPs is shown in Fig. 2.

The operation in the spirit of "risk-limiting" means the operational risk should be mitigated through multiple DSs [5]. At each DS, the input is the prediction of the renewables, and the output is the decision for power buying and selling. At each DP, the power is accumulated from the previous I DSs.
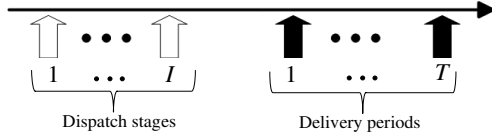

Fig. 2. Operational framework of the RLUC

B. Model formulation of the RLUC in GRIP

Before modeling the RLUC for each target cluster in GRIP, the following assumptions are made for the rest of this paper.

(1) We assume market participants are price-takers, the supply offers of renewables are self-schedules, and the market has sufficient marketability. This assumption has been widely used by textbooks [23], [24], technical report [25], and research papers [26], because it brings a decentralized market solution process, and ensures all buying and selling can be accomplished in the grid infrastructure.

(2) The operational risk in this paper is assumed to be the risk of LOL. This assumption is acceptable because the emergency load shedding is the consequence of other operational risks such as the risk of transmission line overloading. In this regard, LOL is identified as the key concern in the system operation by the standard of North American Electric Reliability Corporation [27]. Guided by this standard, LOL was regarded as the operational risk in many literatures related to the system operation with renewables integration [17].

(3) We assume the line flow equation takes the form of DC power flow, because it is an acceptable approximation of the AC power flow for the UC problem both in the academia [29] and electricity industry [28].

Because the proposed RLUC is multi-stage and multi-period, we assume that DSs run from $i=1,\ldots,I$, and DPs from $t=1,\ldots,T$. The model of RLUC for each target cluster is formulated in (1)-(4).

$$\min \quad \sum_{t=1}^{T}\sum_{i=1}^{I}\mathbf{c}_{it}^{b}\mathbf{p}_{it}^{b} + \sum_{t=1}^{T}\sum_{i=1}^{I}\mathbf{c}_{it}^{s}\mathbf{p}_{it}^{s}$$
$$+ \sum_{t=1}^{T}(\mathbf{c}_{t}^{sp}\mathbf{w}_{t}^{sp} + \mathbf{c}_{t}^{l}\mathbf{l}_{t} + \mathbf{c}_{t}^{u}\mathbf{ON}_{t}) \tag{1}$$

$$s.t. \quad \mathbf{R}_{\min} \le \mathbf{A}_{t}[\mathbf{p}_{it}^{b},\mathbf{p}_{it}^{s},\mathbf{w}_{t}^{sp},\mathbf{l}_{t},\mathbf{ON}_{t},\mathbf{L}_{t},\mathbf{w}_{t}] \le \mathbf{R}_{\max} \tag{2}$$

$$r_{t}(\mathbf{l}_{t}|Y_{i}) \le Risk_{t} \tag{3}$$

$$\left\{ x_{i} = g_{i}(Y_{i}) \middle| x_{i} \in D \right\} \tag{4}$$

where the decision variable is generally denoted by x_i belonging to a set D at DS i. $\mathbf{A}_t$ is a linear mapping matrix for the operational constraints at t, where $\mathbf{R}_{\min}$ and $\mathbf{R}_{\max}$ are the corresponding lower and upper bounds. $r_t(.)$ is the risk index function of LOL. $g_i(.)$ reflects the relationship between the

prediction and the decision. $Risk_t$ represents the risk bound. Reflecting the tradeoff between the expected operational cost and operational risk, the risk bound is a pre-defined parameter in the RLUC. The rule of selecting the risk bound is determined by the risk preference of system operators.

The objective function (1) tries to minimize the total operational cost of each target cluster considering the power trades between clusters. Constraint (2) represents the operational constraints such as the unit capacity, transmission network, and ramping constraint. Constraint (3) means the risk of LOL should be limited. Constraint (4) means the decision variable at DS i is a function of prediction information available at that stage.

This model formulation of the RLUC for each target cluster is generic, so many kinds of distributed energy resources such as demand response and energy storage can be integrated. For example, $\mathbf{p}_{it}^{b}$ and $\mathbf{p}_{it}^{s}$ are the vectors for purchased and sold power at DS i for DP t in a target cluster. The scalar form are $(p_{itn}^{b})_{n \in N}$ and $(p_{itn}^{s})_{n \in N}$ respectively, where n denotes the subordinate cluster number and N is the set of subordinate clusters. For a subordinate cluster representing a traditional generator n_1, we need to confine $p_{itn1}^{b} \ge 0$ and $p_{itn1}^{s}=0$. For a subordinate cluster representing an inelastic load n_2, we need to designate $p_{itn2}^{b}=0$ and $p_{itn2}^{s} \ge 0$. For a subordinate cluster n_3 representing an energy storage or an elastic load, we have $p_{itn3}^{b} \ge 0$ and $p_{itn3}^{s} \ge 0$ in the formulation.

There are three characteristics of the RLUC in GRIP compared with the traditional UC. First, each target cluster is empowered to buy and sell power to maintain the instantaneous power balance. Second, the relation between operational decision and prediction information is explicitly modeled. At last, the operational risk is mitigated by multiple dispatch stages.

While the RLUC in (1)-(4) satisfies the premise and the R1, there are still two technical challenges in satisfying the R2 and R3. For the R2, the challenge is to consider the conditional prediction information in the RLUC. For the R3, the challenge is to choose a risk index which is valid in terms of "risk-limiting" and is computationally tractable for all kind of probability distribution.

V. RLUC IN GRIP BASED ON CONDITIONAL PREDICTION

A. Conditional prediction based RLUC

The first challenge lies in the consideration of the conditional prediction information to model the uncertainty in the RLUC to meet R2.

According to (4), each decision variable x_i depends on the prediction information at that DS i. However, the prediction information at DS i is not known before DS i. For example, we know $Y_1, Y_2, \ldots Y_i$, but we do not know $Y_{i+1}, \ldots Y_I$ at DS i. The missing prediction information can be replaced by the conditional prediction information, assumed to be known in the RLUC. For example, we know $Y_1, Y_2,\ldots Y_i$, and $Y_{i+1}|Y_i$, $Y_{i+2}|Y_i,\ldots, Y_I|Y_i$ at DS i. Therefore, information of both the current and the future prediction can be utilized, if the uncertainty in the RLUC is modeled by the prediction and

conditional prediction simultaneously.

In order to take advantage of the conditional prediction information, the original RLD formulates the operational problem in a recursive form and uses backward recursion to obtain a closed form solution [5].

In the same spirit, (1)-(4) can also be analyzed using backward recursion, and the following relation can be obtained.

$$x_i^* = f_i(Y_i, Y_{i+1}|Y_i, ..., Y_I|Y_i) \qquad (5)$$

This relation further demonstrates the decision variable x_i is a function $f_i(.)$ of the prediction information at DS i and the conditional prediction information at the subsequent DSs.

