Challenges of Model Reduction in Modern Power Grid with Wind Generation

Jagdeep kaur
Department of Electrical Engineering
The Pennsylvania State University
University Park, PA 16802-2705, USA
Email: juk415@psu.edu

Nilanjan Ray Chaudhuri
Department of Electrical Engineering
The Pennsylvania State University
University Park, PA 16802-2705, USA
Email: nuc88@psu.edu

***Abstract*—This paper presents the challenges of model order reduction of modern power grids with inverter-interfaced wind generation. To that end, a widely used model reduction technique called Balanced Truncation (BT) is compared with a relatively new moment matching approach known as the Iterative Rational Krylov Algorithm (IRKA). It is demonstrated that both BT and IRKA can produce an acceptable accuracy during model reduction for power grids with synchronous generators whereas IRKA produces better accuracy when Doubly Fed Induction Generator (DFIG)-based wind generation is considered.**

NOTATIONS

PSS	Power System Stabilizer
$SISO$	Single-Input Single-Output
ROM	Reduced-Order Model
BT	Balanced Truncation
$IRKA$	Iterative Rational Krylov Algorithm
n	Dimension of full order system
r	Dimension of reduced-order system
Im	Image or rank of a matrix

I. Introduction

Power system is a very complex and high dimensional system. Taking the example of the model of the Western Electricity Coordinating Council (WECC) system with about 3,000 generators and 30,000 buses, the number of states becomes about 30,000 when only generators are represented by dynamic equations. The complexity of these models further increases when inverter-interfaced wind farms (WFs) are included. These models are used for designing the controllers of the power grid for applications like damping inter-area oscillations. Modern control design methods such as $\mathcal{H}_2$, $\mathcal{H}_\infty$, and LQG, produces a controller of the order at least equal to that of the system or higher because of inclusion of weights. To resolve this issue, the order of the plant is reduced prior to the controller design.

Model reduction refers to the removal of insignificant states from the system model. This in effect introduces errors that must be minimized. Generally, errors (also called cost function) are expressed in the form of $\mathcal{H}_2$ or $\mathcal{H}_\infty$ norm of the difference between the models of the full order system and the reduced-order system. A lot of research has been done on model reduction [1]–[7]. One of the approaches of model reduction is known as Singular Value Decomposition (SVD), which reserves the dominant modes of the system by focusing on the controllability and observability characteristics. This approach includes balanced truncation, approximate balanced reduction, and singular perturbation methods [1]. The other approach of model reduction is moment matching, which matches r moments of interpolation points in order to preserve the critical modes of the original system. Arnoldi procedure, Lanczos procedure, rational Krylov methods come under this category of model reduction [1]. A few papers [8]–[12] in power system literature address some of the challenges posed by the dimensionality of power grids.

Li *et-al* in [8] proposed the Krylov-Schur method for computing poorly damped oscillatory modes of large power systems. Model order reduction by partitioning the power system into a study area and an external area has been presented in [9]–[11], and [13] where the dimension of the external area system has been reduced. Freitas *et-al* [12] contributed towards the efficient reduction method for the approximation of controllability and observability gramians of large sparse descriptor power system models. They have considered very large power systems, but the effect of inclusion of the renewable sources, for example large inverter-interfaced WFs were not analyzed.

This paper presents the challenges in model order reduction for modern power grids with inverter-interfaced WFs. Two methods are considered in this paper: balanced trunaction (BT) approach, and a relatively new moment matching approach called Iterative Rational Krylov Algorithm (IRKA). To that end, the performance of these approaches has been compared under two different scenarios. In the first scenario conventional power grid with only synchronous generators (SGs) was considered while the other scenario takes into account inverter-interfaced WF.

The paper is organized as follows: after introducing the research problem of model reduction in modern power grid in Section I, the main objectives of model order reduction and two model reduction approaches− BT and IRKA are discussed in Section II. In Section III a test system of conventional and modern power grid is introduced. The result and analysis are presented in Section IV, and finally the Section V highlights the concluding remarks.

