

Effects of Small Variability of Soil Density on the Consequences of Liquefaction

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ABSTRACT

The effects of small inherent variability of soil density on the consequences of liquefaction such as lateral spreading and excessive settlements may have significant practical implications. Here, the effect of variability on soil density is evaluated for mildly sloping soil specimens that are tested in a series of centrifuge tests at different facilities. While every effort was made to prepare the specimens with uniform densities, the achieved densities at different facilities showed a small variation relative to the target density. This paper presents an attempt to investigate the impact of such variability on the expected response of the soil, particularly when it reaches a liquefaction state. A Monte Carlo simulation was conducted using a non-linear effective stress analysis and an elastoplastic constitutive model for sand. The simulation results show that the small variation of density in the soil specimen leads to a relatively small range of variation for lateral displacements, acceleration, and excess pore pressure ratios. However the range of settlements calculated on the ground surface was rather large, with the coefficient of variation ranging from 8% to 30% depending on the type of variability considered. Potential reasons for such drastic differences among various components of the soil response are discussed.

INTRODUCTION

As an inherently heterogeneous material, soil is by nature spatially variable. This variability and the ensuing uncertainty in the soil response should be considered in the design of geotechnical systems. Since the geotechnical engineering practice is moving towards a performance-based design philosophy, stochastic analysis provides a suitable framework for considering the effects of these uncertainties in design. Detailed consideration of the spatial variability of the soil by utilizing a stochastic analysis framework can be particularly insightful when more complex phenomena such as liquefaction and its effects on civil infrastructure are involved. The main objective of this paper is to explore the effects of spatial variability of soil density on the response of a sloping liquefiable ground using probabilistic finite element analysis.

Inherent variability of natural soils have been observed and documented in many previous research works (Lumb, 1966; Lacasse and Nadim, 1997; Jones et al., 2003; Raychowdhury, 2009; Kasama and Whittle, 2016). Lumb (1966) has studied the variability that existed in four typical natural soils. The variability of various properties of these soils such as Atterberg limits, compression index, coefficient of consolidation, as well as the void ratio were investigated. The probability distributions of these properties were observed to be close to a normal or log-normal distribution. Lacasse and Nadim (1997) reported the probability distribution function as well as

the mean and coefficient of variation for different soil properties. For the initial void ratio a coefficient of variation (COV) ranging from 7 to 30 percent was reported. Extensive research have also been reported on the consequences of soil variability on the response of the soil in various problems such as slope stability (Kasama and Whittle, 2016), and soil structure interaction (Raychowdhury, 2009) among others.

With the scarcity of field data and given the time and efforts necessary to perform sufficient number of laboratory experiments to develop a statistically meaningful dataset, numerical simulations offer an alternative that provides insight while maintaining efficiency. Through the use of finite element method, stochastic analysis can be conducted to study the effect of variability in soil properties on the response of geostructures. There are currently two main approaches that utilize the finite element method in stochastic analysis. The first approach is known as the stochastic finite element method (SFEM). The SFEM is based on combining the finite element method with the truncated Taylor series (Beacher and Ingra, 1981; Vanmarcke and Grigoriu, 1983; Sudret and Kiureghian, 2002; Stefanou, 2009). These methods estimate the mean and variance of functions of random variables. The second approach is known as the random finite element method (RFEM). The RFEM method is based on the utilization of Monte Carlo simulation in modeling the random fields using finite element method (Fenton and Griffiths, 1993; Griffiths and Fenton, 1993). While both approaches allow for the evaluation of the mean and variance of random variable functions, the RFEM has an advantage of producing the time history of the random variables for each simulation event (Griffiths and Fenton, 2009).

The effects of spatial variability on the response of liquefiable soil have been investigated in various studies (e.g. Popescu et al. (1996) and Ural (1996), Popescu et al. (2005), Chakraborty et al. (2008), Popescu (2008), Montgomery and Boulanger (2016)). Spatial variability in penetration resistance of liquefiable soil and its impact on settlement has been analyzed by Popescu et al. (1996, 2005). In these studies, the response of horizontally leveled ground has been modeled using 2D and 3D finite elements. More recently, Montgomery and Boulanger (2016) studied the response of sloping ground composed of a liquefiable layer with a layer of non-liquefiable clay layer on top. While Popescu et al. (1996 and 2005) demonstrated the influence of soil spatial variability on the excess pore pressure generation, Montgomery and Boulanger (2016) focused on the settlement and lateral spreading in mildly sloping liquefiable grounds.