However, binary variables and linear constraints are incorporated into the RLUC, so it is difficult to obtain the closed form solutions as [5]. A scenario based RLUC in GRIP is proposed to handle this problem.

B. Scenario interpretation of conditional prediction

For clear illustration, the scenario based RLUC in GRIP composed of 3 DSs and 1 DP is analyzed. If the probability distribution of the random renewables power is represented by a set of discrete scenarios, the scenario tree available at DS1 is shown in Fig. 3(a), and Fig. 3(b) provides the scenario tree of the traditional two-stage operation for the sake of comparison.
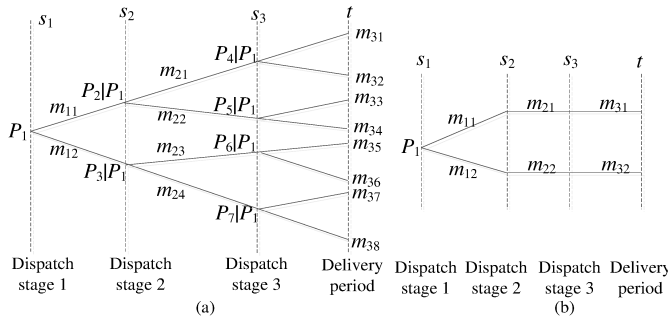

Fig. 3. (a) Scenario tree of the RLUC at DS1. (b) Scenario tree of the traditional two-stage operation at DS1

At DS1 of the RLUC, we know the probability distribution P_1 for the DP, and accordingly we use some sophisticated scenario generation methods to generate samples, for example, two samples $\{m_{11}, m_{12}\}$ in this case. In addition, we have access to two conditional probability distributions $P_2|P_1$ and $P_3|P_1$ of the conditional predictions, conditioned on each sample. With scenario generation tools, four samples $\{m_{21}\ m_{22}\}$ and $\{m_{23}\ m_{24}\}$ at DS2 conditioned on each of m_{11} and m_{12} are generated. Similarly, eight samples $\{m_{31}\ m_{32}\}$, $\{m_{33}\ m_{34}\}$, $\{m_{35}\ m_{36}\}$, and $\{m_{37}\ m_{38}\}$ conditioned on the $\{m_{21}\ m_{22}\}$ and $\{m_{23}\ m_{24}\}$ are generated at DS3 according to the conditional prediction $P_4|P_1$, $P_5|P_1$, $P_6|P_1$ and $P_7|P_1$. In other words, the random variables predicted at DS1 for the DP are $\{m_{31}\ m_{32}\}$, $\{m_{33}\ m_{34}\}$, $\{m_{35}\ m_{36}\}$, and $\{m_{37}\ m_{38}\}$, based on the probability distribution P_1 and conditional probability distribution $P_2|P_1$ to $P_7|P_1$. In addition, the conditional prediction will be updated and the scenario tree will be regenerated in the same way, when the time truly comes to DS2.

In comparison, in the traditional two-stage operation, it is assumed that the prediction at DS1 will remain the same at DS2 and DS3, shown in Fig. 3(b). When the time truly comes to DS2, the scenario tree will be renewed with the updated prediction.

In addition, the scenario tree in Fig. 3(a) has another interpretation which provides some insights on the prediction error. At DS1, the predicted $\{m_{11}, m_{12}\}$ is assumed as the means of the renewables, the $\{m_{21}\ m_{22}\}$, and $\{m_{23}\ m_{24}\}$ is assumed as the second moments and the $\{m_{31}\ m_{32}\}$, $\{m_{33}\ m_{34}\}$, $\{m_{35}\ m_{36}\}$, and $\{m_{37}\ m_{38}\}$ is assumed as the third moments. The optimal decision is made to minimize the expected cost. When the time comes to DS2, either m_{11} or m_{12} is realized as if the mean is certified, which means the prediction updated at DS2 is more accurate. The prediction updated at DS2 is interpreted as the modification on the second and third moment. When the time comes to DS3, the interpretation is similar. Therefore, the conditional prediction can be interpreted as the prediction error.

In a nutshell, the RLUC uses more prediction information than the traditional prediction. Therefore, a better operational decision can be achieved due to the consideration of prediction and conditional prediction simultaneously.

VI. THE VALID RISK INDEX OF THE RLUC IN GRIP

This section deals with the second challenge to meet the R3. The risk index of the RLUC in GRIP is introduced, and the risk chain theorem is mathematically proved for the first time to justify the validity of using this risk index in terms of risk-limiting in the RLUC.

In the risk-limiting operation, system operators should confine the operational risks conditioned on both the current stage and the latter stages at each dispatch stage (DS), provided the prediction for the current stage and the conditional predictions for the latter stages. A valid risk index in terms of risk-limiting means the risk conditioned on the last DS is sufficient for the operational decision [5]. For example, the LOLP is a valid risk index because only the LOLP conditioned on the last DS is sufficient, proved by *Lemma 1* proposed in [5].

Lemma 1: If the information from DS 1 to I satisfy the relationship:

$$Y_1 \supset ... \supset Y_i \supset ... \supset Y_I \qquad (6)$$

Then:

$$P(x \le a|Y_1) \ge ...P(x \le a|Y_i) \ge ... \ge P(x \le a|Y_I) \qquad (7)$$

where a is a parameter.

Thus, if we need each term in (7) to be larger than a given probability P_0, we only need:

$$P(x \le a|Y_I) \ge P_0 \qquad (8)$$

However, LOLP is not capable to precisely describe the operational risk with low probability but high consequences, which is a major concern for operators. In addition, it is not a computationally friendly index. Among the risk indexes frequently used in the engineering discipline, CVaR is a powerful tool to describe the tail loss, being the only coherent and convex risk index [31], and also convenient for computation for any kind of distribution. In addition, CVaR is recommended as the risk index in the electrical engineering [32], [33], and has been widely used in recent works such as [34]-[36]. In this sense, we want to use CVaR as the risk index for the RLUC, so we need to prove the validity of CVaR in terms of "risk-limiting".

CVaR is defined based on the definition of the loss. In power grids, the loss function represents the system failure cost, such as LOL and line overloading cost, which is a random variable x due to the stochastic nature of renewables injection. If the PDF of x is $p(x)$, CVaR of the loss is defined as:

$$CVaR_\alpha(x) = \frac{1}{1-\alpha} \int_{x \geq F_Y^{-1}(\alpha)} xp(x)dx \qquad (9)$$

where α is a confidential level, and $F_Y^{-1}(.)$ represents the inverse function of the cumulative distribution function.

This paper proposes *Theorem 1*, proved in Appendix A, to prove the validity of CVaR in terms of risk-limiting based on *Lemma 1*.

