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Input-Output Characteristics of the Power Transmission Network's Swing Dynamics

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Abstract— **Wide deployment of sensing and actuation capabilities in the electric power grid, along with changing dynamical characteristics, are necessitating analysis of power-system swing dynamics from an input-output perspective. In this article, the input-output properties of the swing dynamics, including the finite and infinite zeros, are characterized from a dynamical-networks perspective. Specifically, an explicit algebraic characterization is given for a matrix whose eigenvalues are the zeros, and in turn structural and graph-theoretic conditions for the absence and presence of nonminimum phase dynamics are developed. Based on these structural results and also an illustrative example, it is demonstrated that the zeros of the swing dynamics are important for analyzing transients and oscillations in the power transmission network, using reduced-order models, and designing controls.**

I. Introduction

New challenges are arising in understanding and controlling transients in the power transmission network [1]. First, a wide array of new devices and technologies are being deployed, which may subject the network to new types of disturbances while also modulating the networks dynamic responses. Second, increasing penetration of intermittent renewables is leading to increasing variability and uncertainty in the networks operating point and inertial characteristics, and hence also in its swing dynamics [2]. Specifically, generation units have increasingly diverse inertias, and the spatial pattern of inertia in the network is both changing from before and becoming more volatile depending on wind and solar conditions. In some cases, the incorporation of renewables is also further stressing the network since these generators are located far from load centers, and myriad other stressors are complicating power-grid operations. The increased stress may lead to more extensive swings [3].

At the same time, new sensors, power-electronics-based actuators, and communication capabilities are being integrated into the electric power grid. In consequence, control of transients in the power transmission network is evolving from a local and specialized paradigm, toward one where many generic sensors and actuators across a wide area are being used in tandem [1], [4]. While this changing paradigm may bring forth many benefits (e.g. in damping oscillations and other transients, addressing fault scenarios, increasing flexibility, etc), it also necessitates new simulations, formal analyses, and control design techniques.

Designing wide-area controls and evaluating propagative transients in the bulk power transmission network requires understanding input-output properties of the network's swing dynamics, as a foundational step. That is, it is necessary to understand the relationship between a putative input at one location in the network (whether an actuation signal or an unknown disturbance) and the swing-dynamics response at another network location (whether a measurement signal used in feedback or a response variable of interest). The main purpose of this study is to explore the input-output properties of the swing dynamics, first from a algebraic standpoint and then from a topological or structural perspective.

Following on the classical analysis of transients, this effort considers linearized models of the power-system swing dynamics [5]. Specifically, a simplified model is considered, which uses two state variables (angle, frequency) for each inertial generator in the network. While traditionally the transient analysis has primarily focused on the internal modal dynamics of the swings, here we impose input and output structures on the canonical model, and characterize the transfer function for the enhanced model. The zeros of the transfer function, in particular, are control invariants that fundamentally limit a systems responses and guide design. Many of the standard controller design techniques and tuning methods used in the power system depend on the transfer function being minimum phase (having left-half-plane zeros); thus, the presence of nonminimum-phase zeros may cause control designs to unexpectedly cause oscillations and instability. Our primary focus here is to determine properties of the zeros, including the presence or absence of nonminimum-phase zeros, from a *structural and graph-theoretic* perspective. In this initial study, we concentrate on single-input single-output (SISO) channels, but approach the analysis in a way that generalizes to more complex input-output structures. A main outcome of the work is that minimum-phase dynamics result when the shortest electrical path between the input and output in the power network is strong compared to longer alternative paths, while nonminimum dynamics result when the longer alternative paths are strong.

There is a very wide literature in the controls community on zeros and their implications on system dynamics and control [6]. While electric power system transients is typically not analyzed from an input-output perspective, Martins and his co-workers have voiced the importance of input-output analyses, and pursued the numerical computation of zeros in a sequence of studies (e.g. [7]). These efforts focus particularly on the computation of zeros for differential-algebraic-equation (singular system) models for the swing dynamics, and follow on analyses of zeros for singular systems in the control community [8], [9]. Relative to both the controls and power literature, the main contribution of