II. Approaches of Model Order Reduction

A SISO linear system is considered in the state-space form:

$$G : \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases} \quad or, \quad G(s) = C(sI - A)^{-1}B + D \tag{1}$$

where,
$A \in \mathbb{R}^{n\times n}, B, C \in \mathbb{R}^n, D = 0, x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}, y(t) \in \mathbb{R}$
It is assumed that the system G is stable, i.e., the eigenvalues of A have strictly negative real parts. The model reduction approaches yield a reduced-order system:

$$G_r : \begin{cases} \dot{x}_r(t) = A_r x(t) + B_r u(t) \\ y_r(t) = C_r x(t) + D_r u(t) \end{cases} \quad or, \quad G_r(s) = C_r(sI - A_r)^{-1}B_r + D_r \tag{2}$$

having much smaller dimension $r << n$ with $A_r \in \mathbb{R}^{r\times r}$, $B_r, C_r \in \mathbb{R}^r$, $D_r = 0$. The main objectives of model reduction are:

1) **Accuracy**: The reduced-order model (ROM) should be able to retain the important characteristic of the original system, which lies in preserving the slow and poorly damped modes for damping control applications.
2) **Stability**: If the original model is stable then the ROM should also be stable.
3) **Scalability**: The model reduction approach should be scalable if the dimension and complexity of the system is further increased, which is the case in modern power system.
4) **Applicability of control theory**: The ROM should be able to capture the critical modes of the system, which is required for designing the controller or implementing other control theories.

Next, the BT and the IRKA approaches are discussed, which are used to achieve these objectives.

A. *Balanced Truncation [12]*

A linear system G in state-space form shown in equation (1) is called balanced if the solutions to the two Lyapunov equations:

$$\begin{aligned} AP + PA^T + BB^T = 0 \\ A^TQ + AQ + C^TC = 0 \end{aligned} \tag{3}$$

are equal and diagonal:

$$\begin{aligned} \mathrm{P} \triangleq \int_0^\infty e^{At}BB^Te^{A^Tt}dt = Q \triangleq \int_0^\infty e^{A^Tt}C^TCe^{At}dt \\ = diag(\sigma_1, \sigma_2, ..., \sigma_n) = \sum \end{aligned} \tag{4}$$

where, $\sigma_1 > \sigma_2 > \cdots > \sigma_m \geq 0$. Here, $\sigma_i \triangleq \sqrt{\lambda_i}(PQ)$, $i = 1...m$, are ordered Hankel singular values of $G(s)$. In balanced system each state is just as controllable as it is observable and the measure of a state's joint observability and controllability is given by its associated Hankel singular value. This property is fundamental to the model reduction using which the states having little effect on the system's input-output behavior, mainly states corresponding to lowest Hankel singular values, are removed. If in G, the state vector x can be partitioned into $[x_1 \ x_2]^T$ where, x_2 is the vector of $n-r$ states representing high frequency modes, which can be removed, then with the appropriate partitioning the state-space representation becomes:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ y \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \\ C_1 & C_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \\ 0 \end{bmatrix} u \tag{5}$$

When A is in diagonalized Jordan form:

$$A = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \lambda_n \end{bmatrix}; B = \begin{bmatrix} b_1^T \\ b_2^T \\ \vdots \\ b_m^T \end{bmatrix}; C = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}^T$$

The reduced-order model is given by:

$$\left[\begin{array}{c|c} A_{11} & B_1 \\ \hline C_1 & D \end{array}\right] \tag{6}$$

which is called the balanced truncation of the full model. This computationally intensive method can be defined in the following steps [12]:

1) Compute the dense $n \times n$ controllability and observability gramians P and Q, respectively, by solving equation (3).
2) Factorize P and Q as $P = UU^T$ and $Q = LL^T$, where U and L are the Choleski factors.
3) Obtain the Hankel singular value by decomposing the product of L and U as follows:

$$U^TL = W\Sigma Y^T = \begin{bmatrix} W_1 & W_2 \end{bmatrix} \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \begin{bmatrix} Y_1^T \\ Y_2^T \end{bmatrix} \tag{7}$$

where W_1 and Y_1 are composed of leading r columns of W and Y and matrix $\Sigma = diag(\sigma_1, \sigma_2, ..., \sigma_n)$ is obtained, which consist of Hankel singular values of the system.
4) Construct the reduced-order model of the order $r << n$

$$A_r = T_L^TAT_R, B_r = T_L^TB, C_r = CT_R, D_r = D \tag{8}$$

where,

$$T_L = LY_1\Sigma_1^{-1/2}, T_R = UW_1\Sigma_1^{-1/2} \tag{9}$$

5) The frequency response of the reduced-order system satisfies global error bound:

$$\|G(jw) - G_r(jw)\|_\infty \leq 2\sum_{i=k+1}^{n} \sigma_i \tag{10}$$

The challenge in this approach is the accuracy and scalability when applied to a complex system like modern power grid with inverter-interfaced wind generation, as will be evident in the case study presented in this paper.