Theorem 1 (Theorem of risk chain): *If* (6) *and* (7) *hold, and the confidential level* α *in each DS is the same, then*:

$$CVaR_\alpha(x|Y_1) \leq ... \leq CVaR_\alpha(x|Y_i) \leq ... \leq CVaR_\alpha(x|Y_I) \quad (10)$$

According to Section V, we have access to the prediction Y_i and the conditional predictions $Y_{i+1}|Y_i$, $Y_{i+2}|Y_i,\ldots$, $Y_I|Y_i$. Therefore, if we want each term in (10) to be smaller than a given risk level $Risk_0$, we only need to ensure (11) holds:

$$CVaR_\alpha(x|(Y_I|Y_i)) \leq Risk_0 \qquad (11)$$

In other words, if system operators want to confine the CVaR in each DS, *Theorem 1* indicates that it is the CVaR that conditioned on the last DS needed to be limited. Taking scenarios in Fig. 3(a) as an example, only $\{m_{31}\ m_{32}\}$, $\{m_{33}\ m_{34}\}$, $\{m_{35}\ m_{36}\}$, and $\{m_{37}\ m_{38}\}$ are sufficient, but $\{m_{11}, m_{12}\}$, $\{m_{21}\ m_{22}\}$, and $\{m_{23}\ m_{24}\}$ is redundant for the CVaR.

VII. DETAILED RLUC MODEL IN GRIP AND SOLUTION

This section gives a detailed model formulation of the RLUC for each target cluster in GRIP which satisfies the premise and the three requirements. Then, two more theorems are applied to solve the RLUC model in GRIP.

A. The detailed model formulation of RLUC

For a certain current dispatch stage $i=C$, the objective function of the RLUC is to minimize the expected social welfare for all DPs in GRIP, denoted in (12):

$$\min \sum_{t=1}^{T} \mathbf{c}_{Ct}^{b}\mathbf{p}_{Ct}^{b} + \frac{1}{k}\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{i=C+1}^{I} \mathbf{c}_{it}^{b}\mathbf{p}_{itk}^{b}$$
$$+ \sum_{t=1}^{T} \mathbf{c}_{Ct}^{s}\mathbf{p}_{Ct}^{s} + \frac{1}{k}\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{i=C+1}^{I} \mathbf{c}_{it}^{s}\mathbf{p}_{itk}^{s} \qquad (12)$$
$$+ \frac{1}{k}\sum_{t=1}^{T}\sum_{k=1}^{K} (\mathbf{c}_t^{sp}\mathbf{w}_{tk}^{sp} + \mathbf{c}_t^{l}\mathbf{l}_{tk} + \mathbf{c}_t^{u}\mathbf{ON}_{tk})$$

The first line in (12) represents the expected power buying cost accumulated from C to I at all DPs. The second line represents the expected profit from selling power accumulated from C to I at all DPs. The last line represents the cost of unit start-up, renewables spillage, and emergency load shedding.

The constraints are shown as follows:

$$0 \leq \mathbf{p}_{Ct}^{b} \leq \mathbf{P}_{max}, \qquad 0 \leq \mathbf{p}_{itk}^{b} \leq \mathbf{P}_{max} \qquad (13)$$

$$-\mathbf{P}_{max} \leq \mathbf{p}_{Ct}^{s} \leq \mathbf{0}, \quad -\mathbf{P}_{max} \leq \mathbf{p}_{itk}^{s} \leq \mathbf{0} \qquad (14)$$

$$\mathbf{ON}_{tk} \cdot \times \mathbf{P}_{min} \leq \mathbf{p}_{Ct}^{b} + \mathbf{p}_{Ct}^{s} + \sum_{i=C+1}^{I}(\mathbf{p}_{itk}^{b} + \mathbf{p}_{itk}^{s}) \leq \mathbf{ON}_{tk} \cdot \times \mathbf{P}_{max} \quad (15)$$

$$\mathbf{p}_{Ct}^{b} + \mathbf{p}_{Ct}^{s} + \sum_{i=C+1}^{I}(\mathbf{p}_{itk}^{b} + \mathbf{p}_{itk}^{s}) + \mathbf{w}_{tk} - \mathbf{w}_{tk}^{sp} - \mathbf{B}\boldsymbol{\theta}_{tk} - \mathbf{L}_{tk} = -\mathbf{l}_{tk} \quad (16)$$

$$-\mathbf{f}_{max} \leq \mathbf{X}\boldsymbol{\theta}_{tk} \leq \mathbf{f}_{max} \qquad (17)$$

$$\mathbf{1}^{T}(\mathbf{p}_{Ct}^{b} + \mathbf{p}_{Ct}^{s} + \sum_{i=C+1}^{I}(\mathbf{p}_{itk}^{b} + \mathbf{p}_{itk}^{s}) + \mathbf{w}_{tk} + \mathbf{l}_{tk} - \mathbf{w}_{tk}^{sp} - \mathbf{L}_{tk}) = 0 \quad (18)$$

$$\mathbf{P}^{D} \leq [\mathbf{p}_{Ct}^{b} + \mathbf{p}_{Ct}^{s} + \sum_{i=C+1}^{I}(\mathbf{p}_{itk}^{b} + \mathbf{p}_{itk}^{s})]$$
$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad (19)$$
$$-[\mathbf{p}_{Ct-1}^{b} + \mathbf{p}_{Ct-1}^{s} + \sum_{i=C+1}^{I}(\mathbf{p}_{i(t-1)k}^{b} + \mathbf{p}_{i(t-1)k}^{s})] \leq \mathbf{P}^{U}$$

$$CVaR(\mathbf{1}^{T}\mathbf{l}_t|(Y_I|Y_C)) \leq Risk_t \qquad (20)$$

where $\mathbf{B}$ is the B-matrix in DC power flow. $\mathbf{X}$ is the matrix for calculating power flow, and $\mathbf{1}^{T}$ is the unit row vector.

(13) and (14) means the buying and selling power at each DS is limited by the unit maximum capacity. (15) means the power accumulated at t from the previous C to I DSs is the actual unit output at t, limited by the unit lower and upper bounds. (16) is the nodal power balance. (17) is the line thermal limit constraint. (18) is the system power balance. (19) is the ramping constraint. (20) means CVaR of LOL for the overall system should be lower than a given bound $Risk_t$.

However, there are two problems in solving (12)-(20). First, the increasing number of DSs will aggravate the computational burden. Second, it is hard to deal with a set of CVaR constraints. These two problems are addressed as follows.

B. Equivalent transformation of the RLUC in GRIP

First, we assume a prerequisite for the objective function in (12):

Prerequisite 1: *The price of buy and sell satisfy the following inequality*:

$$\mathbf{c}_{It}^{s} < ... < \mathbf{c}_{(C+1)t}^{s} < \mathbf{c}_{Ct}^{s} < \mathbf{c}_{Ct}^{b} < \mathbf{c}_{(C+1)t}^{b} < ... < \mathbf{c}_{It}^{b} \qquad (21)$$

Prerequisite 1 represents the relation between the buy and sell prices. If this relation is violated, the operators will not buy or sell any power until the last stage [5], [11], which is not realistic in the practical operations. This prerequisite was originally proposed and justified in [5], [11].

Based on *Prerequisite 1*, this paper proposes *Theorem 2* to simplify the optimization problem, proven in Appendix B.