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this work is the development of structural and graph-theoretic insights into the zeros for the swing-dynamics model. That is, we seek to understand what features in the topology and physics of the power grid, and what placement of sensors and actuators, lead to minimum-phase or nonminimum-phase dynamics. In this sense, the work builds on and contributes to a recent research focus on input-output dynamics of dynamical networks, which has been concentrated on tying the zero locations of canonical linear network models (e.g., models for consensus, disease spread, etc) to the network's topology [10]–[14]. Compared to these previous studies, the research here addresses a more complex and heterogeneous dynamical models, and develops a set of structural results that are specifically relevant to the power-system analysis. Our research also builds on a wide and growing literature that approaches power-system transients analysis from a graph-theory perspective (e.g. [15]–[17], [22]). Finally, our work is connected with an important research thrust that interprets linear system structure, including zeros, from a graph-theoretic perspective (e.g. [18]).

The remainder of the paper is organized as follows. The zeros-analysis problem is formulated in Section 2. A graph-theoretic and structural analysis of the zeros is undertaken in Section 3. In Section 4, examples are used to illustrate the structural results, and to explore their implications on analysis and control of transients. *Due to space constraints, proofs and some details are excluded, see [19].*

II. Problem Formulation

Model-based analyses of transients in the bulk power grid are used for evaluation and design of controls, study of remedial action schemes, system protection, and other reasons. Models at several resolutions are used for analyzing these fast swing dynamics, depending on the needed accuracy, available time and computational resources, and other factors [5]. Detailed models derived from the physics of the transmission network turn out to be nonlinear differential-algebraic equations: simulation of these models is possible but tends to be time-consuming, and formal analysis is difficult. Linearization is routinely used to enable formal analysis and reduce simulation time. Further simplification can be achieved by used reduced state-space representations of generator dynamics, or via aggregation and other network model reduction techniques [20], [21]. The simplest evocative models that represent the networks topological structure only track two state variables (the electrical angle and frequency relative to a reference) at the buses with inertial generators, using a linear differential model. Because the main focus of this work is to gain simple graph-theoretic insights into the networks input-output dynamics, this classical model is considered here. The model is augmented to explicitly represent a single input and a single output, which may be remote from each other. Specifically, noting that both feedback controls and external disturbances often act as injecting or extracting power from a bus, the input is abstractly modeled as a power injection/extraction at a single bus. Meanwhile, noting that electrical frequencies or angles relative to a reference are often the measured responses of interest, the output is chosen as either the angle or frequency at a (possibly different) bus.

Formally, the following model is considered:

$$\begin{bmatrix} \dot{\delta} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -H^{-1}L(\Gamma) & -H^{-1}D \end{bmatrix} \begin{bmatrix} \delta \\ \omega \end{bmatrix} + \begin{bmatrix} 0 \\ \mathbf{e}_i \end{bmatrix} u \quad (1)$$

$$y = \begin{bmatrix} 0 & \mathbf{e}_j^T \end{bmatrix} \begin{bmatrix} \delta \\ \omega \end{bmatrix}$$

where $\delta(t) = \begin{bmatrix} \delta_1 \dots \delta_n \end{bmatrix}^T$ represents the differential electrical angles at the n buses at time t (relative to a nominal trajectory), $\omega(t) = \begin{bmatrix} \omega_1 \dots \omega_n \end{bmatrix}^T$ represents the differential electrical frequencies at the buses, the notation $\mathbf{e}_q$ represents a 0-1 indicator vector with qth entry equal to 1, the scalar input $u(t)$ is a power-injection signal at bus i, and the scalar output $y(t)$ is the frequency at bus j. The model is defined by the following parameters: the positive diagonal matrix H represents the inertias of the generators at the buses, the positive diagonal matrix D captures the dampings of the generators, and the matrix $L(\Gamma)$ is a symmetric positive-definite or positive semi-definite matrix that entirely specifies the interactions among the buses. Importantly, the zero pattern and nonzero entries in the matrix $L(\Gamma)$ are commensurate with the topology of the power transmission network (equivalently, electrical connectivity among the buses), as specified by the graph Γ. Specifically, Γ is defined to be an undirected weighted graph whose vertices represent the buses, and whose edge weights are the susceptances of the lines connecting the buses. Each off-diagonal entry of the matrix $L(\Gamma)$ equals the negative of the edge weight between the corresponding vertices if there is an edge, and equals zero otherwise. The diagonal entries of $L(\Gamma)$ are positive, and at least as large as the absolute sum of the off-diagonal entries on the corresponding row or column. We assume throughout the article that Γ is connected.