B. *Moment Matching [6], [7]*

A linear system G in state-space form mentioned in (1) whose transfer function can be represented as:

$$G(s) = C(sI - A)^{-1}B + D \tag{11}$$

This can be expanded in a Laurent series around a given point $s_0 \in \mathbb{C}$ in the complex plane as:

$$G(s_0 + \sigma) = \eta_0 + \eta_1\sigma + \eta_2\sigma^2 + \eta_3\sigma^3 + ... \quad (12)$$

where, η_k is the k^{th} moment of system G at s_0, defined as:

$$\eta_k = \frac{(-1)^k}{k}\left[\frac{d^k}{ds^k}(C(sI - A)^{-1}B)\right]_{s=s_o} \quad (13)$$

with integer $k \geq 1$. The idea is to seek a reduced-order system G_r, such that the Laurent expansion of the corresponding transfer function at s_0 has the form:

$$G_r(s_0 + \sigma) = \hat{\eta}_0 + \hat{\eta}_1\sigma + \hat{\eta}_2\sigma^2 + \hat{\eta}_3\sigma^3 + ... \quad (14)$$

where r moments are matched: $\eta_j = \hat{\eta}_j$, $j = 1, 2, ..., r$ for appropriate $r << n$.

If s_0 is infinity, numerically efficient solution is given by means of the $rational\ Lanczos/Arnoldi$ procedures. The rational Krylov method is a generalized version of the Arnoldi and Lanczos methods. Given a dynamical system G, a set of interpolation points and an integer r, the rational Krylov algorithm produces a reduced-order system G_r that matches r moments of the interpolation points. One such approach is called the $Iterative\ Rational\ Krylov\ Algorithm\ (IRKA)$ [6], which addresses the optimal $\mathcal{H}_2$ approximation of the stable, SISO large-scale dynamical system G. This system is converged to a stable r^{th} reduced-order system with $r < n$, such that $G_r(s)$ minimizes the $\mathcal{H}_2$ error norm (also called cost function), i.e.,

$$G_r(s) = \arg \min_{\deg(G_r)=r} \|G(s) - G_r(s)\|_{\mathcal{H}_2} \quad (15)$$

This technique uses Krylov projection matrices $V \in \mathbb{R}^{n \times r}$ and $Z \in \mathbb{R}^{n \times r}$ that span certain Krylov subspaces with the property that $Z^TV = I_r$ that leads to the ROM as:

$$A_r = Z^TAV, B_r = Z^TB, C_r = CV \quad (16)$$

$Theorem$ 1 [6]: Let $G_r(s)$ solves the optimal $\mathcal{H}_2$ problem and let $\hat{\lambda}_i$ denotes eigenvalues of A_r (i.e., $\hat{\lambda}_i$ are the Ritz values), then the first-order necessary conditions for $\mathcal{H}_2$ optimality are:

$$\frac{d^k}{ds^k}G(s)\bigg|_{s=-\hat{\lambda}_i} = \frac{d^k}{ds^k}G_r(s)\bigg|_{s=-\hat{\lambda}_i}, k = 0, 1. \quad (17)$$

Theorem 1 states that the reduced-order model interpolates $G(s)$ and its first derivative at the mirrored Ritz values, which is analyzed in Grimme's [7] work on Krylov-based model reduction.

$Theorem$ 2: Given $G(s) = C(sI - A)^{-1}B + D$ and r interpolation points $\{\sigma_i\}_{i=1}^r$, let $V \in \mathbb{R}^{n \times r}$ and $Z \in \mathbb{R}^{n \times r}$ are obtained as follows:

$$\begin{aligned} \text{Im}(V) &= Span\{(\sigma_1 I - A)^{-1}B, ..., (\sigma_r I - A)^{-1}B\} \\ \text{Im}(Z) &= Span\{(\sigma_1 I - A)^{-T}C^T, ..., (\sigma_r I - A)^{-T}C^T\} \end{aligned} \quad (18)$$

with $Z^TV = I_r$. Then the reduced-order model $G_r(s)$ can be obtained as in (8), that interpolates $G(s)$ and its derivative at $\{\sigma_i\}_{i=1}^r$. This approach exploits the connection between Krylov-based reduction and interpolation [6].