Theorem 2: *The buying and selling power from dispatch stage C+2 to I equal to zero for all clusters, all delivery periods and all samples, shown as*:

$$\mathbf{p}_{itk}^{b} = \mathbf{0}, \mathbf{p}_{itk}^{s} = \mathbf{0}, \quad \forall i \geq C+2, \forall t, \forall k, \qquad (22)$$

Theorem 2 means in each DS, we only need to generate a deterministic decision for the current DS and a random decision for the next DS in terms of scenarios, because the decision variables for the rest DSs must be zero. The optimization problem becomes (12)-(20) and (22), so a large number of decision variables are eliminated.

Second, CVaR is usually regarded as an objective function [34]. Faced with a set of CVaR constraints, we apply *Theorem 3* to transform a set of CVaR constraints into piece-wise linear constraints based on the work in [37], [38].

Theorem 3: *Define F(.) as a function of variable β and g(x,y) in (23). g(x,y) is a loss function of decision variable x and*

random variable y with PDF $p(y)$. α is the confidential level, and $[x]^+$ means $max(x,0)$:

$$F(g(x,y),\beta) = \beta + \frac{1}{1-\alpha}\int [g(x,y) - \beta]^+ p(y)dy \quad (23)$$

If $g(x,y)$ is a convex function, then the following two optimization problems have the same efficient frontier:

$$
\begin{array}{ll}
\text{(a) } \min_{x} f(x,y) & \text{(b) } \min_{x,\beta_i} f(x,y) \\
\text{s.t. } CVaR(g_i(x,y)) \le Risk_i & \text{s.t. } F_i(g_i(x,y),\beta_i) \le C_i
\end{array}
\quad (24)
$$

where $f(.)$ is the objective function. $Risk_i$ and C_i are parameters.

Thus, we can define $F(.)$ based on scenarios for the RLUC:

$$F_t(\mathbf{1}^T \mathbf{l}_t, \beta_{lt}) = \beta_{lt} + \frac{1}{k(1-\alpha)}\sum_{k=1}^{K}[-\mathbf{1}^T(\mathbf{p}_{Ct}^b + \mathbf{p}_{Ct}^s)$$

$$+ \sum_{i=C+1}^{I}(\mathbf{p}_{itk}^b + \mathbf{p}_{itk}^s) + \mathbf{w}_{tk} - \mathbf{w}_{tk}^{sp} - \mathbf{B}\theta_{tk} - \mathbf{L}_{tk}) - \beta_{lt}]^+ \quad (25)$$

and (20) can be transformed as:

$$F_t(\mathbf{1}^T \mathbf{l}_t, \beta | (Y_I | Y_C)) \le Risk_t \quad (26)$$

In (26), according to the definition of CVaR, the risk bound $Risk_t$ should be selected to be larger than the maximum loss of load associated with the confidential level α, when operators want to draw the efficient frontier.

Finally, each target cluster should solve the RLUC in GRIP composed of (12)-(19), (22), (25)-(26), which is a piece-wise mixed integer linear programming problem.

VIII. CASE STUDY

In this section, three cases are studied. The first case is based on a 9-bus system, which includes 3 clusters. The second case is based on a realistic provincial system: Gansu power grid in China, which is regarded as one cluster. The third case is based on a realistic regional power grid in China, composed of five provincial power systems and regarded as five transmission level clusters. The models for these cases are coded in CVX 2.1 in MATLAB 2013 and solved by GUROBI 7.0. All experiments are conducted on a PC Dell OPTIPLEX 9010 with Intel Dual Core i5 at 3.30, 3.30 GHz and 128 GB RAM in a 64-bit Windows 7 operating system.

A. Case on a nine-bus system

The system topology of the nine-bus system is shown in Fig. 4. Cluster 1 represents a cluster for a transmission system, cluster 2 represents a cluster for a distribution system, and cluster 3 represents a cluster for a micro-grid. The wind power is injected in bus 1, 4 and 7 in different clusters. The detail data for the system and wind prediction are given in [40]. The confidential level α is 95%, and the risk bound is 500$/hr. According to the prediction data in [40], 1000 scenarios are generated by the proposed approach in Section V, and are reduced to 100 scenarios with sophisticated scenario reduction package in GAMS [30]. We need to mention that professional tools and expertise on scenario generation and reduction are assumed to be available, so details on the scenario generation and reduction are out of scope of this work.
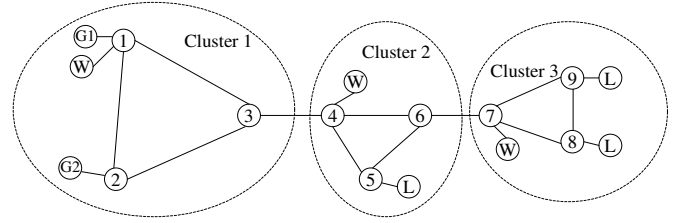

Fig. 4. Topology of the nine-bus system

The objective of this case is fourfold: (1) Show the operational schedule of cluster 1 given by the RLUC. (2) Compare the system operational costs among the traditional operation, the 2-stage RLUC, and the 3-stage RLUC. (3) Demonstrate the efficient frontier reflecting the tradeoff between the risk bounds on the operational cost. (4) Illustrate the power interchange between clusters using the RLUC in GRIP. The first to the third objectives are demonstrated on cluster 1 (the transmission level grid) on three DPs, so cluster 1 is the target cluster. The last objective studies the whole system on one DP, so the target cluster is cluster 1, 2 or 3, depending on which one is the operational object of RLUC.

Objective 1: In Table II, the operational schedule of the target cluster 1 at DS1 is directly calculated by (12)-(19), (25)-(26) according to the data in [40]. At DS2, the operators run the RLUC again and obtain the operational schedule at DS2 based on the updated prediction. The same calculation is done at DS3.

Because the mean value of the wind power decreases [40], unit 1 is scheduled to offset the power shortage at DS2 and DS3 due to its cheaper price at DS2 and DS3. The total operational cost of is 686$/hr. The CPU time is 2.31s.

TABLE II
OPERATIONAL SCHEDULE OF EACH UNIT IN CLUSTER 1

	Unit 1			Unit 2		
	DP1	DP2	DP 3	DP 1	DP 2	DP 3
DS 1	15MW	15MW	15MW	21MW	26MW	23MW
DS 2	5MW	5MW	7MW	0MW	0MW	0MW
DS 3	6MW	4MW	4MW	0MW	0MW	0MW
Status	ON	ON	ON	ON	ON	ON
Total	26MW	24MW	26MW	21MW	26MW	23MW

Objective 2: The prediction error (%) represents the difference between the true value and the measured value over the true value. Fig. 5(a) shows the comparison on the PDFs of the operational cost among the traditional operation, the 2-stage and the 3-stage RLUC when prediction error equals to 20%. These PDFs are simulated by Monte Carlo simulation on 1000 times. In addition, Fig. 5(b) shows the average operational cost in different prediction errors.