For convenience, we use the notation A for the state matrix of the system, i.e. $A = \begin{bmatrix} 0 & I \\ -H^{-1}L(\Gamma) & -H^{-1}D \end{bmatrix}$. We also find it convenient to define the state vector of the swing-dynamics model as $\mathbf{x} = \begin{bmatrix} \delta \\ \omega \end{bmatrix}$. It can easily be checked that the matrix A is stable, in these sense that all eigenvalues are in the closed left half plane with no defective eigenvalues on the $j\omega$-axis. In fact, it can be checked that all eigenvalues of A are in the open-left half plane (OLHP), except that there will be one eigenvalue at the origin in the special case that $L(\Gamma)$ is a true Laplacian matrix (all row sums are zero). The graph Γ is referred to as the **network graph**. Also, the nodes in the network where the input is applied and the output is measured (i and j, respectively) are referred to as the input and output nodes, and the corresponding vertices in the graph are referred to the the input and output vertices. The simplified model for the swing dynamics considered here is widely in power-engineering community [15], and constitutes a linearization of nonlinear Kuramoto oscillator-type model for the swing dynamics [22].

The main purpose of this work is to characterize the zeros of the input-output swing-dynamics model (1), in terms of its structural parameters (the graph Γ, the inertias in H, the dampings in D) and the input and output locations (i and j, respectively). Specifically, we develop conditions that guarantee minimum-phase dynamics, or alternatively permit the possibility for nonminimum-phase behaviors. These structural characterizations are used to explore how physical characteristics of the power system modulate zero locations, and also to support numerical computation of zeros. Additionally, via an example, the structural characterizations are illustrated, and their use in analysis/control of power-system transients is briefly explored.

Notes: 1) The formulation represents a considerable simplification of the models used in practice for simulations, in that detailed generator models are not used, and reduction of the algebraic equations (corresponding to load-only buses) in advance is assumed. 2) An angle output could be considered instead of a frequency output, but the only difference is that the transfer function to frequency has one additional zero at the origin. See [19] for details.

III. Structural and Graph-Theoretic Characterizations

A graph-theoretic and structural characterization is undertaken of the zeros of the swing-dynamics model (1). This structural characterization is developed using an algebraic expression for the state matrix of the *system's zero dynamics*. This approach contrasts with the standard numerical approaches used to compute zeros, which typically are based on solving a generalized eigenvalue problem (e.g. [7]). While the generalized-eigenvalue formulation is convenient for algorithmically finding zeros, it does not allow easy insight into the connection between a network's topological structure and its zeros. Instead, we approach the analysis of zeros using the *special coordinate basis* (SCB) transformation, which expresses a linear system as an integrator chain (or infinite-zero structure) along with a feedback block which captures the finite zero dynamics [23]. Importantly, the SCB allows an explicit algebraic characterization for the state matrix of the finite zero dynamics, whose eigenvalues (not generalized eigenvalues) are the zeros. Specifically, as shown below, the SCB allows expression of the *zero state matrix* as a submatrix of A plus a structured perturbation, which also can be related to the swing model's parameters. This explicit expression for the zero state matrix allows the development of structural results on minimum-phase and nonminimum-phase dynamics. It is worth noting that the approach taken here is tied to classical invariants notions (e.g. [24]) and the associated geometric theory for linear systems, but the algebraic approach is more convenient for obtaining structural results.

As a preliminary step, the relative degree of the transfer function is determined. Specifically, the following theorem shows that the relative degree is entirely governed by the distance d between the input and output, which is defined as the minimum number of directed arcs from the input to the output locations in the network graph Γ:

Theorem 1: The relative degree of the input-output swing-dynamics model (1), and hence the number of infinite zeros, is $n_d = 2d + 1$. The number of finite zeros is $n_a = 2n - 2d - 1$.

The number of infinite zeros, which equals the relative degree, indicates the number of diverging branches on the positive root locus of the transfer function. From the classical control theory, the infinite-zero structure of a system guides controller architecture selection and control design. Theorem 1 shows that this number is entirely decided by the distance between the input and output in the graph, for the swing-dynamics model.