Since the interpolation points in (17) depend on the final reduced-order model, which is not known a $priori$, the initial interpolation points are chosen as r mirrored images of eigenvalues of the original system. The reduced-order model is iteratively corrected using Krylov steps, such that in the next iteration the reduced-order model interpolates the full-order model at the mirrored Ritz values $-\lambda_i(Ar)$ from the previous reduced-order model. The process terminates when the Ritz values from the consecutive iterations stagnate.

The steps of the IRKA algorithm are shown below [6]:

1) Make an initial shift selection σ_i = for $i = 1, ..., r$
2) $Z = [(\sigma_1 I - A^T)^{-1}C^T, ..., (\sigma_r I - A^T)^{-1}C^T]$
3) $V = [(\sigma_1 I - A)^{-1}B, ..., (\sigma_r I - A)^{-1}B]$
4) $Z = Z(Z^TV)^T$ To make $(Z^TV = I_r)$
5) while (not converged)
   a) $A_r =\ Z^TAV,$
   b) $\sigma_i \leftarrow\ -\lambda_i(Ar)$ for i = for $i = 1, ..., r$
   c) $Z = [(\sigma_1 I - A)^{-T}C^T, ..., (\sigma_r I - A)^{-T}C^T]$
   d) $V = [(\sigma_1 I - A)^{-1}B, ..., (\sigma_r I - A)^{-1}B]$
   e) $Z = Z(Z^TV)^T$ To make $(Z^TV = I_r)$
6) $A_r = Z^TAV,\ B_r = Z^TB,\ C_r = CV$

There are several methods to find the updated Ritz values or interpolation points to correct the reduced-order model in each iteration by minimizing the cost function. Some of these techniques are line search [3] and trust region [14]. In the above algorithm the simplest way of getting the updated interpolation points is considered. There is no guarantee that in each iteration the reduced-order system will be stable. Therefore at times the $\mathcal{H}_2$ error norm might go to infinity. However, the strength of this approach is its applicability for very large and complicated systems since it doesn't have to solve large Lyapnuov equations. It uses the Krylov projection matrices for finding the reduced-order model.

III. TEST SYSTEM

To study the challenges of model reduction on power grid, a 68-bus, 16-machine, 5-area system, shown in Fig. 1, is considered with two scenarios [15]: conventional power system with only synchronous generators (PS-SG), and modern power system with inverter-interfaced DFIG-based WF (PS-DFIG).

PS-SG model: All SGs were represented by subtransient models and eight of them ($G1$-$G8$) were equipped with IEEE DC1A excitation systems. A static excitation system with a PSS was installed at $G9$, the rest of the SGs were under manual excitation control. In this case we consider the input as the voltage reference to the PSS while the tie-line power between bus $54-53$ was consider as the output. The dimension of this model is 133.

PS-DFIG model: In this model the SG, G9 was replaced by a DFIG-based WF as described in [15] to represent the modern power grid, which includes the complexities of the renewable sources. In this case also the output is the same as in PS-SG model and the input is the modulating signal of the DFIG rotor current I_{dr} [15]. The total number of states in PS-DFIG model is 148.

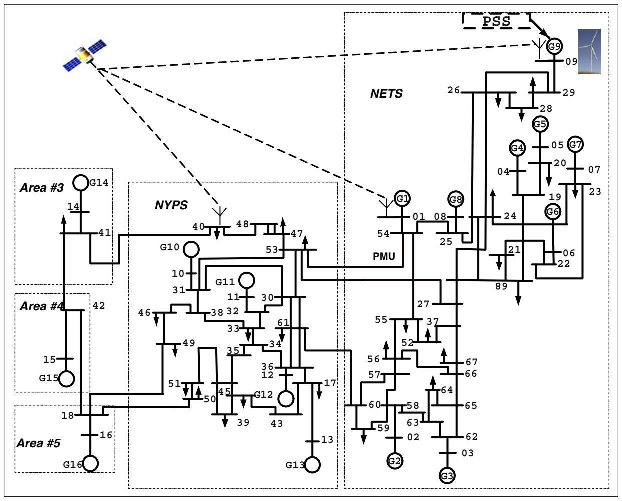

Fig. 1. Single line diagram of the 68-bus, 16-machine, 5-area, New England and New York Inter-connected power system.