The main objective of this paper is to assess the effects of inevitable variations in the density of the soil specimens prepared for centrifuge testing. While in all the planned centrifuge experiments, a specific density is targeted, previous experience shows that small to modest levels of variation are quite common in the achieved densities (Kutter et al., 2016). Here, two stochastic analyses are performed to investigate the effects of these variabilities. The first analysis investigates the epistemic variability resulting from the difficulty in achieving the target density while preparing soil specimen. The second analysis evaluates the spatial variability that may exist in the specimen. These analyses are designed to study the variability of the soil response in terms of displacements, accelerations, and excess pore water pressures that are observed during a centrifuge experiment. Observations regarding the probability distributions of the displacements at different locations on the soil surface are presented. In the following sections, the experimental setup and the assumptions considered in the stochastic analyses will be discussed first. Afterwards the simulation results are presented along with a discussion of the observed trends.

LEAP Centrifuge Experiment

The centrifuge experiment considered here is a part of project LEAP (Liquefaction Experiment and Analysis Project) which aims at developing a database of high quality centrifuge tests that can be used to assess the validity of current and future constitutive/numerical modeling techniques for the analysis of soil liquefaction and its consequences. The centrifuge experiments conducted in the planning phase of LEAP (LEAP-2015) investigate the seismic response of a mildly sloping ground. The experiments were performed at multiple facilities in the US, England, Japan, China, and Taiwan (Kutter et al., 2016). Figure 1 shows the geometry of the experiment in the prototype scale. The specimen is made of Ottawa F65 sand with a slope of 5-degrees. The height of the soil specimen is 4.0 meters at the center with a length of 20 meters and a width of at least 9 meters (depending on the soil box used at different facilities). The soil was prepared with a target dry density of 1652 kg/m^3 . Three arrays of pore pressure transducers were placed at the center and at the locations that are 3.5 meters away from the sides of the rigid box (Fig. 1). Similarly three arrays of accelerometers were placed at the same locations and at various depths of the soil. The base excitation consists of a ramped sinusoidal motion with a peak acceleration of $0.15g$ and a frequency of 1 Hz . Additional details regarding the experiment setup are given in Kutter, et al. (2016).

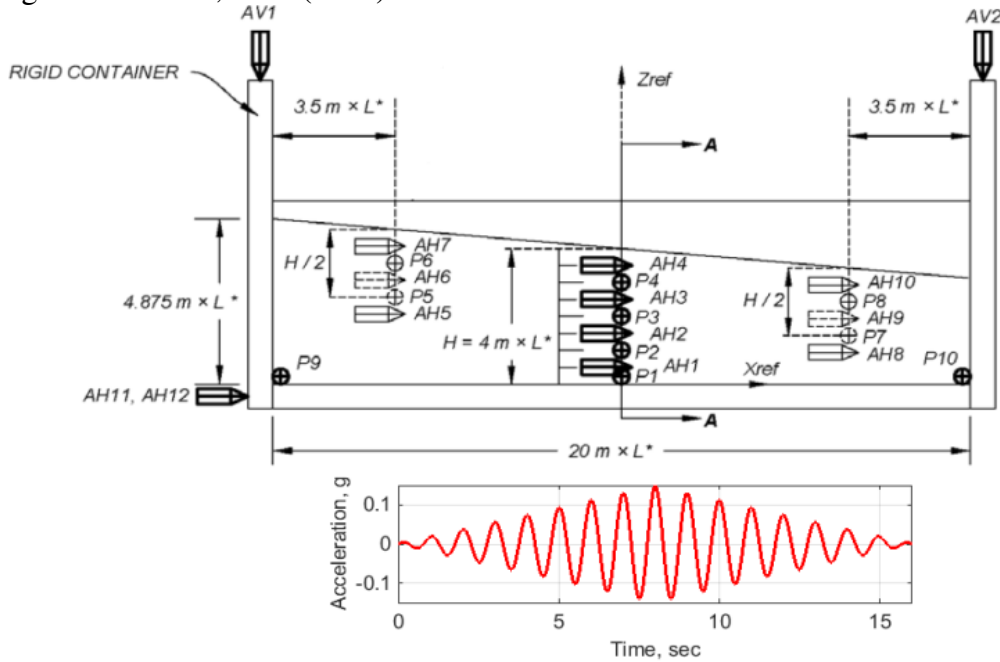


Figure 1: Configuration of the Centrifuge Tests and the Time History of the Base Excitation