On the other hand, the locations of a system's finite zeros in the complex plane dictate dynamical-response characteristics (e.g., undershoot), and place essential limits on control [6]. This motivates structural and graph-theoretic analysis of the finite zero locations for the swing model, in terms of its parameters and the input and output locations. As a stepping stone toward these structural analyses, first an algebraic expression for the zero state matrix is obtained. The eigenvalues of this matrix, which we denote A_{aa}, exactly specify the $2n - (2d + 1)$ finite zeros of the model. The algebraic expression for A_{aa} follows from the SCB transformation of (1). As Theorem 1 makes clear, the infinite zeros are essentially tied to the shortest path between the input and output vertices in Γ. We find it convenient to define some notation related to this path. In particular, we choose a path of minimum length (least number of edges) between the input and output, and refer to it as the *special input-output path*. In addition, the nodes in the network corresponding to the vertices on the special input-output path are referred to as the nodes associated with the special input-output path. Likewise, the state variables (angle, frequency) at these nodes or buses are referred to as the state variables associated with the special input-output path, and the rows and columns of the state matrix corresponding to these state variables are also referred to as being associated with the special input-output path. Corresponding terminology is used to refer to the vertices, nodes, state variables, and matrix entries that are not on the special input-output path.

From here on, we assume (without loss of generality) a particular ordering of the original state vector and the corresponding graph vertices. Specifically, the input location in Γ is labeled as vertex n, and hence the corresponding state variables are δ_n and ω_n. Also, the $d + 1$ vertices along the special input-output path are labeled as follows: the vertex at a distance k from the output along the special input-output path is labeled as vertex $i = n-k$ ($k = 1, 2, ..., d$). The states corresponding to each vertex are δ_{n-k} and ω_{n-k}. Hence, the input location is at vertex $i = n - d$. The remaining vertices, which are not on the special input-output path, are labeled $i = 1, ..., n-d-1$. For this labeling of the vertices, the state space form of the swing-dynamics model becomes:

$$\dot{\mathbf{x}} = A\mathbf{x} + (\mathbf{e}_i \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix})u \tag{2}$$

$$y = (\mathbf{e}_j \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix})^T \mathbf{x}, \tag{3}$$

where

$$A = \tilde{L} \otimes \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + \tilde{D} \otimes \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} + I \otimes \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \tag{4}$$

and where $\tilde{L} = -H^{-1}L$ and $\tilde{D} = -H^{-1}D$. The matrix A can also be naturally partitioned as $A = \begin{bmatrix} A_{n_a} & A_{n_{ad}} \\ A_{n_{da}} & A_{n_d} \end{bmatrix}$, where A_{n_a} is a matrix of dimension $(2n - (2d + 1)) \times (2n - (2d + 1))$. We note that rows and columns of A_{n_a} are associated with the vertices (and corresponding state variables) that are not on the special input output, and in addition the angle variable associated with the input vertex.

The algebraic expression for the state matrix A_{aa} of the zero dynamics is presented in the following theorem:

Theorem 2: The finite zeros of the system (1) are the eigenvalues of matrix $A_{aa} = A_{n_a} - A_{n_{ad}} Z_{n_d}^{-1} Z_{n_{ad}}$, where A_{n_a} and $A_{n_{ad}}$ are submatrices of A as defined above, where

$$Z_{n_{ad}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \{A\}_{n_a+1,1} & \{A\}_{n_a+1,2} & \cdots & \{A\}_{n_a+1,n_a} \\ \{A^2\}_{n_a+1,1} & \{A^2\}_{n_a+1,2} & \cdots & \{A^2\}_{n_a+1,n_a} \\ \vdots & \vdots & \cdots & \vdots \\ \{A^{n_d-1}\}_{n_a+1,1} & \{A^{n_d-1}\}_{n_a+1,2} & \cdots & \{A^{n_d-1}\}_{n_a+1,n_a} \end{bmatrix} \tag{5}$$

and where Z_{n_d} is the following lower triangular matrix:

$$Z_{n_d} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \{A\}_{n_a+1,n_a+1} & \{A\}_{n_a+1,n_a+2} & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ \{A^{n_d-1}\}_{n_a+1,n_a+1} & \{A^{n_d-1}\}_{n_a+1,n_a+2} & \cdots & \{A^{n_d-1}\}_{n_a+1,n} \end{bmatrix} \tag{6}$$

Remark: The matrix $Z_{n_d}^{-1}$ is a lower triangular matrix. An iterative formula for its entries can be developed in a similar fashion to the analysis in [12]. This computation is omitted to save space.