TABLE I
COMPARISON OF RELATIVE $\mathcal{H}_2$ ERROR NORM IN POWER SYSTEM WITH TWO MODEL REDUCTION APPROACHES (r = 32)

Type of PS	Model reduction approach	
	BT	IRKA
PS-SG	1.7737×10^{-4}	4.1345×10^{-5}
PS-DFIG	∞	5.9×10^{-3}

The objective is to reduce these models to a suitable order r such that it is able to correctly capture the poorly damped modes of the full order PS-SG and PS-DFIG models, which are listed in Table II and Table III, respectively.

IV. RESULTS AND ANALYSIS

In this section the model reduction techniques mentioned in Section II are applied on the PS-SG and PS-DFIG models.

□ **Conventional power grid with SGs (PS-SG):** Both BT and IRKA were used to reduce the system dimension from 133 to 32. As shown in Table I the IRKA was able to converge to a lower value of relative $\mathcal{H}_2$ error norm, defined as: $||G - G_r||_{\mathcal{H}_2}/||G||_{\mathcal{H}_2}$ compared to BT. Table II compares the inter-area modes captured by the reduced order models obtained from BT and IRKA. The eigenvalue plot of full system and reduced systems are shown in Fig. 2(a) and the inter-area modes are zoomed in Fig. 2(b). Comparison from Table II and Fig. 2(b) shows acceptable accuracy in estimating all inter-area modes. Singular value plots of the full order and the reduced-order systems are compared in Fig. 3, which shows good match between the full order and the reduced order models. Fig. 4 shows the zoomed view of Fig. 3 highlighting the inter-area modes, which shows both the ROMs are able to capture the modes accurately. This analysis shows that IRKA also performs satisfactorily compared to widely used BT approach in the conventional power grids.

TABLE II
COMPARISON OF INTER-AREA MODES CAPTURED BY REDUCED-ORDER MODEL (r = 32) IN CONVENTIONAL POWER GRID WITH SGS

Full order model		Reduced-order model			
		BT		IRKA	
$\xi, \%$	f, Hz	$\xi, \%$	f, Hz	$\xi, \%$	f, Hz
6.50	0.382	6.50	0.382	6.50	0.382
4.40	0.502	4.30	0.502	4.40	0.502
5.70	0.618	5.70	0.618	5.70	0.618
5.00	0.791	5.20	0.791	4.80	0.791

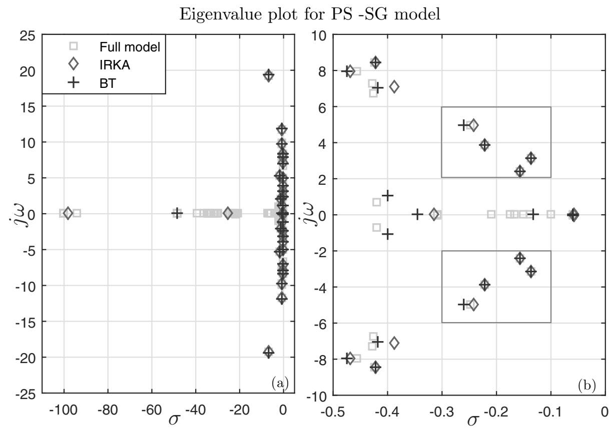

Fig. 2. a): Eigenvalue plot for full system and reduced-order systems (r = 32) in conventional power grid with SGs. b): Zoomed inter-area modes.

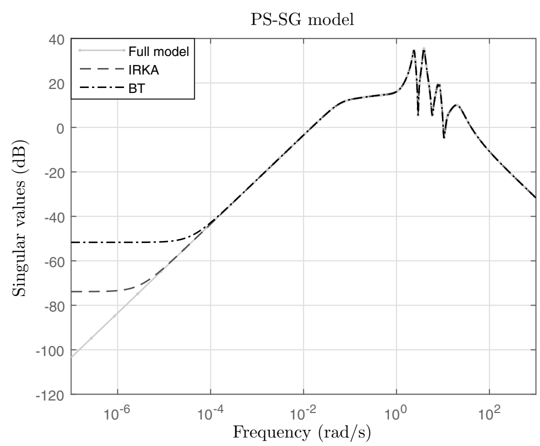

Fig. 3. Singular value plot of full order system and reduced-order systems in the PS-SG case.