From these figures, it can be concluded that the mean and variance of the operational costs among the traditional operation, the 2-stage RLUC, and the 3-stage RLUC decrease gradually. Thus the 3-stage RLUC has the lowest operational cost and is more robust to the prediction error. In addition, larger prediction errors leads to bigger differences among the average operational cost.
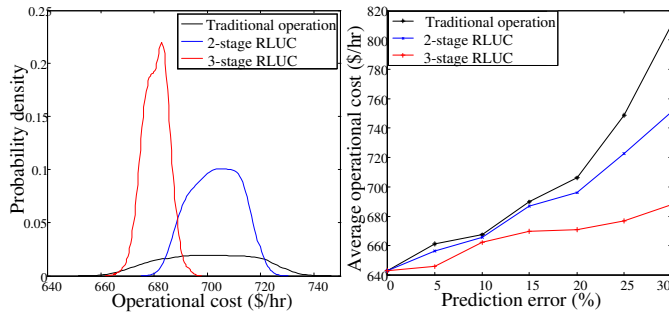
Fig. 5. (a) Comparison on PDFs. (b) Comparison on average operational cost

Objective 3: The result of the third objective is given in Fig. 6. The red, black, and blue curves indicate the confidential level of 95%, 90% and 85% respectively. The efficient frontiers of DS 1 and DS 2 are given in Fig. 6 (a) and (b). There are two observations in Fig. 6. First, a risk-averse system operator will procure a high operational cost, preferring a low risk bound. By contrast, the risk neutral operators will achieve a lower operational cost by accepting a higher risk bound. Second, the operational cost is lower at DS 2 than at DS 1 for the same risk level, because the uncertainty in the second stage is realized partially, and the accuracy of the conditional prediction also increased in the second stage. In other words, there is less operational risk in the second stage due to the risk-limiting operation.
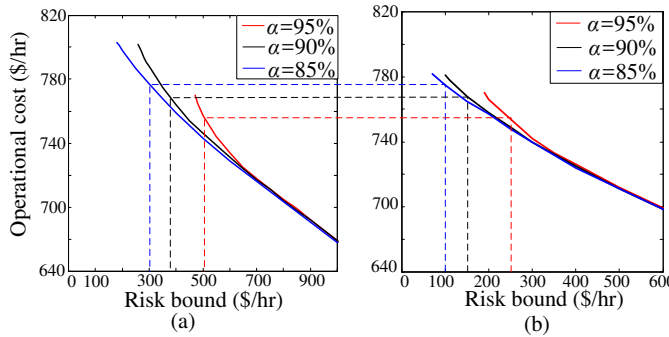

Fig. 6. (a) Efficient frontier at DS 1 (b) Efficient frontier at DS 2

Therefore, guided by the efficient frontier, system operators in each target cluster with various risk preferences can make different operational plans by selecting risk bound values.

Objective 4: The detailed power interchange schedule between clusters is given in Table III, and the comparison of the operational cost and wind spillage level with and without bi-directional power interchange is compared in Table IV

In Table III, the operation in each target cluster determined by the RLUC is illustrated in the objective 1, and then the power interchange schedule is formulated to maintain the power balance.

Specifically, Table III shows the bi-directional power interchange between clusters in GRIP using the RLUC. At DS1, cluster 3 decides to buy 13MW power from cluster 2 after knowing its own wind injection at bus 7. Cluster 2 buys 79MW from cluster 1, after the wind prediction at bus 4 and the load at bus 7 become available. Similarly, cluster 1 buys 63MW. At DS2, cluster 3 predicts that the wind will be higher than the prediction at DS1, so it decides to sell 3MW to cluster 2. In cluster 2 and cluster 1, they buy additional 7MW and 1 MW because they predict the wind will be lower than the value

predicted at DS1. Following the same logic, cluster 3 and 2 sell 3MW and 2MW, and cluster 1 buys 1 MW at DS3.

TABLE III
POWER INTERCHANGE SCHEDULE IN GRIP

	DS 1		DS 2		DS 3	
	Buy	Sell	Buy	Sell	Buy	Sell
Cluster 1	30MW	0MW	2MW	0MW	1MW	0MW
Cluster 2	87MW	0MW	3MW	0MW	0MW	4MW
Cluster 3	13MW	0MW	0MW	3MW	0MW	2MW

Table IV illustrates advantages of permitting the power interchange in GRIP. First, there is no wind spillage because the extra power at the real-time, accumulated from previous DSs due to the inaccurate prediction, can be traded to the upper clusters through market, if the transmission network and unit ramping are adequate. Second, the operational cost is lower because the free wind power can be fully assimilated by means of this power interchange between clusters.

TABLE IV
COMPARISON ON THE OPERATION WITH AND WITHOUT INTERCHANGE

	Without interchange		With interchange	
	Operational cost ($/hr)	Wind spillage (%)	Operational cost ($/hr)	Wind spillage (%)
Cluster 1	146	0	146	0
Cluster 2	350.4	9.7	348.4	0
Cluster 3	45	18.4	42.7	0

In sum, one remark can be drawn from case 1.

Remark 1: For each target cluster, the RLUC can effectively reduce the operational cost compared with the traditional operation. The increased number of DSs brings lower operational cost and extra robustness against prediction error. For the entire GRIP, the RLUC takes advantage of the power interchange and hence reduces the operational cost and wind spillage.

B. Case on Gansu power grid in China

The system can be viewed as a transmission level cluster of which the system topology and detail data are given in [40]. It is an equivalent transmission grid of Gansu provincial system in 330kV and 750kV. There are 132 buses, 177 transmission lines, 25 traditional generators and 6 wind farms, and the delivery periods are composed of 3 DPs. The confidential level α is 90%, and the risk bound is 8000$/hr. In this case, 1000 scenarios are generated and are reduced to 50 scenarios. The computational time of the 3-stage RLUC for this transmission cluster on 3 DPs is 1416.79s. The average costs of the traditional operation, the 2-stage RLUC and the 3-stage RLUC are 28,113$/hr, 27,694$/hr and 26,823$/hr on this system. Similar to the case 1, this result also indicates the cost-saving of the RLUC compared with the traditional operation.

The objective of this case is to analyze the impact factors of the wind power integration in this system using the RLUC. First, the congestion situation for the current system is simulated. Second, the impact of transmission, ramping capacity and wind penetration level is discussed.

Objective 1: Table V shows the result of running the RLUC in the Gansu power grid. The numbers of congested (100% loaded), 90%-100% loaded and 80%-90% loaded lines are counted for this system with and without wind integration.