The algebraic expression for the zero state matrix A_{aa} in Theorem 2 enables the development of structural and graph-theoretic insights. To develop these results, it is useful to recognize that A_{aa} is in the form $A_{aa} = A_{n_a} + A_q$, where A_{n_a} is a principal submatrix of the state matrix A and A_q is a perturbation matrix which has a special sparse structure. The following theorem gives structural insight:

Theorem 3: The matrix A_{aa}, whose eigenvalues are the zeros of the swing-dynamics model, can be expressed in the form $A_{aa} = A_{n_a} + A_q$. Let us define $[A_{aa}]_{i,j}$ (respectively $[A_{n_a}]_{i,j}$) to refer to the 2×2 submatrix of A_{aa} (respectively A_{n_a}) whose rows are associated with vertex i, and whose columns are associated with vertex j. Also, let d_i be the distance from the input location to the vertex i in Γ, and let d_j be the distance from vertex j to the output location in Γ. We have that $[A_{aa}]_{i,j} = [A_{n_a}]_{i,j}$, unless $d_i + d_j \leq d+1$ and i is adjacent to a vertex in the special input-output path other than the output. For $d_i + d_j \leq d+1$, $[A_{aa}]_{i,j}$ may differ from $[A_{n_a}]_{i,j}$. However, the row of $[A_{aa}]_{i,j}$ corresponding to δ_i is equal to this row of $[A_{n_a}]_{i,j}$ (these entries in the perturbation are alway 0). Also, the entry of $[A_{aa}]_{i,j}$ corresponding to ω_i and ω_j differs from this entry for $A_{n_a i,j}$ only if $d_i + d_j \leq d$.

Theorem 3 expresses that the matrix A_{aa} can be viewed as a perturbation of the principal submatrix A_{n_a} of A associated with the vertices that are not on the special input-output path. Since this is the case, we also identify the rows and columns of A_{aa} by their associated vertices in the graph Γ, specifically the vertices off the special input-output path whose state variables correspond to these rows and columns. The main outcome of Theorem 3 is that the perturbation A_q is structured, in the sense only certain entries of A_{aa} differ from those of A_{n_a} based on the network graph Γ. Specifically, consider an entry in A_q whose row corresponds to vertex i and whose column corresponds to vertex j (where i and j are not on the special input-output path). The entry can be non-zero only if the distance of j from the output in Γ plus the distance of i from the input in Γ is at most $d+1$ (the length of the special input-output path plus 1). Additionally, the vertex i must be adjacent to the special input-output path. Thus, only the entries whose rows and columns correspond to vertices near the input-output path are perturbed.

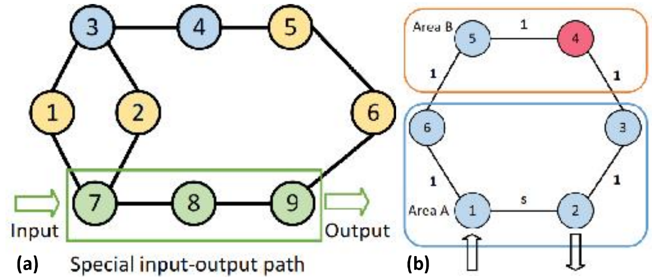

Fig. 1: (a) The result of Theorem 3 is diagrammed. In this example, the zeros state matrix A_{aa} will be perturbed from the principal submatrix A_{n_a} only on rows associated with vertices $i = 1, 2$ (next to the special input- output path), and columns associated with vertices $j = 5, 6$ (distance from i to the input and from the output to j is less than or equal to $d+1$). (b) A 6-bus example is developed to gain further insight into the dependence of zeros on structural parameters, and to understand their implications on transient analysis and control for the power transmission network. One particular implication that is discussed is the possible loss of nonminimum-phase dynamics in reduced-order models. Bus 4 gen. has inertia h, other buses have inertia 1. Generators at all buses have common damping d.

Remark: Finding the zeros by computing A_{aa} and then finding its eigenvalues is also attractive from a computational standpoint, see [19] for some further discussion.