Stochastic Analysis Set up

There are two types of uncertainties discussed in this study. The first type is the epistemic uncertainty which arises from the errors in measurements. In the LEAP-2015 centrifuge tests, the specimens were prepared by dry pluviation. The difficulty in preparing a sloped soil profile, the inaccuracy in the measurement of the final geometry of the specimen, and the ability of the experimenter to keep the height of fall constant during the dry pluviation process usually lead to variability in the reported achieved dry density. Hence, the first set of stochastic simulations investigates the effects of the epistemic variability in the soil dry density (or void ratio). The initial void ratio is considered as a random variable that is uniformly assigned to the finite

element model. As shown in Table 1, the mean value of the initial void ratio is taken to be 0.606 which corresponds to the target dry density of 1652 kg/m^3 . Kutter et al. (2016) reported that the range of variability of the achieved dry densities of the centrifuge specimens in LEAP-2015 project was $\pm 54 \text{ kg/m}^3$. Considering this range as the standard deviation of the soil dry density yields a COV of 7.78% for the initial void ratio (assuming the a constant specific gravity of 2.65). This COV falls within the reported range of 7 to 30% by Lacasse and Nadim (1997) for natural soils.

Table 1: Stochastic Analyses Parameters

Analysis	Mu	C.O.V.
Epistemic Variability	0.606	7.78%
Spatial Variability	$0.5588+0.0244y$	7.78%

The second type of variability considered in this study is the spatial variability of initial void ratio. Although the centrifuge specimens are prepared in a closed and controlled environment, preparing a specimen with a void ratio that is constant with the depth is rather difficult. For the second stochastic analysis, the initial void ratio is considered to be a Gaussian random field. The mean value of the initial void ratio is assumed to be dependent on depth with higher void ratio closer to the surface representing looser soil conditions and lower void ratio at the base representing a denser soil. A constant coefficient of variation is considered (the same value as the one used in the first stochastic analysis). The random field is considered to be spatially uncorrelated. For both cases the permeability is considered to be a function of the void ratio (Eq. 1) that is based on the relationship obtained from the permeability tests performed on samples of Ottawa sand F65 prepared with various densities (Vasko 2015, Calderon et al 2017).

$$k(\text{cm/s}) = 0.02 * e^{-0.003} \quad (1)$$

Figure 2 shows the contours of initial void ratio for one of the realization of the spatial variability case. Figure 3 shows the empirical cumulative distribution of the initial void ratio for both the epistemic and spatial variability cases. The achieved distributions are compared to the target distribution presented in the plot with diamond points at different percentiles.

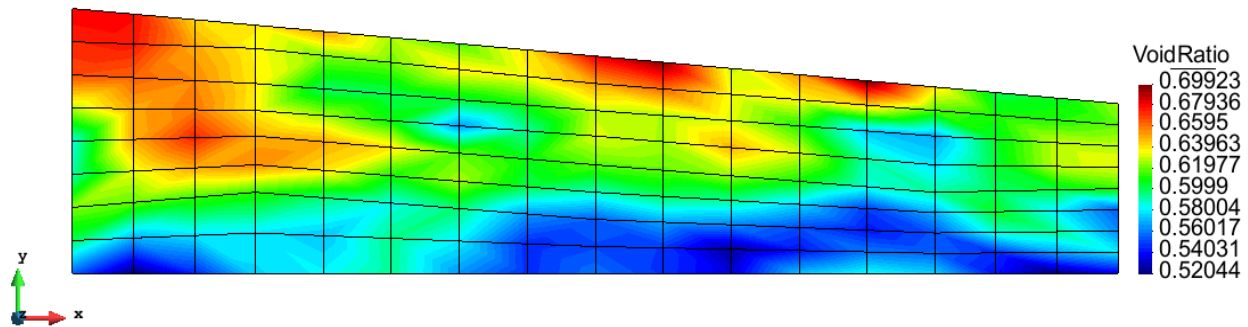


Figure 2: Distribution of Initial Void Ratio of a Single Realization of the Stochastic Analysis