TABLE V
LINE FLOW AND WIND SPILLAGE IN GANSU POWER GRID

	100% loaded lines	90%-100% loaded lines	80%-90% loaded lines	Wind spillage
System without wind	0 line	Line (36-57), Line (110-121)	Line (4-17), Line (10-37), Line (41-53), Line (57-108)	0%
System with wind	Line (36-57), Line (51-60), Line (51-62)	Line (51-59), Line (57-108), Line (62-72), Line (110-121)	Line (16-17)	28.35%

In Table V, Line (*N-M*) represents the transmission line from bus *N* to bus *M*. If a line is 100% loaded, it is a congested line. If a line is between 80%-100% loaded, it is regarded as potentially congested. In the system without wind power integration, there is no congested line, and there are totally 6 lines beyond 80% loaded. However for the system with wind power integration, there are 8 lines beyond 80% loaded, and three of them are congested. If we trace back to the geographical region for the congested and potentially congested lines in this realistic system, we find these lines concentrate in a narrow corridor spanning over 900km where the wind power is transmitted from the top to the bottom of the corridor. For this system, the long transmission distance and the limit on transmission capacity are the main reasons for the transmission congestion and the high percentage of wind spillage. These test results are in consistency with the realistic situation in China [39].

Objective 2: The interrelation among the transmission capacity, unit ramping capacity and wind penetration is discussed using the RLUC, in order to analyze the impact factors and their sensitivities on this realistic system.

For the Gansu power grid, Fig. 7 (a) shows contours of the operational costs with 20% wind penetration, where the transmission and ramping capacity are parameters. The factor of transmission capacity means the multiplier of the transmission capacity in the RLUC model for the potential congested transmission lines, which are given by Table V. The factor of ramping capacity means the multiplier of the ramping capacity in the RLUC model for the top 10 largest units in the system at bus 4, 6, 7, 8, 10, 14, 18, 19, 46 and 80.

In Fig. 7 (a), the status of the current system is denoted by point A, and the gradient at A is also drawn in the figure. The operational cost decreases faster on the vertical direction than on the horizontal direction, so transmission expansion contributes more than ramping capacity on integrating wind power for the system under study. However for other system status, for example point B, it is suggested to increase the ramping capacity of the units.

Fig. 7 (b) shows contours of the operational costs, where the transmission and wind penetration rate are parameters, when the ramping capacity factor is 1. For the current system status in point A, either transmission expansion or increasing wind penetration level brings benefit to the system. However for point C, increasing the wind penetration level is not a wise choice, because the incremental benefit is comparatively low at such a high penetration level.
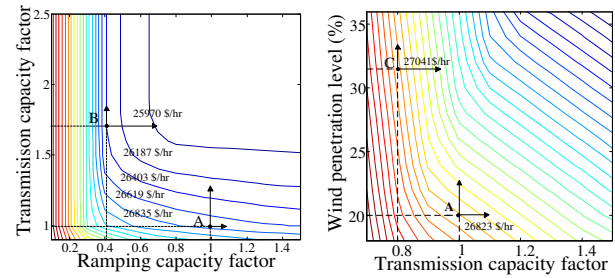

Fig. 7. (a) Relation between ramping and transmission. (b) Relation between transmission and wind penetration level

For the current Gansu power grid, we can draw one remark.

Remark 2: Transmission expansion is the most effective and critical way for the wind power integration for the current system. But for the future system, the wind penetration level and the system ramping capacity may be the critical issues.

C. Case on a regional power grid in China

The objective of this case is to demonstrate the RLUC in a realistic large-scale power system. The GRIP in this case is a regional power grid composed of five provincial power systems, which are regarded as five clusters. There are totally 647 buses, 913 transmission lines, 130 units, and 25 wind farms. The detail data of each cluster is given in [40]. Fig. 8 shows the grid topology and the power interchange schedule between clusters. The RLUC is used in each cluster of this grid on 12 DPs. The confidential level α is 90%, and the risk bound is 10,000$/hr. 1000 scenarios are generated and are reduced to 50 scenarios.
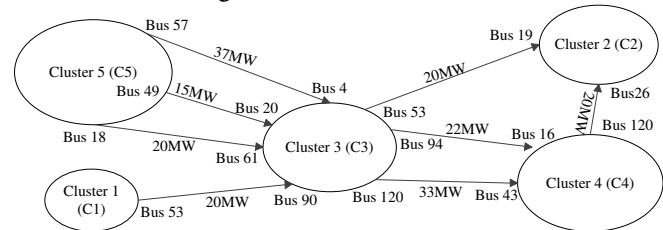

Fig. 8. Topology of the regional power grid

System operators use RLUC for each target cluster to make the operational plan, based on its internal load and generation and external power trade schedules. For example, the unit on/off results for cluster 1 on 12 DPs are given in Table VI, when cluster 1 is regarded as the target cluster, and other clusters as the subordinate clusters.

TABLE VI
UNIT ON/OFF RESULT FOR CLUSTER 1

Bus number of units	Unit on/off result on 12 DPs
Bus 3	000000000000
Bus 7	111011111111
Bus 8	111111111111
Bus 10	111111111111
Bus 16	111111111111
Bus 18	010000000000
Bus 25	000001000000
Bus 38	000000000000
Bus 43	111111111111
Bus 48	000000000000
Bus 61	000000000000
Bus 71	000000000000
Bus 82	000001000000
Bus 85	111111111111
Bus 90	011001110000

After obtaining the operational schedule for each cluster, system operators can determine the operational plan for the

whole grid. The final operational result for the whole grid on 12 DPs is given in Table VII.

TABLE VII
OPERATIONAL RESULT FOR THE WHOLE GRID ON 12 DPS

Cluster	C1	C2	C3	C4	C5
Operational cost ($)	86,243	165,920	122,902	158,270	180,744
Total traditional generation (MW)	8,104	10,745	12,630	13,022	14,997
Total wind power (MW)	237	0	571	573	509
Total load (MW)	8,341	10,745	13,201	13,595	15,560
Wind spillage (%)	25.3	0	29.97	21.42	30.05

The problem scales of the RLUC faced by operators are summarized in Table VIII.

TABLE VIII
PROBLEM SCALE OF THE RLUC FOR EACH CLUSTER ON 12 DPS

Cluster	C1	C2	C3	C4	C5
Number of binary variables	9,000	264	15,000	16,800	24,000
Number of continuous variables	86,442	2,760	139,112	145,858	189,292
Number of total variables	95,442	3,024	154,212	331,650	415,412
Number of constraints	180,312	5,116	283,112	162,648	213,292
CPU time	478s	11s	1,598s	6,389s	9,253s
Gap	0.00%	0.00%	0.01%	0.1%	0.5%

Because cluster 2 has no wind power injection, the RLUC for cluster 2 is deterministic and the CPU time is very short. Being larger than 5000s, the computational times of cluster 4 and 5 are acceptable, because the computational times in related works that have similar problem scales are in the same order of magnitude [18]. Provided more advanced solvers and computers, the scenarios and the problem scale can be further enlarged to render a better operational plan, and the computational time can be further reduced.

IX. CONCLUSION

The proposed RLUC is a comprehensive operational method in GRIP, because it satisfies the basic premise and three requirements. Thus it can be directly utilized in the realistic operation for all levels of clusters in GRIP. The characteristics of the RLUC in GRIP are threefold. First, it considers the power trades and the operational constraints in the UC problem in the spirit of "risk-limiting". Second, it takes advantage of the conditional prediction information so as to obtain the globally optimal solution. At last, it utilizes the CVaR as the valid and computational friendly risk index in the RLUC, based on the mathematical proof on its validity in terms of risk-limiting.