Expressing the matrix A_{aa} as a perturbation of A_{n_a} enables graph-theoretic analysis of the zeros, as developed in the following theorems. These analyses requires first noting that the eigenvalues of the matrix A_{n_a} are in the closed left half plane. Precisely, the matrix A_{n_a} has a single eigenvalue at $s = 0$ (associated with the angle dynamics of the input bus), and the remaining eigenvalues are strictly in the OLHP. Since the matrix A_{n_a} is stable, the eigenvalues of the matrix A_{aa} and hence the zeros of the swing models can be guaranteed to be in the left half plane if the perturbation A_q either does not change the eigenvalues of A_{n_a}, or is sufficiently small. The following three theorems use this idea to give structural conditions under which the swing-dynamics model is minimum phase.

The first of these structural results addresses the case that the input and output are at the same vertex in Γ (the same bus in the network):

Theorem 4: The input-output swing-dynamics model (1) has all zeros in the OLHP, except one zero at $s = 0$, if the input and output locations are at the same vertex.

The second of these structural results addresses the case that there is only a single path between the input and output:

Theorem 5: The input-output swing-dynamics model (1) has all zeros in the OLHP, except one zero at $s = 0$, if there is a single path between the input and output vertices in the network graph Γ.

The next result shows that minimum-phase dynamics are maintained even when there are multiple paths between the input and output, provided that the special input-output path is sufficiently strong (has high susceptances) compared to the other paths.

Theorem 6: Consider the zeros of the input-output swing-dynamics model (1) for an arbitrary graph Γ. Now consider scaling up all the edge weights on a special input-output path by a factor κ. For sufficiently κ, the zeros are in the OLHP except one zero at $s = 0$.

Two other structural characteristics of the input-output model are worth discussing. First, we note that the model is minimum phase, provided that the dampings throughout the network are sufficiently scaled up; we leave a full development of this result to later work. Conversely, the swing-dynamics model is nonminimum-phase if the special input-output path is sufficiently weak compared to other longer paths between the input and output. A proof of the nonminimum-phase result is rather involved, see [13] for a similar proof for a simpler synchronization model.

The graph-theoretic analysis of zeros developed here can potentially support power-system analysis and controller design in several ways. First, the results give insight into estimation and control of the dynamics. It is well known that the finite and infinite zero structure of a system, and particularly the presence of nonminimum-phase zeros, place essential limits on estimator and control performance and guide control design. For instance, for control channels, the locations of zeros determine whether or not high-gain control is viable and place restrictions on reference tracking and disturbance rejection. Likewise, the zeros of a disturbance-input-to-sensor transfer function influence whether or not dynamic state filtering is possible in the presence of unknown disturbance inputs. In current power-grid operations, control designs are often simplistic in nature, perhaps using simple proportional-integral-derivative controllers with manually-tuned parameters. Our work shows that the network's topology modulates whether such simple control schemes are likely to work or fail. Specifically, if the input and output are collocated, or the shortest path between then is the dominant one, then the channel of interest is minimum-phase and simple control/filtering algorithms may be apt. On the other hand, if the network has alternate long paths between the input and output, caution is needed to ensure that the dynamics is indeed minimum phase, and more sophisticated designs are needed if it is not. This intuition further leads to shortest-path-type algorithms for screening for non-minimum-phase channels, and for placing sensors or actuators to avoid nonminimum-phase characteristics. Details are omitted in the interest of space.

IV. Example: Illustrations and Implications

A small-scale example is developed, both to illustrate the structural analyses of zeros developed in Section 3 and to further explore their relevance to power grid operations and analysis. Specifically, a network with six buses is considered (see Figure 1(b)), which are viewed as forming two areas (Buses 1, 2, 3, and 6 form Area A; Buses 4 and 5 form Area B). The buses in Area A are aligned in a straight line, however there is an alternate path for power flow via the two buses in Area B. In the example, we focus on the case that an input is applied at Bus 1 and the output is taken at Bus 2. The parameters of the model are shown in the figure. Three studies are undertaken for the example. First, the dependencies of the zeros on structural parameters of the network are determined, and compared with the formal results developed in Section 3. Second, the implications of the zeros analysis on model reduction are explored. Third, other uses of the structural and graph-theoretic analyses of zeros are envisioned.