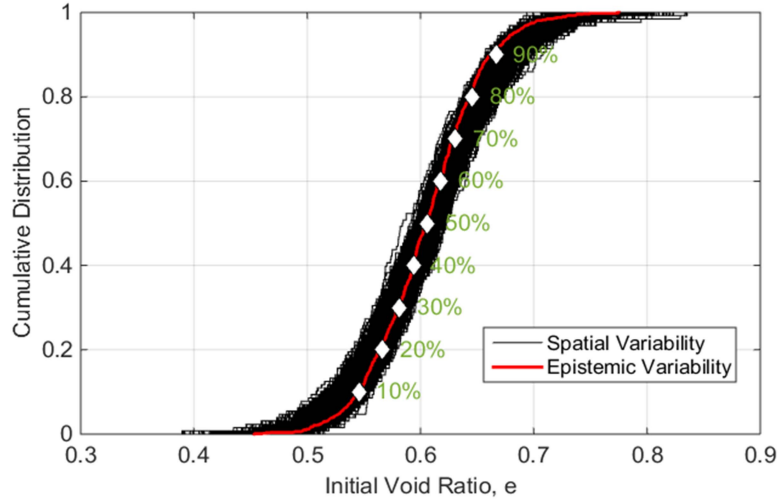


Figure 3: Cumulative Distribution of Initial Void Ratios for both Stochastic Analyses

Finite Element Model and Calibration of the Constitutive Model

The finite element mesh used in the analyses reported here is shown in Figure 2. The soil is modeled in plane strain condition with 128 quadrilateral elements with displacements and pore pressure as independent degrees of freedom at each node. The displacements are fixed at the base of the model and lateral displacements are fixed on the sides. Free drainage is assumed at the soil free surface. Ottawa F65 sand is modeled using the critical state two-surface plasticity model proposed by Dafalias and Manzari (2004). The model is designed to capture the stress-strain-strength behavior of sands under monotonic and cyclic loading conditions. The model is calibrated using strain-controlled cyclic triaxial tests performed by Vasko (2015) on Ottawa F-65 sand. Figure 4 the results of model simulations against the liquefaction strength curve reported by Vasko (2015).

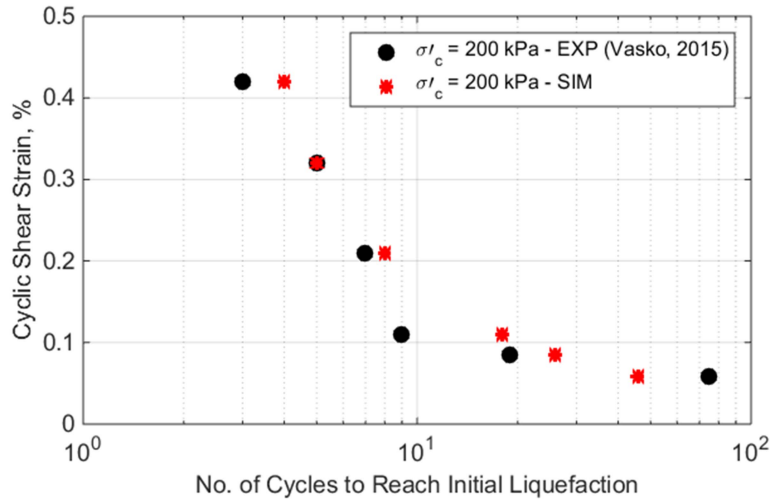


Figure 4: Comparison of the observed and simulated liquefaction strength curves for undrained strain-controlled cyclic triaxial tests on Ottawa F65 sand.

Results of the Stochastic Analyses

Figure 5 shows the results of the excess pore water pressure ratio time histories computed at a point near the bottom of the soil specimen where the pore pressure sensor P1 (Fig. 1) is placed.

The results are illustrated in terms of the mean, the mean plus (or minus) one standard deviation, and the range of the simulations results for both epistemic and spatial variability cases.