With the proposed RLUC in GRIP, we draw the following conclusions. First, it reduces the operational cost compared with the traditional method. Second, it reduces the wind spillage by the power trades between clusters. At last, it has the potential to serve as a guide for future renewables integration in GRIP, because it models the most crucial factors in the system such as transmission lines, ramping, and wind penetration.

APPENDIX

A. Proof of Theorem 1

Because the confidential level at each DS is the same α, we have:

$$\alpha = F_{Y_i}(x = F_{Y_i}^{-1}(\alpha)) = F_{Y_{i+1}}(x = F_{Y_{i+1}}^{-1}(\alpha)) \tag{27}$$

Because of the probability relation in (7), we have:

$$F_{Y_{i+1}}(x = F_{Y_{i+1}}^{-1}(\alpha)) \le F_{Y_i}(x = F_{Y_{i+1}}^{-1}(\alpha)) \tag{28}$$

Thus:

$$F_{Y_i}^{-1}(\alpha) \le F_{Y_{i+1}}^{-1}(\alpha) \tag{29}$$

By the definition of CVaR, with (29) and (7), we have:

$$CVaR_\alpha(x|Y_i) - CVaR_\alpha(x|Y_{i+1})$$
$$= [xF_{Y_i}(x)]_{F_{Y_i}^{-1}(\alpha)}^{+\infty} - \int_{F_{Y_i}^{-1}(\alpha)}^{+\infty} F_{Y_i}(x)dx - [xF_{Y_{i+1}}(x)]_{F_{Y_{i+1}}^{-1}(\alpha)}^{+\infty} + \int_{F_{Y_{i+1}}^{-1}(\alpha)}^{+\infty} F_{Y_{i+1}}(x)dx \tag{30}$$
$$= \alpha[F_{Y_{i+1}}^{-1}(\alpha) - F_{Y_i}^{-1}(\alpha)] - \int_{F_{Y_i}^{-1}(\alpha)}^{F_{Y_{i+1}}^{-1}(\alpha)} F_{Y_i}(x)dx + \int_{F_{Y_{i+1}}^{-1}(\alpha)}^{+\infty} [F_{Y_{i+1}}(x) - F_{Y_i}(x)]dx$$
$$= \int_{F_{Y_i}^{-1}(\alpha)}^{F_{Y_{i+1}}^{-1}(\alpha)} [\alpha - F_{Y_i}(x)]dx + \int_{F_{Y_{i+1}}^{-1}(\alpha)}^{+\infty} [F_{Y_{i+1}}(x) - F_{Y_i}(x)]dx \le 0$$

Thus:

$$CVaR_\alpha(x|Y_i) \le CVaR_\alpha(x|Y_{i+1}) \tag{31}$$

B. Proof of Theorem 2

Lemma 2: *In the optimal solution of the following optimization problem, there must be y=w=0.*

$$\min \ ax + by + cz + dw$$
$$s.t. \ R_{min} \le A(x + y) + B(z + w) \le R_{max},$$
$$0 \le x + y \le M, \ x \ge 0, \ y \ge 0, \tag{32}$$
$$0 \le z + w \le N, \ z \ge 0, \ w \ge 0$$

where a,b,c and d are parameters satisfying 0<a<b and 0<c<d. R_{min}, R_{max}, A, B, M and N are parameters. x, y, z and w are variables.

Proof of Lemma 2:

Assume $(x, 0, z, 0)$ (x, α, z, β) to be two feasible solution of (32) where $\alpha > 0$ and $\beta > 0$. Substitute $(x, 0, z, 0)$ and (x, α, z, β) into (32) after some manipulation, we have (33) and (34):

$$\min \ ax + cz$$
$$s.t. \ R_{min} \le Ax + Bz \le R_{max},$$
$$0 \le x \le M, \ x \ge 0, \tag{33}$$
$$0 \le z \le N, \ z \ge 0$$

$$\min \ a(x + \alpha) + (z + \beta) + (b - a)\alpha + (d - c)\beta$$
$$s.t. \ R_{min} \le A(x + \alpha) + B(z + \beta) \le R_{max},$$
$$0 \le x + \alpha \le M, \ x + \alpha \ge \alpha, \tag{34}$$
$$0 \le z + \beta \le N, \ z + \beta \ge \beta$$

Because $(b-a)$ and $(d-c)$ in (34) are larger than zero, $(x,0,z,0)$ must have lower value than (x,α,z,β). Thus the lemma is proven.

For DS $C+1$ to I, the decisions must be either buy or sell, and cannot be purchase in one stage and sell it in another, otherwise the *Prerequisite 1* is violated. Thus it is easy to reformulate (12) -(20) to the same form in (32), and *Theorem 2* can be proved.

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REFERENCES

[1] A. Q. Huang, M. L. Crow, G. T. Heydt, J. P. Zheng, and S. J. Dale, "The future renewable electric energy delivery and management (FREEDM) system: the Energy Internet," *Proc. IEEE,* vol. 99, no. 1, pp. 133-148, Jan. 2011.

[2] F. F. Wu, K. Moslehi, and A. Bose, "Power system control centers: past, present, and future," *Proc. IEEE,* vol. 93, no. 11, pp. 1890-1908, Nov. 2005.

[3] C. H. Hauser, D. E. Bakken, and A. Bose, "A failure to communicate: next generation communication requirements, technologies, and architecture for the electric power grid," *IEEE Power Energy Mag.,* vol. 3, no. 2, pp. 47-55, Mar. 2005.

[4] F. F. Wu, P. P. Varaiya, and R. S. Y. Hui, "Smart grids with intelligent periphery: an architecture for the Energy Internet," *Engineering,* vol. 1, no. 4, pp. 436-446, Dec. 2015.

[5] P. P. Varaiya, F. F. Wu, and J. W. Bialek, "Smart operation of smart grid: risk-limiting dispatch," *Proc. IEEE,* vol. 99, no. 1, pp. 40-57, Jan. 2011.

[6] Allen J. Wood, B. F. Wollenberg, and G. B. Sheble, *Power Generation, Operation and Control*: Wiley, 2013.

[7] E. Lannoye, D. Flynn, and M. O'Malley, "Transmission, variable generation, and power system flexibility," *IEEE Trans. Power Syst.,* vol. 30, no. 1, pp. 57-66, Jan. 2015.

[8] C. Peng, S. Lei, Y. Hou and F. Wu, "Uncertainty management in power system operation," *CSEE J. Power Energy Syst.,* vol. 1, no. 1, pp. 28-35, May 2015.

[9] V. Borkar and R. Jain, "Risk-constrained markov decision processes," *IEEE Trans. Autom. Control,* vol. 59, no. 9, pp. 2574-2579, Sept. 2014.