Dependencies of Zeros on Structural Parameters: For the example, the relationships of the zero locations on three structural parameters – the susceptance s of the line between Buses 1 and 2, the inertia h of the generator at Bus 4, and the common damping d of the generators—are studied. For each structural parameter, the largest real part among the zeros is plotted as a function of the parameter in each case, to highlight the dominant zero (see Figure 2(a,b,c)). As expected, the system is nonminimum phase when the susceptance s between Buses 1 and 2 is sufficiently small, and becomes minimum phase for larger susceptances. The dependence conceptually matches the expectation that the system would be minimum phase if the shortest input-output path is dominant, and nonminimum phase if the shortest path is weak. More specifically, the real part of the dominant zero decreases monotonically with s until it reaches -0.05, and then remains at that value. Also, the real part of the dominant zero decreases with increasing damping d, which is expected the primary path between input and output becomes prominent compared to the longer secondary path. The dependence of the zeros on the inertia h of generator 4 is much more sophisticated. To gain further insight into this case, the zeros of the input-output swing model are traced in the complex plane as a function of the parameter h (Figure 2(d)). This plot is akin to a root locus, though not exactly in the sense that the characteristic polynomial of interest does not show a linear dependence on the parameter. The locus plot shows that different zeros become dominant as h is changed, reflecting changes in how oscillations at different frequencies propagate through the alternate input-output path.

Zeros and Model Reduction: Reduced-order models are commonly used for simulation and analysis of power-grid

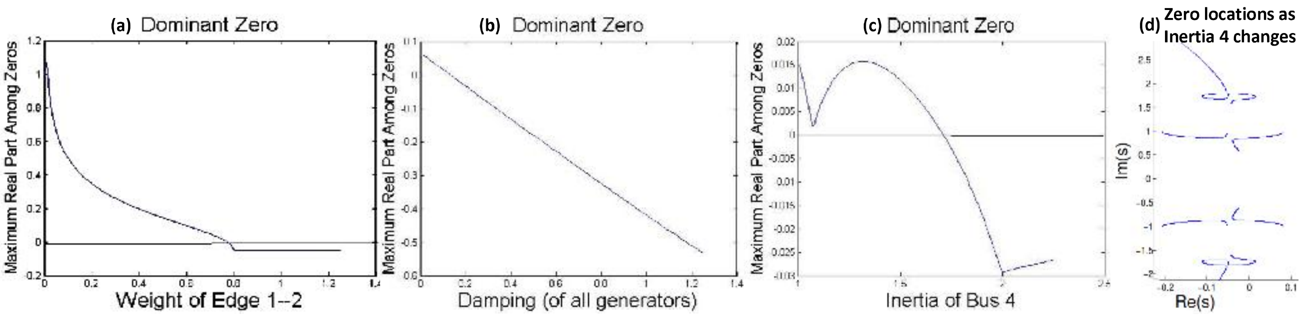

Fig. 2: (a,b,c):The dependences of the dominant zero location (the largest real part among the zeros) on three structural parameters are shown. (d) The movement of the zeros in the complex plane as a function of the inertia of generator 4 are shown. The zeros exhibit a complex behavior, with two pairs moving into the right half plane for different values of the inertia.

transients, to reduce computational burden, simplify representation of network components governed by other authorities, and for other reasons. Several established techniques are available for model reduction [20], [21], including balanced-truncation- and coherency- based methods. These techniques are adept at preserving wide-area oscillatory modes. Some techniques, such as the slow-coherency-based methods, also preserve the network's topology in an area of interest while aggregating the topology in other areas. However, very little work has been done to understand how model reduction influences input-output dynamics, and model reduction techniques are not known to preserve input-output characteristics like minimum-phase dynamics. A few recent results have studied controllability-preserving model reduction [25], but none address input-output dynamics (specifically zeros) to the best of our knowledge.