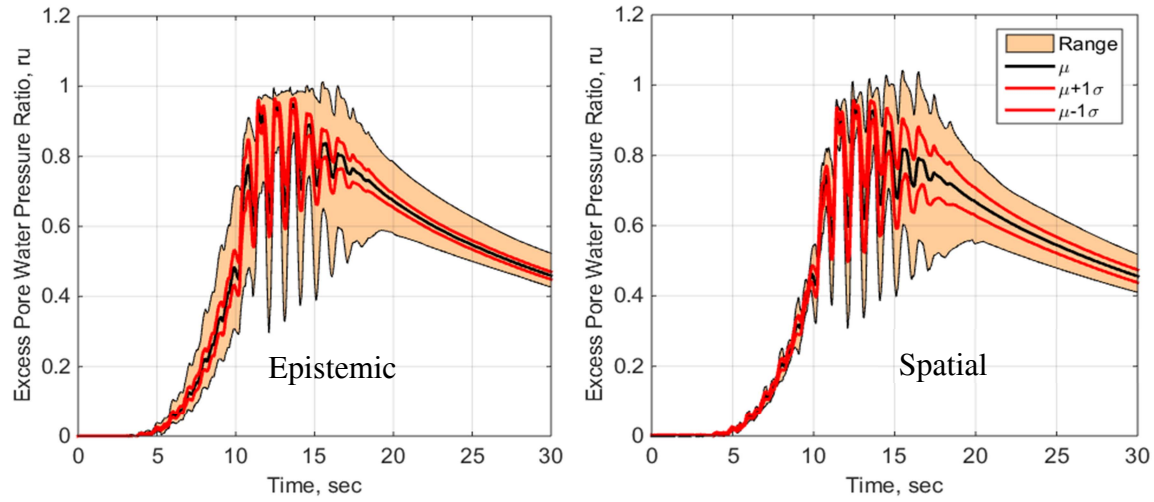


Figure 5: Excess Pore Water Pressure Time Histories computed at the Location of P1

Figures 6 and 7 show the surface settlement and lateral displacement time histories at the top of slope (3.5 meters away from the walls of the soil box) and center of slope respectively.

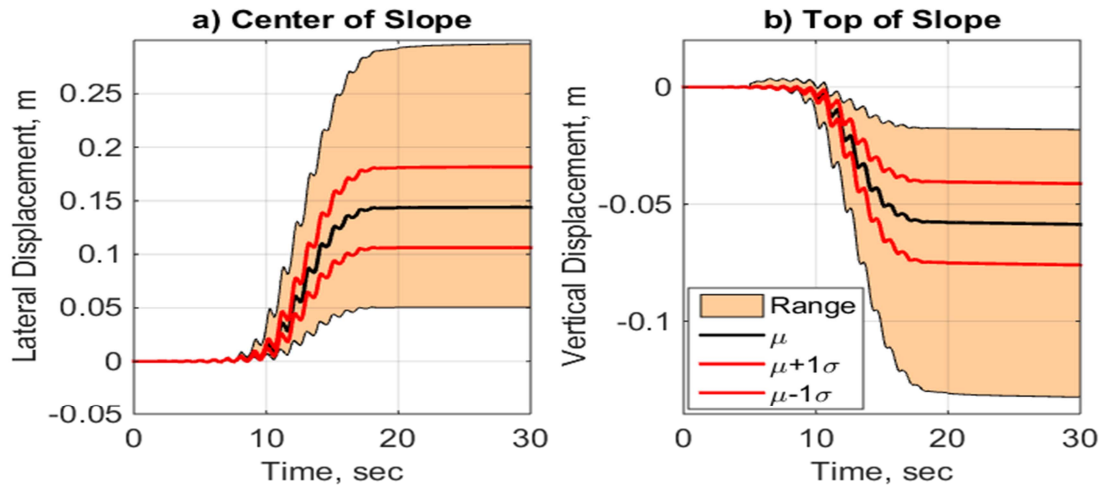


Figure 6: a) Surface Lateral Displacement at the Center of Slope b) Surface Settlement at the Top of Slope for the Epistemic Variability Analysis

Figures 8 and 9 illustrate the final values of vertical displacements obtained at the various locations along the free surface for each of the analyses cases, respectively. It is noted that mainly due to the presence of lateral rigid walls, the surface profile of the soil specimen at the end of the shaking period is uneven. This phenomenon significantly affects the vertical displacement at the center of the soil specimen, which may range from a settlement to a heave, as noted by the ranges of the settlements shown in Figures 8 and 9. For this reason, it is more reasonable to compare the settlement time history at the top of the slope as presented in Figures 6 and 7.