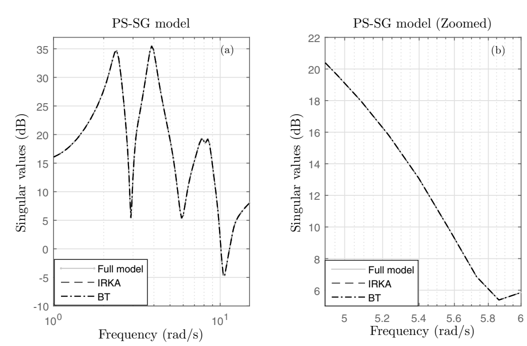

Fig. 4. Zoomed view of Fig. 3 highlighting the inter-area modes in PS-SG case.

TABLE III
COMPARISON OF INTER-AREA MODES CAPTURED BY REDUCED-ORDER MODEL (r = 32)IN MODERN POWER GRID WITH WIND GENERATION

Full order model		Reduced-order model					
		BT		IRKA		IRKA(LI)	
$\xi, \%$	f, Hz	$\xi, \%$	f, Hz	$\xi, \%$	f, Hz	$\xi, \%$	f, Hz
1.40	0.400	1.40	0.400	1.40	0.400	1.40	0.400
4.30	0.502	3.30	0.502	4.30	0.502	4.30	0.502
4.50	0.622	4.50	0.622	4.50	0.622	4.50	0.622
5.00	0.791	-12.30	0.824	4.70	0.790	4.70	0.790

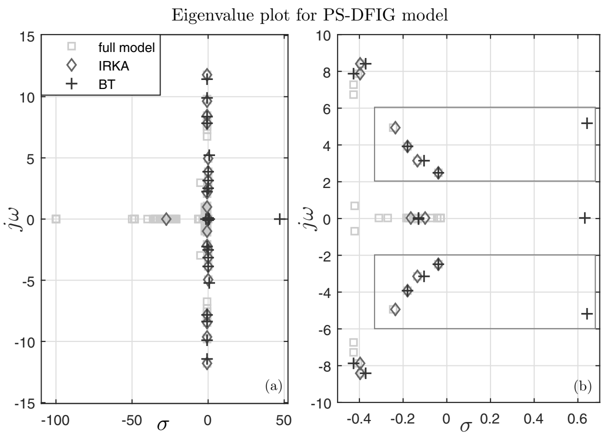

Fig. 5. a): Eigenvalue plot for full system and reduced-order system (r=32) in modern power grid with inverter-interfaced WF. b): Zoomed inter-area modes.

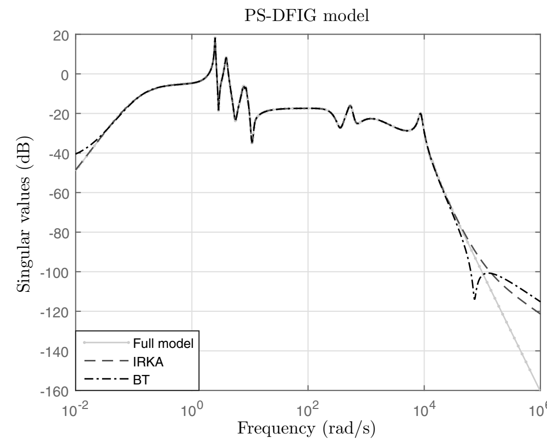

Fig. 6. Singular value plot of full order system and reduced-order systems in the PS-DFIG case.

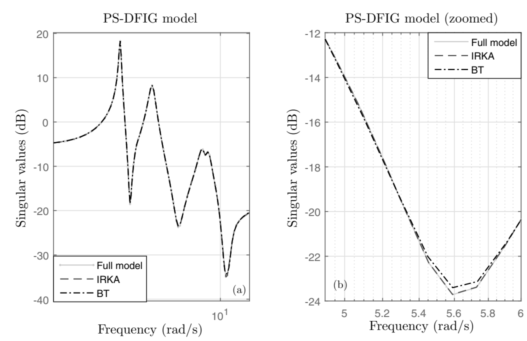

Fig. 7. Zoomed view of Fig. 6 highlighting the inter-area modes in PS-DFIG case.