The formal analyses in Section 3 indicate that model reduction may alter the finite-zero structure of the swing-dynamics model even when the the input and output are in the same area. Specifically, the aggregation of buses outside the area of interest may change characteristics of alternate input-output paths, and hence cause nonminimum-phase dynamics to become minimum phase or vice versa. In fact, the impact of model reduction on zeros is easily demonstrated in the 6-bus example considered. Specifically, we consider the 6-bus model with the following parameters: the susceptance between lines 1 and 2 is $s = 0.75$, the inertia of bus 4 is $h = 1$, and the common damping is $d = 1$. The input-output dynamics of interest is nonminimum phase in this case. In a study of transients in Area A, the two generators in Area B (buses 4 and 5) may be reduced to a single aggregated generator. This model reduction to has been undertaken for the six-bus model, using the slow-coherency approach. The reduced model closely preserves 10 of the 12 modes of dynamics (errors of $< 10\%$ in each modal frequency and damping), while removing one pair of localized modes. However, the input-output dynamics of the reduced model is minimum phase, and in fact the zero locations are drastically changed compared to the full model. Thus, the nonminimum-phase characteristics of the system are destroyed by the model reduction. Figure 3 shows the impulse responses and frequency responses before and after model reduction. The impulse response and Bode magnitude plot are closely preserved, but the Bode phase plot shows a drastic change which reflects the nonminimum-phase vs minimum-phase dynamics.

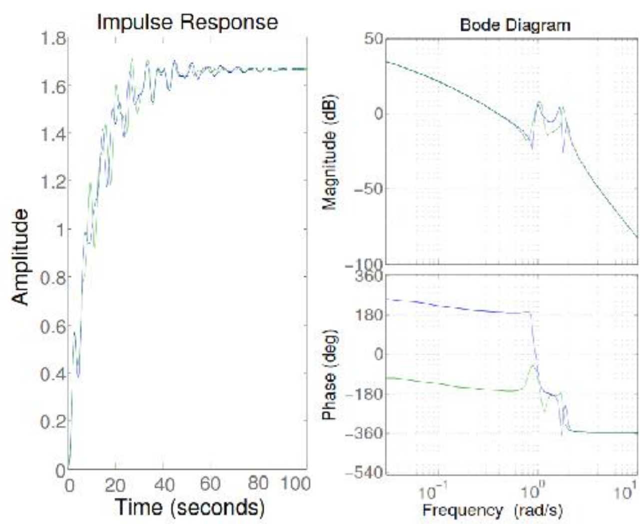

Fig. 3: The impulse response and frequency response for the input-output dynamics of the original model (shown in blue) and reduced model (green) are shown. The impulse response and Bode magnitude plot remain essentially unchanged, while the Bode phase plot changes drastically reflecting the change from a nonminimum-phase to a minimum-phase dynamics.

Conceptually, the change in zero structure results from the fact that the alternate input-output path has a changed structure compared to the original. Since the alternate path plays a crucial role in determining the zero locations, it is not surprising that the zero locations change significantly upon model reduction. This example indicates that caution is needed in undertaking model reduction, when input-output characteristics need to be preserved. Zero-preserving model-reduction techniques will be developed in future work.

Other Implications: The developed structural results support numerical analysis, control, and sensor and actuator selection. It is instructive to consider the example from this perspective. With regard to numerical computation of zeros, the state matrix of the zero dynamics A_{aa} can be found by perturbing only two entries of a submatrix of A. Thereafter, standard eigenvalue-analysis methods for sparse matrices can be brought to bear.

The formal results also help in screening for channels with nonminimum-phase dynamics, which is useful for sensor/actuator selection, controllers selection, etc. In the 6-bus example, nonminimum-phase dynamics are anticipated whenever the input and output are adjacent and the susceptance of the direct link is sufficiently weak. If the input and output have a distance of 2 between them, one would guess that nonminimum-phase dynamics could also result provided that the shorter path is sufficiently weak; in fact, this is the case. Thus, control design in these cases must be undertaken with the possibility for nonminimum-phase dynamics in mind, and sensor and actuator placement can be undertaken to avoid configurations that cause nonminimum-phase dynamics. It is worth noting that the structural results also give insight into how the nominal power flow may impact transients. To this point, our analysis has assumed a linearization of the swing dynamics around a zero-power-flow solution, hence the edge weights in the graph are exactly the line susceptances. As power flow on a line increases, it is easy to check that the corresponding virtual "susceptance" at nominal voltage level in the linearized model is reduced (specifically, scaled by the cosine of the nominal angle difference across the line). This indicates that increased congestion on the shortest path between the input and output may yield nonminimum-phase dynamics. As an illustration, it is interesting to study how the zeros change, when the nominal power injection at bus 1 and the nominal load at bus 2 are increased. The congestion in all lines increases in consequence, but the direct line from bus 1 to bus 2 is disproportionately impacted. In consequence, as the flow is increased, the effective susceptance of this line decreases more than others, and the system's dominant zero moves right in the complex plane.

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