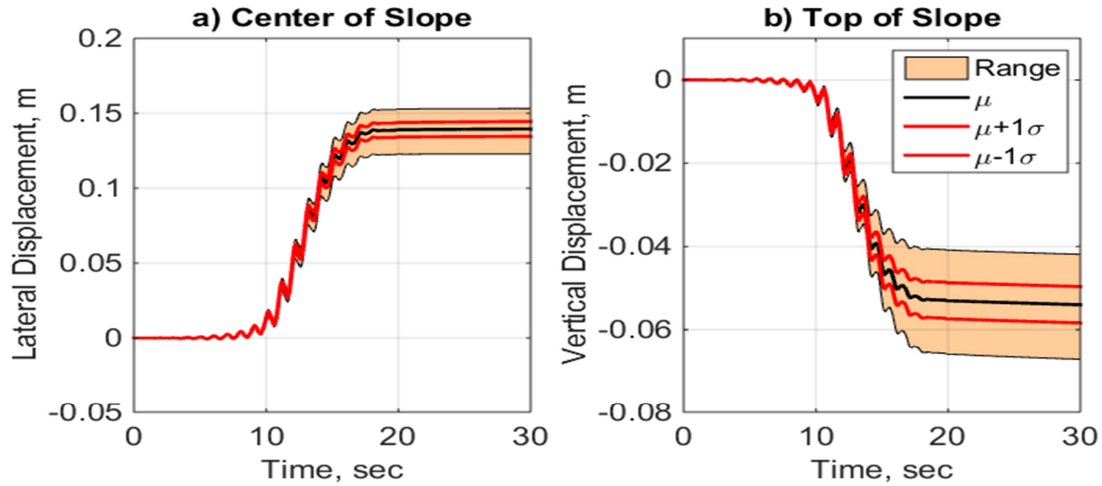


Figure 7: a) Surface Lateral Displacement at the Center of Slope b) Surface Settlement at the Top of Slope for the Spatial Variability Analysis

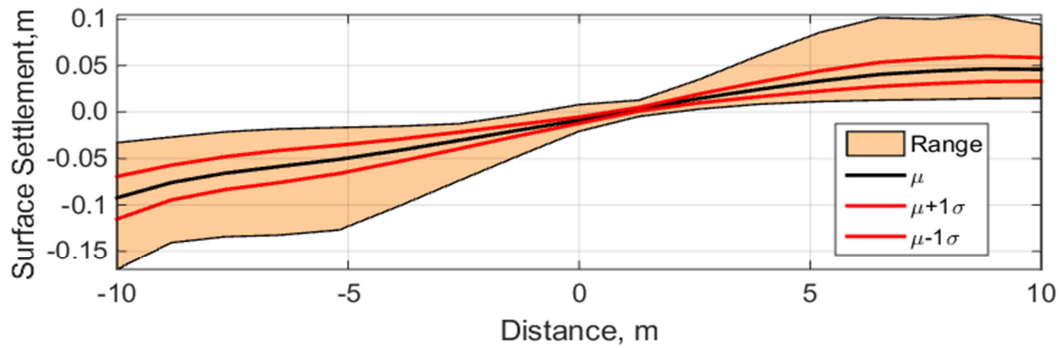


Figure 8: Surface Settlement of the Soil Model for the Epistemic Variability Analysis

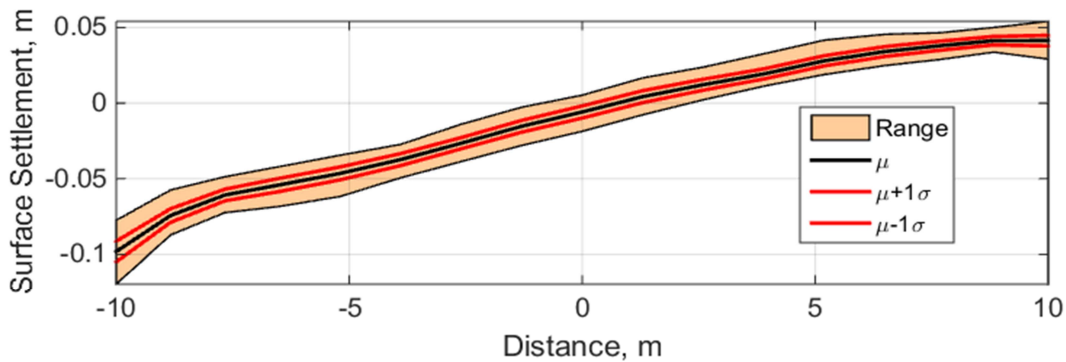


Figure 9: Surface Settlement of the Soil Model for the Spatial Variability Analysis