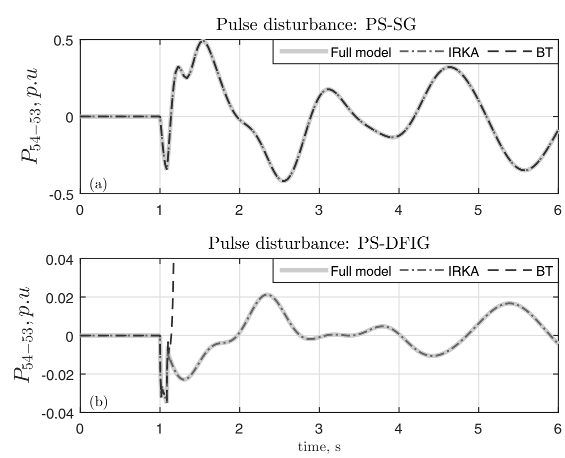

Fig. 8. Dynamic response following a pulse disturbance at the input of the conventional and modern power grid when represented with their ROMs (r = 32) with different model reduction techniques.

□ **Modern power grid with DFIG (PS-DFIG):** In this system the complexity of the power grid is increased by replacing G9 in Fig. 1 with the WF while the rest of the system remained same as in PS-SG case. In this case the model is reduce to the same order r = 32 using both the approaches. As shown in Table I the IRKA was able to converge to a low valued relative $\mathcal{H}_2$ error norm while BT produced an unstable ROM. Table III compares the inter-area modes captured by the ROMs obtained from BT and IRKA where BT captured the 0.791 Hz mode incorrectly with a negative damping. IRKA captures all the modes correctly. The eigenvalue plot of the full system and the reduced systems are shown in Fig. 5(a) and the inter-area modes are zoomed in Fig. 5(b). Comparison from Table III and Fig. 5(b) shows the inaccuracy of BT in capturing the original eigenvalue corresponding to the 0.791 Hz mode, i.e., $-0.2476 \pm 4.968i$ eigenvalue in the original model is mapped $0.6405 \pm 5.179i$ that makes the ROM unstable. Considering the singular value plots of the full order and the reduced-order systems shown in Fig. 6, it appears that both the ROMs have similar response. The zoomed version of Fig. 6 is shown in Fig. 7, which clearly shows the deviation of the BT-based ROM from the full order model. On the other hand, the singular value plot of the IRKA-based ROM overlaps with that of the full order model.

□ **Validation through time-domain analysis:** Figure 8 shows the response of the full order PS-SG and PS-DFIG model and both of its ROMs when pulse disturbance is applied to the input. Figure 8(a) shows the ROMs from both the approaches are able to imitate the behavior of the original model by retaining its critical modes in PS-SG case. When the complexity of the power grid is increased in PS-DFIG case, the BT-based ROM produces unstable response, see

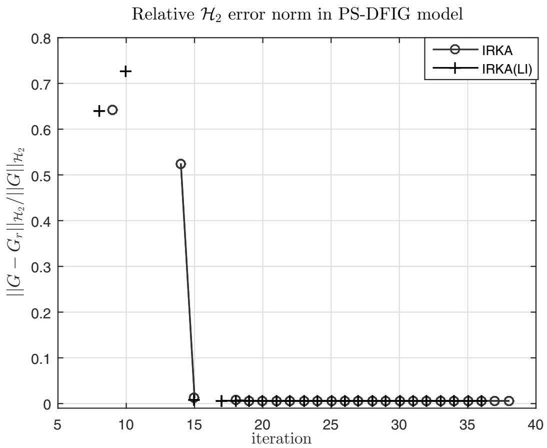

Fig. 9. Relative $\mathcal{H}_2$ error norm in reduced-order system (r = 32) with different initial interpolation points in PS-DFIG case.

Fig. 8(b). On the other hand the IRKA base ROM produces response overlapping that of the full order model. These results are in line with the frequency-domain analysis and confirms the accuracy of IRKA when the complexity of the system increases due to the inclusion of DFIG-based WF.

□ **Discussion:** As discussed in Section II, BT, after balanced realization, retains only $r = 32$ states which have the highest Hankel singular values. To further analyze the performance of the BT approach, participation factor analysis was performed on both the study systems. To ensure the controllability and observability gramians matrices are equal balanced realization was performed prior to participation factor analysis [16]. From the analysis it was observed that in PS-DFIG case the relative participation of the state in 0.791 Hz mode w.r.t. the maximum participation of the same state in the other inter-area modes was very low while in PS-SG case it was significant. The low relative participation of the state in 0.791 Hz could be the reason behind the failure of BT to capture this mode in PS-DFIG case. Our research is focused on developing further insight into this subject matter.