[10] S. S. Soman, H. Zareipour, O. Malik, and P. Mandal, "A review of wind power and wind speed forecasting methods with different time horizons," in *North American Power Symposium (NAPS), 2010*, Sept. 2010, pp. 1-8.

[11] R. Rajagopal, E. Bitar, P. Varaiya, and F. Wu, "Risk-limiting dispatch for integrating renewable power," *Int. J. Elec. Power,* vol. 44, no. 1, pp. 615-628, Jan. 2013.

[12] J. Qin, B. Zhang, and R. Rajagopal, "Risk limiting dispatch with ramping constraints," in *Proc. 2013 IEEE International Conference Smart Grid Communications (SmartGridComm)*, Oct. 2013.

[13] J. Qin, I. S. Han, and R. Rajagopal, "Storage in risk limiting dispatch: control and approximation," in *Proc. 2013 American Control Conference (ACC)*, Jun. 2013.

[14] B. Zhang, R. Rajagopal, and D. Tse, "Network risk limiting dispatch: optimal control and price of uncertainty," *IEEE Trans. Autom. Control,* vol. 59, no .9, pp. 2442-2456, Sep. 2014.

[15] C. Wu, G. Hug, and S. Kar, "Risk-limiting economic dispatch for electricity markets with flexible ramping products," *IEEE Trans. Power Syst.,* vol. 31, no. 3, pp. 1990-2003, May 2016.

[16] G. Huang, Y. Wen, Y. Bao, C. Guo, S. Ma, and Q. He, "Comprehensive decoupled risk-limiting dispatch," in *Proc. 2015 IEEE Power & Energy Society General Meeting*, Jul. 2015.

[17] Z. Wu, P. Zeng, X. Zhang and Q. Zhou, "A solution to the chance-constrained two-stage stochastic program for unit commitment with wind energy integration," *IEEE Trans. Power Syst.*, vol. 31, no. 6, pp. 4185-4196, Jan. 2016.

[18] N. Zhang et al., "A convex model of risk-based unit commitment for day-ahead market clearing considering wind power uncertainty," *IEEE Trans. Power Syst.*, vol. 30, no. 3, pp. 1582-1592, May 2015.

[19] C. Zhao, Q. Wang, J. Wang, and Y. Guan, "Expected value and chance constrained stochastic unit commitment ensuring wind power utilization," *IEEE Trans. Power Syst.*, vol. 29, no. 6, pp. 2696–2705, Nov. 2014.

[20] M. Asensio and J. Contreras, "Stochastic unit commitment in isolated systems with renewable penetration under CVaR assessment", *IEEE Trans. Power Syst.s*, vol. 7, no. 3, pp. 1356-1367, May 2016.

[21] Y. Huang, Q. Zheng, and J. Wang, "Two-stage unit commitment model including non-generation resources with conditional value-at-risk constraints," *Elect. Power Syst. Res.*, vol. 116, pp. 427–438, Nov. 2014.

[22] J. Rifkin, *The Third Revolution*, Palgrave Macmillan: New York, 2011.

[23] M. Shahidehpour, H. Yamin, Z. Li, *Market Operations in Electric Power Systems: Forecasting, scheduling, and Risk Management*, Wiley, 2002

[24] D. S. Kirschen, G. Strbac, *Fundamentals of Power System Economics*, Wiley, 2004

[25] National Renewable Energy Laboratory, Argonne National Laboratory, "Evolution of Wholesale Electricity Market Design with Increasing Levels of Renewable Generation", 2014. [Online]. Available: http://www.nrel.gov/docs/fy14osti/61765.pdf

[26] M. Rahimiyan, L. Baringo, "Strategic bidding for a virtual power plant in the day-ahead and real-time markets: a price-taker robust optimization approach", *IEEE Trans. Power Syst.*, vol. 31, no. 4, pp. 2676-2687, Jul. 2016

[27] North American Electric Reliability Corporation, "Reliability Standards for the Bulk Electric Systems of North America," 2016. [Online], Available:http://www.nerc.com/pa/Stand/Reliability%20Standards%20Complete%20Set/RSCompleteSet.pdf

[28] California Independent System Operator, "Technical Status Report on: AC Power Flow under Convergence Bidding-Summary of Factory Test Results,"2010.[Online],Available:https://www.caiso.com/Documents/ConvergenceBiddingTechnicalStatusUpdate.pdf

[29] C. Uckun, A. Botterud, and J. Burge, "An improved stochastic unit commitment formulation to accommodate wind uncertainty," *IEEE Trans. Power Syst.*, vol. 31, no. 4, pp. 2507-2517, Jul. 2016.

[30] The General Algebraic Modeling System (GAMS), 2014. [Online]. Available: http://www.gams.com/docs/document.htm

[31] R. T. Rockafellar and J. O. Royset, "Risk measures in engineering design under uncertainty," in *Proc. 12th International Conference on Applications of Statistics and Probability in Civil Engineering*, 2015.

[32] North American Electric Reliability Corporation, "Integrated Bulk Power System Risk Assessment Concepts," 2010. [Online]. Available: http://www.nerc.com/docs/pc/rmwg/Integrated_Bulk_Power_System_Risk_Assessment_Concepts_Final.pdf.

[33] Northwest Power and Conservation Council, "Review of Supply Adequacy Criteria in the Northwest," 2011. [Online]. Available: https://www.nwcouncil.org/media/10358/vfinal_Review_of_Power_SupplyAdequacy_Criteria_in_the_Northwest_Region.pdf

[34] Y. Zhang and G. B. Giannakis, "Distributed stochastic market clearing with high-penetration wind power," *IEEE Trans. Power Syst.,* vol. 31, no. 2, pp. 895-906, Mar. 2016.

[35] N. Yu, A. Somani and L. Tesfatsion, "Financial risk management in restructured wholesale power markets: Concepts and tools, " in *Proc. 2010 IEEE Power & Energy Society General Meeting*, Minneapolis, MN, Jul. 2010, pp. 1-8.

[36] N. Yu, L. Tesfatsion and C. C. Liu, "Financial bilateral contract negotiation in wholesale electricity market using Nash bargaining theory, " in *IEEE Trans. Power Syst.*, vol. 27, no. 1, pp. 251-267, Feb. 2012.

[37] R. T. Rockafellar and S. Uryasev, "Optimization of conditional value at-risk," *J. Risk,* vol. 2, pp. 21-41, 2000.

[38] C. Peng, Y. Hou, N. Yu, J. Yan, S. Lei and W. Wang, "Multi-period risk-limiting dispatch in power systems with renewables integration," to appear in *IEEE Trans. Ind. Informat.*

[39] R. Piwko, P. Meibom, H. Holttinen, B. Shi, N. Miller, Y. Chi, *et al.*, "Penetrating insights: lessons learned from large-scale wind power integration," *IEEE Power Energy Mag.,* vol. 10, no. 2, pp. 44-52, Feb. 2012.

[40] Wind Power Simulation Data. [Online] Available: http://www.eee.hku.hk/~yhhou/DataV2.pdf

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