Figure 10 shows the histograms of the final surface settlements and lateral displacements at the top (Fig. 10-a) and center of the slope (Fig. 10-b) for the case with spatial variability. The plots also show the best fit distribution for the computed displacements using the Maximum Likelihood Estimation technique. The lateral displacement can be modeled using a normal distribution with mean of 0.1415m and standard deviation of 0.005m. Relative to the computed histogram, the normal distribution introduces very small errors in the mean and standard deviation (0.082% and 2.22%, respectively). Similarly, the vertical displacement can be

modeled using a normal distribution with very small error in the values of mean and standard deviation.

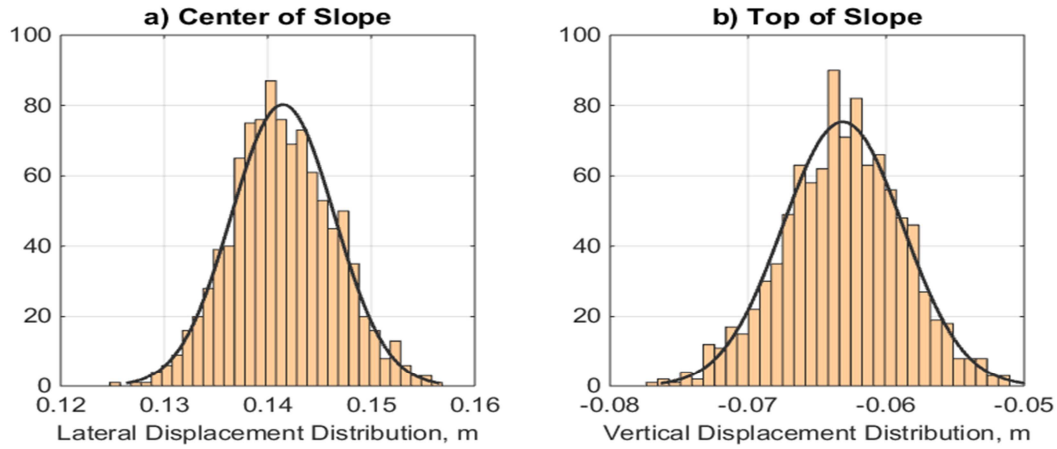


Figure 10: Histograms of a) Surface Lateral Displacement at the Center of Slope b) Surface Settlement at the Top of Slope for the Spatial Variability Analysis

Figure 11 shows the lateral displacements of the points located on the central line of the soil specimen for the spatial variability case. The results of the finite element analyses for soil specimens with uniform initial void ratios corresponding to various percentiles of the target distribution are also shown in Figure 11.