□ **Applicability for the larger systems:** IRKA uses r mirrored images of eigenvalues of the original system as the initial interpolation points. When the dimension of the system is very large it becomes computationally expensive to find all the eigenvalues [8], which can be handled using partial eigenvalue decomposition. To demonstrate the applicability of this approach, a partial eigenvalue decomposition was performed to obtain r eigenvalues from the original system with the largest imaginary part (indicated by 'LI'), which was used to generate the interpolation points. As shown in Fig. 9 IRKA converges to the same ROM (see Table III) with the same relative $\mathcal{H}_2$ error norm when these interpolation points were used.

V. Conclusion

This paper highlights the challenges of model reduction in modern power grids with inverter-interfaced WFs. To that end, the performance of the two model reduction techniques: BT and IRKA has been compared under two different scenarios of power systems – conventional power grid with only synchronous generators and modern power grid with inverter-interfaced WF. It is shown that both BT and IRKA can produces comparable accuracy during model reduction for conventional power grids. For modern power grid BT results in an unstable reduced ROM whereas IRKA produces an accurate reduced model. We are working on the applicability of IRKA on larger grid models with multiple DFIG based WFs.

Acknowledgmentnt

Financial support from NSF under grant award # 1464208 is gratefully acknowledged.

References

[1] S. Gugercin and A. Antoulas, "A comparative study of 7 algorithms for model reduction," in *Proceedings of the 39th IEEE Conference on Decision and Control*, vol. 3, 2000, pp. 2367–2372.

[2] A. Astolfi, "Model reduction by moment matching for linear and nonlinear systems," *IEEE Trans. on Automatic Control,*, vol. 55, no. 10, pp. 2321–2336, 2010.

[3] C. Beattie and S. Gugercin, "Krylov-based minimization for optima H2 model reduction," *46th IEEE conference on Decision and Control,*, 2007.

[4] A. V. D. P. Gallivan, K. Vandendorpe, "Sylvester equations and projection-based model reduction," *Journal of Computational and Applied Mathematics*, vol. 162, no. 1, pp. 213–229, 2004.

[5] N. Martins, L. Lima, and H. Pinto, "Computing dominant poles of power system transfer functions," *IEEE Transactions on Power Systems,*, vol. 11, no. 1, pp. 162–170, 1996.

[6] S. Gugercin, A. Antoulas, and C. A. Beattie, "A rational Krylov iteration for optimal H2 model reduction," in *Proceedings of the 17th International Symposium on Mathematical Theory of Networks and Systems,*, 2006.

[7] E. Grimme, "Krylov projection methods for model reduction," Ph.D. dissertation, Unversity of Illinois, Urbana-Champaign, 1997.

[8] Y. Li, G. Geng, and Q. Jiang, "An efficient parallel Krylov-Schur method for eigen-analysis of large-scale power systems," *IEEE Trans. on Power Systems*, no. 99, pp. 1–11, 2015.

[9] G. Scarciotti, "Model reduction of power systems with preservation of slow and poorly damped modes," in *IEEE PES General Meeting*, 2015, pp. 1–5.

[10] D. Chaniotis and M. Pai, "Model reduction in power systems using Krylov subspace methods," in *IEEE PES General Meeting*, vol. 2, 2005.

[11] C. Wang, H. Yu, P. Li, C. Ding, C. Sun, X. Guo, F. Zhang, Y. Zhou, and Z. Yu, "Krylov subspace based model reduction method for transient simulation of active distribution grid," in *IEEE PES General Meeting*, 2013, pp. 1–5.

[12] F. Freitas, J. Rommes, and N. Martins, "Gramian-based reduction method applied to large sparse power system descriptor models," *IEEE Trans. on Power Systems,*, vol. 23, no. 3, pp. 1258–1270, 2008.

[13] C. Sturk, L. Vanfretti, Y. Chompoobutrgool, and H. Sandberg, "Coherency-independent structured model reduction of power systems," in *IEEE PES General Meeting*, 2015.

[14] C. Beattie and S. Gugercin, "A trust region method for optimal H2 model reduction," *Joint 48th IEEE conference on Decision and Control,*, 2009.

[15] Y. Amirthagunaraj, K. Jagdeep, and N. R. Chaudhuri, "Modeling adequancy for studying power oscillation damping in grids with wind farms and networked control system (NCS)," in *IEEE PES General Meeting, accepted and to be presented*, 2016, pp. 1–5.

[16] P. Kundur, *Power system stability and control*, ser. The EPRI power system engineering series. New York; London: McGraw-Hill.