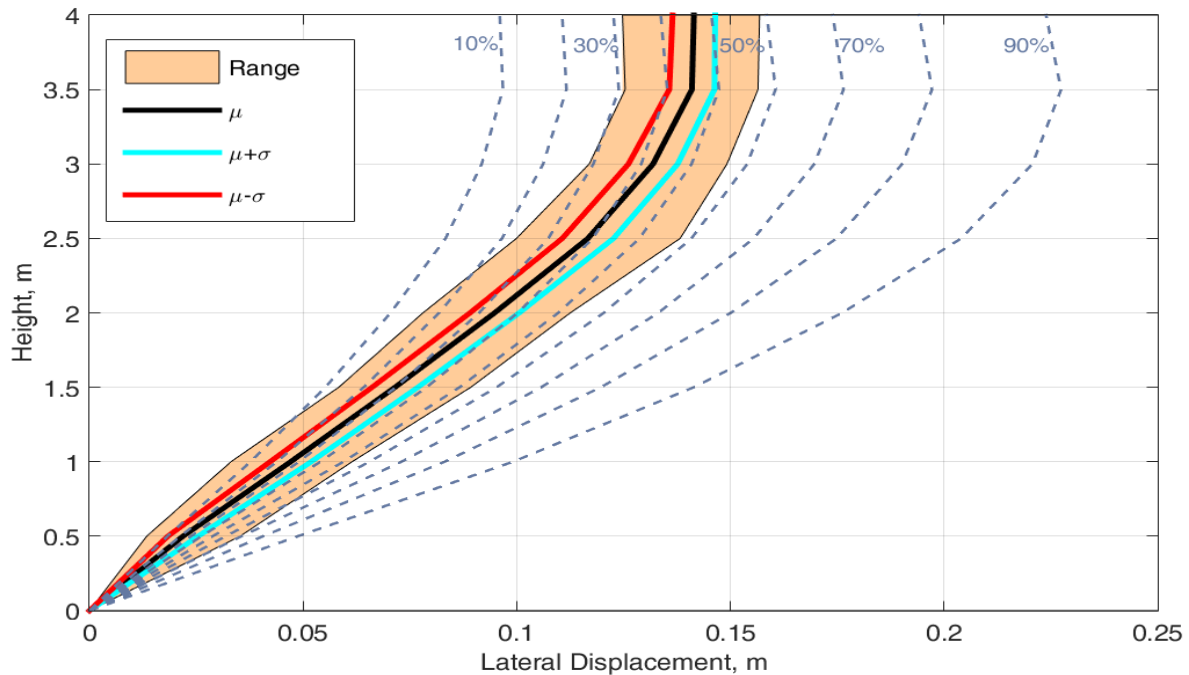


Figure 11: Comparison of Soil Lateral Displacements at the Center of the Model computed for the case with Spatial Variability and the Uniform Models corresponding to different percentile of the initial void ratio for target distribution

Discussion

The results shown in Figure 5 indicate that the mean excess pore water pressure ratios obtained for the epistemic and spatial variability analyses are similar in terms of the maximum

excess pore pressure generated, and the rates of generation and dissipation. However, a smaller range of variability is observed during the excess pore pressure generation phase for the spatial variability case. Moreover, the variability of the maximum values of excess pore pressures and rate of dissipation are higher for the cases with spatial variability than those obtained for the epistemic variability case. The smaller range of variability in the rate of generation corresponds to the smaller range of variability in the average soil stiffness in the spatial variability case.

Figures 6 and 7 show that range of variation obtained for lateral displacements and settlements in the spatial variability case is smaller than those for the epistemic variability. For the epistemic variability case, lateral displacement at the end of the simulation has a mean of 14.4 cm and a COV of 25%. While for the spatial variability case, the mean and COV of the lateral displacement are 13.96 cm and 3.4% respectively. Similarly, the settlement for the epistemic variability case has a mean of 5.86 cm and COV of 29.59%, while the spatial variability case shows a mean of 5.40 cm and COV of 8.10%. Although both cases have shown heaving in the soil surface near the right wall of the soil box, a lower mean settlement and a higher tendency to heave are observed in the spatial variability case. Figures 8 and 9 illustrate the final surface profiles at the end of numerical simulations. The results confirm the observed trend in Figs. 6 and 7.

The histograms, shown in Fig. 10, show the shape of the distribution of the final settlements and lateral displacements. It is observed that both the settlement and lateral displacement show a distribution similar to a normal distribution function. Figure 11 shows that the average response of the soil profile in the spatial variability case corresponds to the response of a soil specimen with a uniform initial void ratio that falls between the 20th and 30th percentile of the target distribution for the locations deeper than 2.5 m. For the locations at depth between 1.3m to 2.5m the soil response corresponds to response of a soil with a uniform initial void between the 30th and 40th percentile of the target distribution. Similarly for the soil with depth less than 1.3m the soil response corresponds to that with a uniform initial void ratio between the 40th and 50th percentile of the target distribution.

Conclusion

The effects of small variability of soil density on the response of liquefiable sloping ground were investigated. By considering the initial void ratio as a random variable with normal distribution, two sets of stochastic analyses were performed. The first case investigates the effects of epistemic variability of initial void ratio caused by the inability to achieve the target density. The second case investigates the effects of the spatial variability in initial void ratio due to the soil inherent heterogeneity and specimen preparations. It is noted that the analyses presented in this paper are related to freshly deposited clean sand deposits and do not encompass other types of uncertainty in soil density such as those caused by soil ageing soil.

The results showed that the average excess pore pressure response was similar for the two cases, while the variations at the rate of generation, maximum excess pore pressure and rate of dissipation are different. The lateral displacements and surface settlements showed a higher degree of variation in the epistemic variability cases than in the spatial variability cases. Finally the lateral displacements of the soil profile for the spatial variability were compared to the responses of different soil specimens that are prepared with uniform initial void ratios corresponding to various percentiles of the target distribution.

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